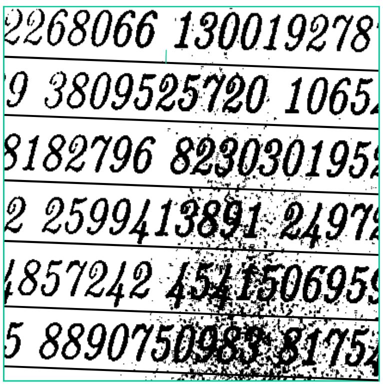
INF 4300
09.11.2011 – Anne Solberg

Morphology

- Gonzalez and Woods, Chapter 9
- Except sections 9.5.7, 9.5.8, 9.5.9 and 9.6.4
- Repetition of binary dilatation, erosion, opening, closing
- Binary region processing: connected components, convex hull, thinning, skeleton.
- Grey-level morphology: erosion, dilation, opening, closing, smoothing, gradient, top-hat, bottom-hat, granulometry.

Example

- Text segmentation and recognition.
- Binary morphological operations useful after segmentation to get better segmentation of the objects.



Some symbols have been fragmented.

Some symbols are connected with
background noise.

Symbols can be connected with neighboring symbols.

Find and remove lines or frames.

Simple set theory

- Let A be a set in $\mathbb{Z}^2$ (integers in 2D) . If the point $a=(a_1,a_2)$ is an element in A we denote: $a \in A$
- If a is not an element in A we denote: $a \notin A$
- An empty set is denoted $\varnothing$.
- If all elements in A also are part of B, A is called a subset of B and denoted: $A \subseteq B$
- The union of two sets A and B consists of all elements in either A or B, and is denoted: $A \cup B$
- The intersection (=snitt) of A and B consists of all elements that are part of both A and B and is denoted: $A \cap B$
- The complement of a set A is the set of elements not in A: $A^C = \{w \mid w \notin A\}$

$A^C = \{w \mid w \notin A\}$

- The difference of two sets A and B is:

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^C$$

Set theory on binary images

- The complement of a binary image

$$g(x,y) = \begin{cases} 1 \text{ if } f(x,y) = 0 \\ 0 \text{ if } f(x,y) = 1 \end{cases}$$

- The intersection of two images f and g is

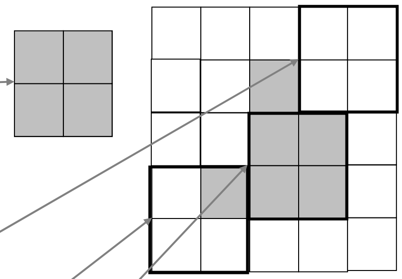
$$h = f \cap g = h(x,y) = \begin{cases} 1 \text{ if } f(x,y) = 1 \text{ and } g(x,y) = 1 \\ 0 \text{ otherwise} \end{cases}$$

- The union of two images f and g is

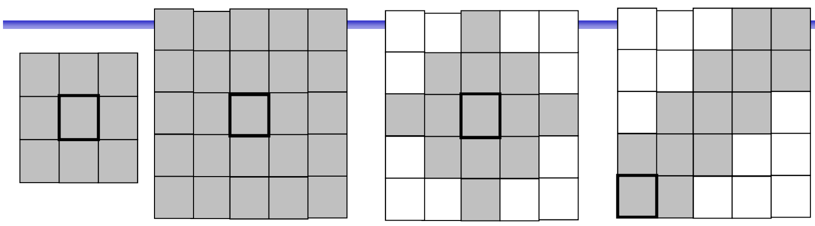
$$h = f \cup g = h(x,y) = \begin{cases} 1 \text{ if } f(x,y) = 1 \text{ or } g(x,y) = 1 \\ 0 \text{ otherwise} \end{cases}$$

Repetition - Fit vs. Hit

- A structure element for a binary image is a small matrix of pixels
- Let the structure element overlay the binary image containing an objects at different pixel positions. The following cases arise:
  - Positions where the element does not overlap with the object.
  - Positions where the element parly overlaps the object – where the element hits the object
  - Positions where the element fits inside the object – where the element fits the object



