UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 – Digital image analysis**
Date:	**Friday December 11, 2009**
Exam hours:	**14.30-17.30**
Number of pages:	**7 pages plus 1 page enclosure**
Enclosures:	**1 sheet containing plots**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the “spirit” of the exercise. In such a case, you should make it clear what assumptions you have made.

- Please note that all parts of the exercises have equal weight. You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

- Some of the questions are based on printed figures or plots included in the exam text. An extra copy of this sheet is included at the end of the exam text. Please draw your solution on this sheet, mark it with your candidate number and include it in your solution.

- Your answers should be **short**, typically 1-3 sentences or a sketch should be sufficient.

Good luck!!

Exercise 1.Using 1D projection histograms

a) How are 1D projection histograms computed? Explain either using formulas or by words if you do not remember exact formulaes.

1D horizontal projection of the region:

$$p_h(x) = \sum_y f(x,y)$$

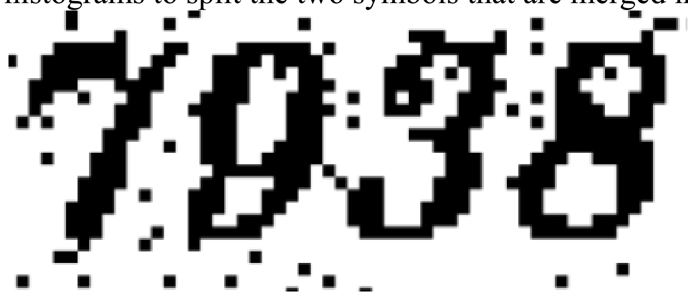
1D vertical projection of the region:

$$p_v(y) = \sum_x f(x,y)$$

b) You are given images like the music sheet below. How can you use 1D projection histograms to determine if the page is rotated? (Not to find the rotation angle, just IF it is rotated)? Discuss shortly how you could do this.

Consider a projection onto the vertical axis. If the page is not rotated, you will see distict peaks in the histogram at the locations of the horisontal note lines. If the page is rotated, the peaks will not be distinct, more of a gradual increase/decrease.

c) 1D projection histograms can also be useful for splitting connected objects. Consider the segmented symbols below. Describe briefly how you could use 1D projection histograms to split the two symbols that are merged in the image below.



There are several ways to do this. One alternative is to use projection onto the horisontal axis. The gaps between the numbers will be mimimum values of the histograms. Minimum values are then good candidates for splitting locations, maube combined with other criteria like location of the split.

Exercise 2. Shape descriptors

a) How can you compute the width and height of an object in a segmented binary image? We have presented several alternatives, list the alternatives shortly.
Examples are bounding box (object-oriented would be better), major axis/minor axis, thickness from skeleton, dimensions of a fitted ellipse etc.

Assume that we have a binary image f(x,y). A general discrete moment is defined as

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y)$$

b) What is the definition of central moments?
Central moments are defined as

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x,y)$$

c) What is the main advantage of using central moments?

Position invariance.

d) How could second order moments be useful for discriminating between the two symbols **0** and **1**?

For example using $\mu 20$ the spread along the x-axis.

Exercise 3: Hough transform

You are given two point is (x,y)-space: (1,1) and (0,2).

a) Give the equations for the (a,b) representation of these two points.

(1,1): b=1-a
(0,2): b=2

b) Plot them in (a,b)-space.

c) Explain the algorithm for finding lines in the polar representation ((ρ,θ)-space).

Exercise 4. Texture

a) Describe in short how to calculate a GLCM for an entire image. Explain the necessary concepts, using a sketch if you find it useful.

Stikkord som må med for full score her er glidende vindu, inputparametre til algoritmen (retning og avstand), samt skisse av selve algoritmen. Kvantisering spørres om under, så det gir ikke poeng. Diskusjon om kanteffekter kan trekke opp.

b) From an unknown texture sample you calculate a couple of GLCM features for three different step lengths and the four directions 0°, 45°, 90°, 135°. Looking at the output you notice that most features are nearly identical for all step lengths for the 0° direction, but wildly varying between step lengths for the other directions. What does this tell you about the texture?