Repetition – structure elements



- Can have different sizes and shapes
- A structuring element has an origo
  - Origo is a pixel
  - Origo *can* be outside the element.
  - Origo is often marked on the structuring element using
  - Origo can be flat or non-flat (have different values)
    - We will only work with a flat structure element

Repetition -Erosion of a binary image Simplified notation

- To compute the erosion of pixel (x,y) in image f with the structure element S: place the structure elements such that its origo is at (x,y). Compute

$$g(x,y) = \begin{cases} 1 \text{ if S fits f} \\ 0 \text{ otherwise} \end{cases}$$

- Erosion of the image f with structure element S is denoted

ε (f|S) = f ɵ S

- Erosion of a set A with the stucture element B is defined as the position of all pixels x in A such that B is included in A when origo of B is at x.

$$A \theta B = \{ x | B_x \subseteq A \}$$

0 1 0 0 0 0 0 1 1 0 0
1 1 1 0 0 0 1 1 1 1 0
0 1 1 1 0 1 1 1 1 0 0
0 0 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 1 0 0
0 0 1 1 1 1 0 1 1 1 0
0 1 1 1 1 0 0 0 1 1 1
0 0 1 1 0 0 0 0 0 1 0

erodert med

1 1 1
1 $\boxed{1}$ 1
1 1 1

gir

0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0

0 1 0
1 $\boxed{1}$ 1
0 1 0

gir

0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 1 1 0 0
0 0 1 0 0 0 1 1 0 0 0
0 0 0 1 0 1 1 0 0 0 0
0 0 0 0 1 1 0 1 0 0 0
0 0 0 1 1 0 0 0 1 0 0
0 0 1 1 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0

Edge detection using erosion

- Erosion removes pixels on the border of an object.
- We can find the border by subtracting an eroded image from the original image: g = f - (f ɵ s)
- The structure element decides if the edge pixels will be 4-neighbors or 8-neighbors

0 0 1 1 1 1 0 1 1 1 0
0 1 1 1 1 1 1 1 1 1 0
0 1 1 1 1 1 1 1 1 1 0
1 1 1 1 0 1 1 1 1 1 1
0 1 1 1 1 1 1 1 1 1 1
0 1 1 1 1 1 1 1 1 1 0
0 1 1 1 1 1 1 1 1 1 0
0 0 0 0 0 1 1 1 0 0 0

Eroded by

0 1 0
1 $\boxed{1}$ 1
0 1 0

gives

0 0 0 0 0 0 0 0 0 0 0
0 0 1 1 1 1 0 1 1 0 0
0 0 1 1 0 1 1 1 1 0 0
0 1 1 0 0 0 1 1 1 1 0
0 0 1 1 0 1 1 1 1 1 0
0 0 1 1 1 1 1 1 1 0 0
0 0 0 0 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0

=>

difference

0 0 1 1 1 1 0 1 1 1 0
0 1 0 0 0 0 1 0 0 1 0
0 1 0 1 0 0 0 0 0 1 0
1 0 1 0 1 0 0 0 0 0 1
0 1 0 1 0 0 0 0 0 0 1
0 1 0 0 0 0 0 0 0 1 0
0 1 1 1 1 0 0 0 1 1 0
0 0 0 0 0 1 1 1 0 0 0

1 1 1
1 $\boxed{1}$ 1
1 1 1

0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 0 0 0 1 0 0
0 0 1 0 0 0 1 1 1 0 0
0 0 1 0 0 0 1 1 1 0 0
0 0 1 0 0 0 1 1 1 0 0
0 0 1 1 1 1 1 1 1 0 0
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0 0 0 0 0 0 0 0 0 0 0

0 0 1 1 1 1 0 1 1 1 0
0 1 1 0 0 1 1 1 0 1 0
0 1 0 1 1 1 0 0 0 1 0
1 1 0 1 0 1 0 0 0 1 1
0 1 0 1 1 1 0 0 0 1 1
0 1 0 1 1 1 0 0 0 1 0
0 1 1 1 1 1 0 1 1 1 0
0 0 0 0 0 1 1 1 0 0 0

Edge detection

$$f - (f \ominus S)$$

Example use: find border pixels in a region

Dilation of a binary image

- Place S such that origo lies in pixel (x,y) and use the rule

$$g(x,y) = \begin{cases} 1 \text{ if } S \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

- The image f dilated by the structure element S is denoted:

$$f \oplus S$$

- Dilation of a set A with a structure element B is defined as the position of all pixels x such that B overlaps with at least one pixel in A when origo is placed at x.

$$A \oplus B = \{x \mid B_x \cap A \neq \emptyset\}$$

$$\begin{matrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&1&1&0&0 \\ 0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&1&0&1&1&0&0&0&0 \\ 0&0&0&0&1&1&0&1&0&0&0 \\ 0&0&0&1&1&0&0&0&1&0&0 \\ 0&0&1&1&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{matrix}$$

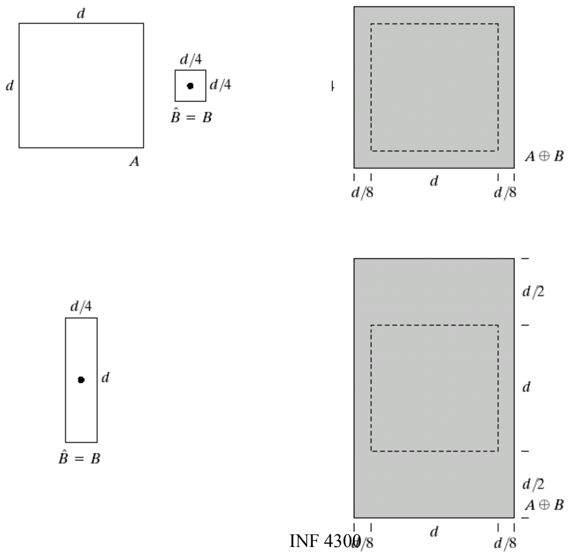
Dilated by

$$\begin{matrix} 1&1&1 \\ 1&\boxed{1}&1 \\ 1&1&1 \end{matrix} \qquad \begin{matrix} 0&1&0 \\ 1&\boxed{1}&1 \\ 0&1&0 \end{matrix}$$

gives

$$\begin{matrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&1&1&1&1&1&0 \\ 0&1&1&1&1&1&1&1&1&1&0 \\ 0&1&1&1&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&1&1&1&0 \\ 0&1&1&1&1&1&1&1&1&1&0 \\ 0&1&1&1&1&1&0&1&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{matrix} \qquad \begin{matrix} 0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&0&0&0&1&1&1&1&0 \\ 0&1&1&1&0&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&1&0&0&0 \\ 0&0&0&1&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&0&1&1&1&0 \\ 0&0&1&1&1&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0 \end{matrix}$$

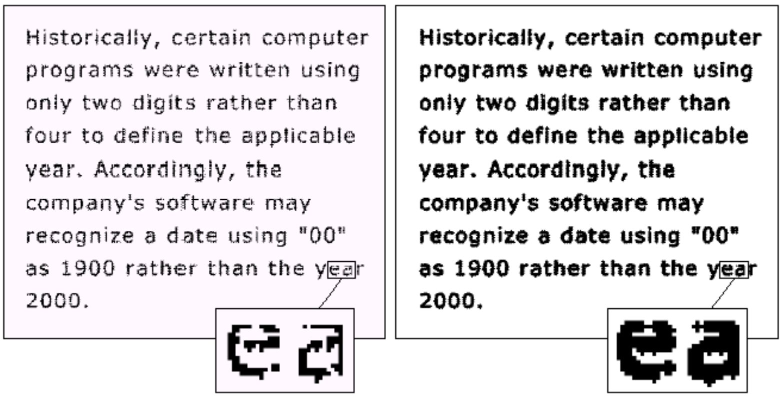
Dilation



Effect of dilation

- Expand the object
- Both inside and outside borders of the object
- Dilation fills holes in the object
- Dilation smooths out the object contour
- Depends on the structure element
- Bigger structure element gives greater effect

Example of use of dilation – fill gaps



0	1	0
1	1	1
0	1	0

Opening

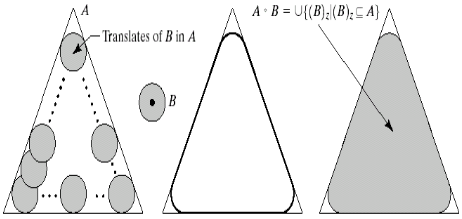
- Erosion of an image removes all structures that the structuring element can not fit inside, and shrinks all other structures.
- If we dilate the result of the erosion with the same structuring element, the structures that survived the erosion (were shrunken, not deleted) will be restored.
- This is calles morphological opening:

$$f \circ S = \left(f \, \theta \, \mathrm{S} \right) \oplus S$$

- The name tells that the operation can create an opening between two structures that are connected only in a thin bridge, without shrinking the structures (as erosion would do).