Teksturen er horisontal

c) When analyzing texture it is often useful to quantize the graylevels in the image. Explain why. Comment on one problem with this approach.

Kvantisering reduserer størrelsen på GLCM bl.a. Ved for grov kvantisering vasker en ut lavkontrast tekstur.

d) For the following texture a coocurrence matrix is calculated (ignore problems with the edge pixels, i.e., do not evaluate coocurrences across the edges)

j →

i ↓

2	**1**	**1**	**3**	**2**	**1**
1	**2**	**3**	**3**	**2**	**2**
2	**2**	**1**	**3**	**3**	**1**
1	**2**	**2**	**1**	**1**	**2**
3	**3**	**3**	**2**	**1**	**3**
3	**3**	**3**	**1**	**1**	**2**

The coocurrence matrix given ***h=(i,j)*** is j

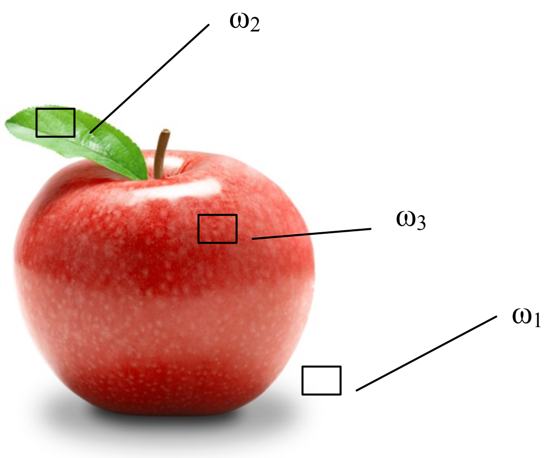
i

	1	2	3
1	2	2	4
2	3	5	1
3	2	2	4

What is ***h***?

1. (0,1)
2. (1,0)
3. (1,1)
4. (1,-1)
5. None of the above

Exercise 5. Classification using Bayes rule



pixel	class	channel 1	channel 2	channel 3
1	ω1	248	244	243
2	ω1	247	243	240
3	ω1	248	244	243
4	ω1	249	245	249
MEAN	ω1	248	244	243.75
VAR	ω1	0.67	0.67	14.25
5	ω2	174	164	49
6	ω2	177	169	36
7	ω2	184	170	65
8	ω2	195	189	91
MEAN	ω2	182.5	173	60.25
VAR	ω2	87	120.67	560.92
9	ω3	170	46	48
10	ω3	156	68	46
11	ω3	142	22	34
12	ω3	118	19	39
MEAN	ω3	146.5	38.75	41.75
VAR	ω3	491.67	526.25	41.58

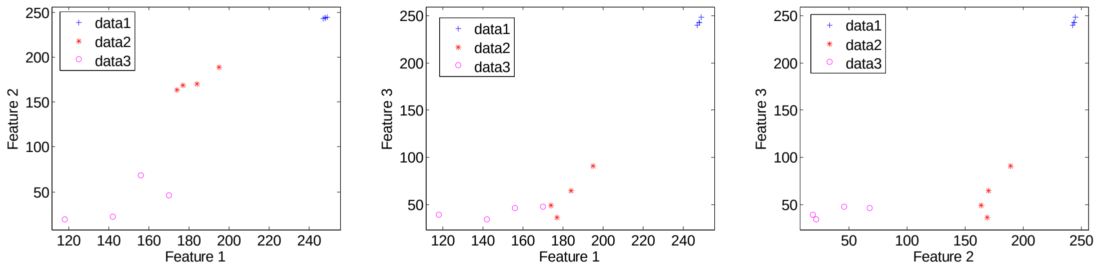
Figur 1: Training image

For the multispectral image (3 channels) above, we have the training data picked from the indicated regions as given in the table

Your task is to classify the pixels in the image below into class ω1 (background), ω2 (leaf), and ω3 (apple) based on the information from the image in Figur 1. You are only allowed to use two channels.

a) Evaluate by visual inspection (using 2D feature spaces) which two channels are the most suited.