Visualizing opening

- Visualize opening by imagening that the structure element traverses the edge of the object.
  - First on the inside of the object. The object shrinks.
  - Then the structure element is traversing the outside of the resulting object from the previous passage. The object grows, but small branches removed in the last step will not be restored.



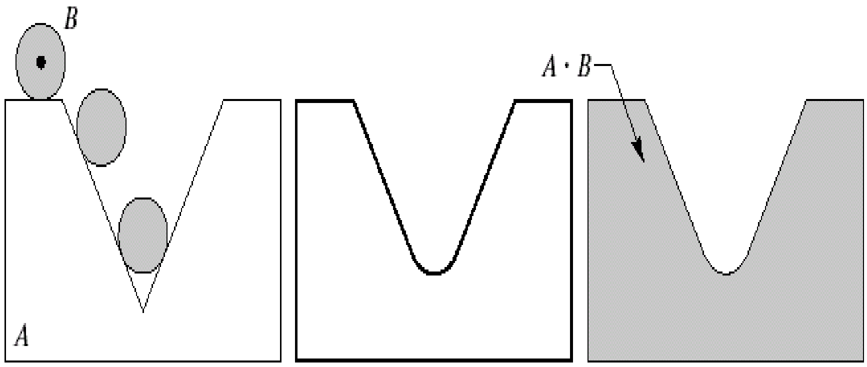
Closing

- A dilation of an object grows the object and can fill gaps.
- If we erode the result after dilation with the rotated structure element, the objects will keep their structure and form, but small holes filled by dilation will not appear.
- Objects merged by the dilation will not be separated again.
- Closing is defined as

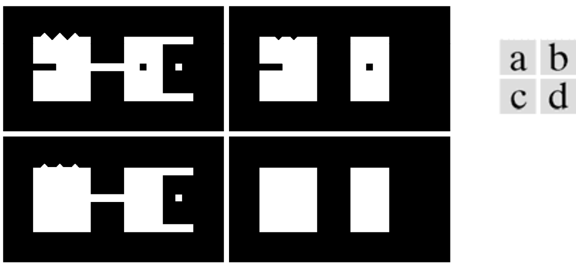
$$f \bullet S = \left(f \oplus \hat{S}\right) \theta\, \hat{S}$$

- This operation can close gaps between two structures without growing the size of the structures like dilation would.

Closing



Example of opening and closing



- **a) Original image I**
- **b) Opening of I**
- **c) Closing of I**
- **d) Closing of b)**

Closing is a dual operation to opening

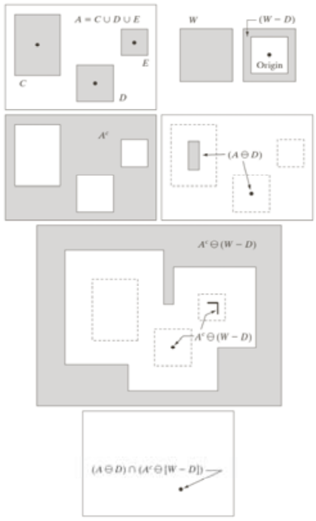
$$f \circ S \subseteq f \subseteq f \bullet S$$

“Hit or miss”- transformation

- Transformation used to detect a given pattern in the image – template matching
- Subject: find exactly the shape given by the object D.
- D can fit inside many objects, so we need to look at the local background W-D.
- First, compute the erosion of A by D, AθD (all pixels where D can fit inside A)
- To fit also the background: Compute A^C, the complement of A. The set of locations where D exactly fits is the intersection of AθD and the erosion of A^C by W-D, A^C θ(W-D).
- Hit-or-miss is expressed as A ⊛ D:

$$(A\theta D)\cap[A_c\theta(W-D)]$$

Main use: Detection of a given pattern or removal of single pixels

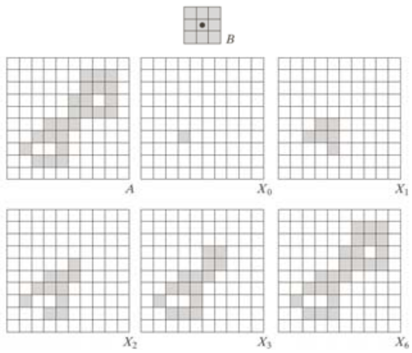


Morphology for extracting connected components

- Detects a connected object Y, in image A, given a point p i Y
  - Start with X_0, a point in Y
  - Dilate X_0 with either a square or plus
  - Let X_1 be only those pixels in the dilation that are part of the original region.
  - Continue dilation X_1 to give X_2 until X_k=X_k-1

$X_k = X_{k-1} \oplus B \cap A$

X_0 = p, k=1,2,3,



Computing convex hull using morphology

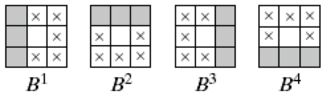
- Convex hull C of a set of points A may be estimated using the Hit-or-Miss transformation
- Consider the four structuring elements B^1-B^4.
- Apply hit-or-miss with A using B^1 iteratively until no more changes occur. Let D^1 be the result.
- Then do the same with B^2,..,B^4 do compute D^2...D^4 in the same manner.
- Then compute the convex hull by the union of all the D^is.

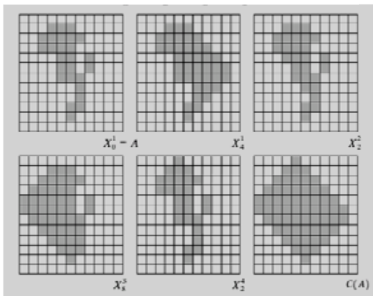
$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A;\ i = 1, 2, 3, 4;$$
$$k = 1, 2, 3, \ldots;\ X_0^i = A; \text{ and}$$
$$D^i = X_{\text{conv}}^i.$$

C (A) = $\bigcup$ D_i

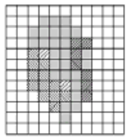
- Gives too big area to guaranty convexity
- Can be corrected by taking the intersection to the maximum dimension in x and y direction

C_res = C (A) ∩ ROI(A)



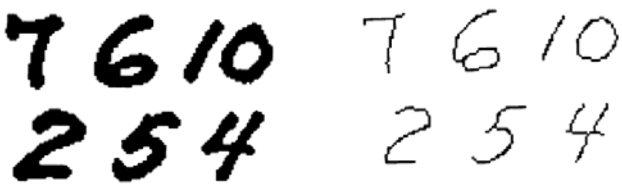


C_res



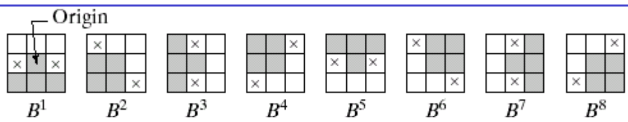
x in B means don't care

Region thinning and skeletons

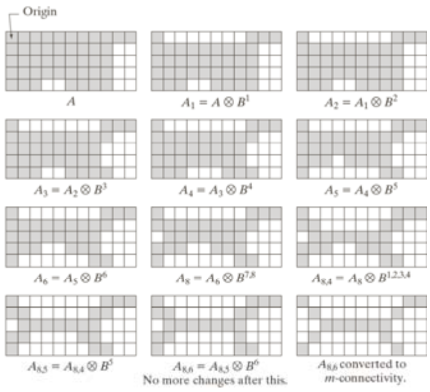


- Let the region object be described using an intrinsic coordinate system, where every point is described by its distance from the nearest boundary point.
- The skeleton is defined as the set of points whose distance from the nearest boundary is locally maximum.
- Many different methods for computing the skeleton exist.
- Thinning is a procedure to compute the skeleton.
- Shape features can later be extracted from the skeleton.

Thinning



- Thinning of a set A with structure element B (a set of 8 structure elements)
- First, thin A by one pass of B1, then thin the result by one pass of B2, until A is thinned with one pass of Bn.
- Then repeat the entire process untill no further changes.