a)
1 &2 eller 2&3 er begge brukende, men 2&3 er best for naturlig valg av klassevise avstandsmål.



b) Based on this you are designing a classifier using Bayes rule. The data is modeled with gaussian distributions having the same covariance matrix $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Write the generic form of the discriminant function.

b)

$$d_j(x) = x^T m_j - \frac{1}{2} m_j^T m_j$$

tilordnes til klasse j if $d_j(x) > d_i(x), \forall i$
evt er direkte avstand til mean også akseptabelt

$$D_j(x) = \left\| x - m_j \right\| = \sqrt{(x - m_j)^T (x - m_j)}$$

c) Classify the pixels below using the functions as in (b). Note that just giving the class for the pixels will not give any score even if correct. Whether you answer the classification problem graphically or numerically is equivalent.

pixel	channel 1	channel 2	channel 3
1	200	45	50
2	169	136	55
3	231	218	201
4	203	176	131

for alle tre kanaler

$$d_1 = 248.00x_1 + 244.00x_2 + 243.75x_3 - 90227$$

$$d_2 = 182.50x_1 + 173.00x_2 + 60.25x_3 - 33433$$
$$d_3 = 146.50x_1 + 38.75x_2 + 41.75x_3 - 12353$$

ved bruk av 1 & 2

$$d_1 = 248.00x_1 + 244.00x_2 - 60520$$
$$d_2 = 182.50x_1 + 173.00x_2 - 31618$$
$$d_3 = 146.50x_1 + 38.75x_2 - 11481.9$$

1: class 3 as 60 12667.375 19561.8438
2: class 2 as 14576 22752.875 18546.5938
3: class 1 as 49960 48253.875 30807.0938
4: class 2 as 32768 35877.875 25077.5938

ved bruk av 2 & 3

$$d_1 = 244.00x_2 + 243.75x_3 - 59475.03$$
$$d_2 = 173.00x_2 + 60.25x_3 - 16780$$
$$d_3 = 38.75x_2 + 41.75x_3 - 1622.3$$

1: class 3 as -36307.5312 -5982.03125 2208.9375
2: class 2 as -12884.7812 10062.2188 5943.9375
3: class 1 as 42710.7188 33044.7188 15216.9375
4: class 2 as 15400.2188 21561.2188 10666.9375

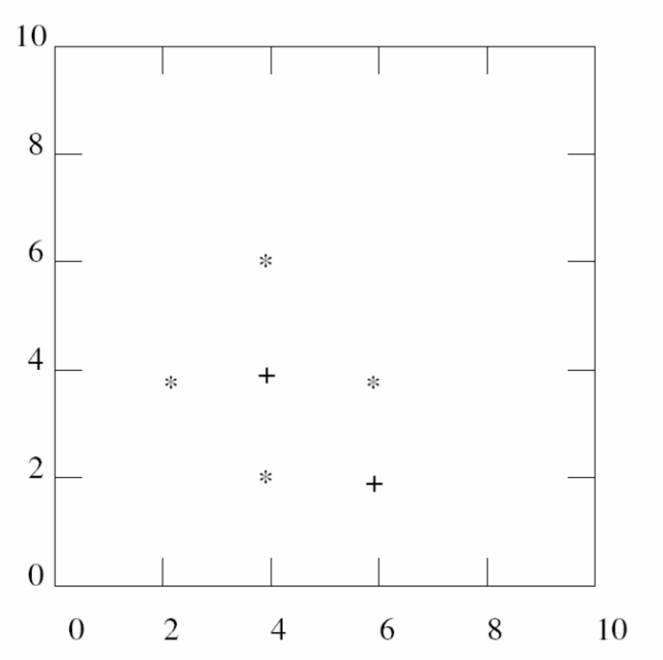
Remember that, when assuming a 2D Gaussian distribution, the point probability of a point $x=[x_1, x_2]^T$ can be written on the form

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \frac{1}{(2\pi)^{1/2}\,|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}\right)^T \Sigma^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}\right)\right)$$

where the probability distribution function takes as parameters the vector $\mu=[\mu_1, \mu_2]^T$ and a matrix $\sum$.