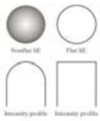
- $A \otimes B = A - (A \circledast B)$
- $A \otimes \{B\} =$
$((((A \otimes B1) \otimes B2)\ldots) \otimes Bn)$

Thickening

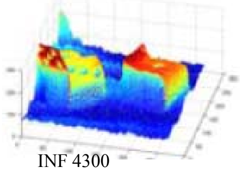
- Thickening is the dual operator of thinning.
- It can be computed as a separate operation, but it is normally computed by thinning the background and then complementing the result: from $C=A^C$, thin C, then form C^C.

Gray level morphology

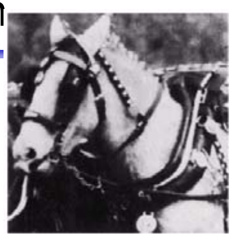
- We apply a simplified definition of morphological operations on gray level images
  - Grey-level erosion, dilation, opening, closing
- Image f(x,y)
- Structure element b(x,y)



- Assume symmetric, flat structure element, origo at center (this is sufficient for normal use).
- Erosion and dilation then correspond to local minimum and maximum over the area defined by the structure element



Gray level erosion /dilation



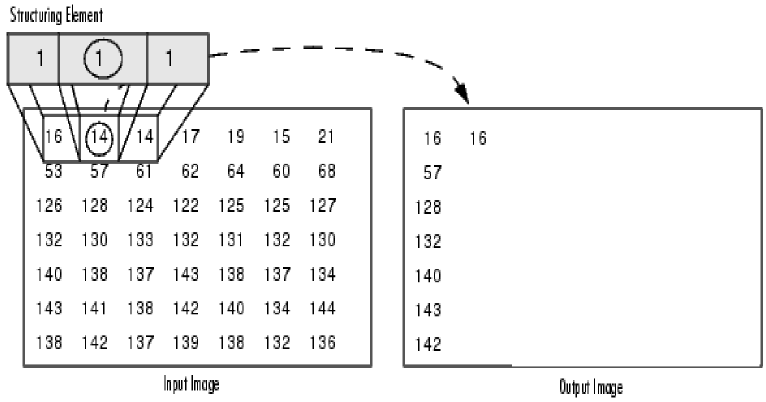
- **Erosion**:
  - Place the structure element with origo at pixel (x,y)
  - Chose the local **minimum** grey level in the region defined by the structure element
  - Assign this value to the output pixel (x,y)
  - Results in darker images and light details are removed

$$[f \ominus b](x,y) = \min_{(s,t)\in B}\{f(x+s, y+t)\}$$

- **Dilation**:
  - Chose the local **maximum** over the region defined by the structure element
  - Let pixel (x,y) in the outimage have this maximum value.
  - Results in brighter images and dark details are removed

$$[f \oplus b](x,y) = \max_{(s,t)\in B}\{f(x-s, y-t)\}$$

Gray level morphology



Morphological Dilation of a Grayscale Image

Gray level opening and closing

- Corresponding definition as for binary opening and closing
- Result in a filter effect on the intensity
- Opening: Bright details are smoothed
- Closing: Dark details are smoothed

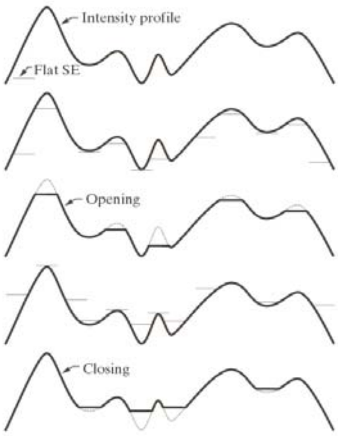
$$f \circ S = (f \,\theta\, S) \oplus S$$

$$f \bullet S = (f \oplus S) \,\theta\, S$$

Interpretation of grey-level opening and closing

- Intensity values are interpreted as heigh curves over the (x,y)-plane.
- Opening of f by b: push the structure element up from below towards the surface of f. The value assigned is the highest level b can reach. (smooth bright values)

- Closing: push the structure element from above down at the surface of f. (smooth dark values)