Exercise 6. K-nearest neighbor classification

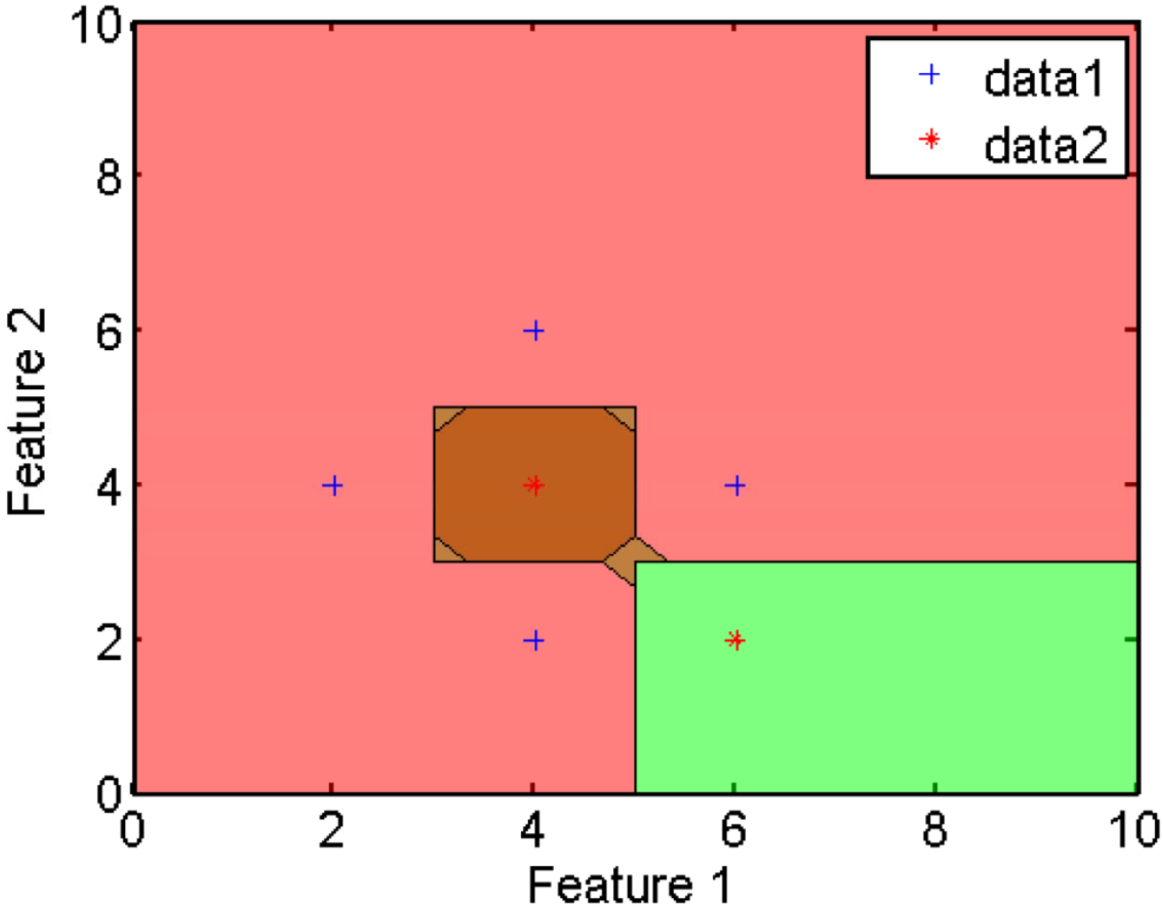
In the following questions you will consider a *k*-nearest neighbor classifier using Euclidean distance metric on a binary classification task. We assign the class of the test point to be the class of the majority of the *k* nearest neighbors. Note that a point can be its own neighbor.



Figur 2: k-nn classification task, a plus indicates a positive example and a star a negative example.

a) In the Figur 2, sketch the *1*-nearest neighbor decision boundary for this dataset.
b) How would the point (8,1) be classified using 1-nn?

a)



b) Point (8,1) will be classified to class *

Candidate number: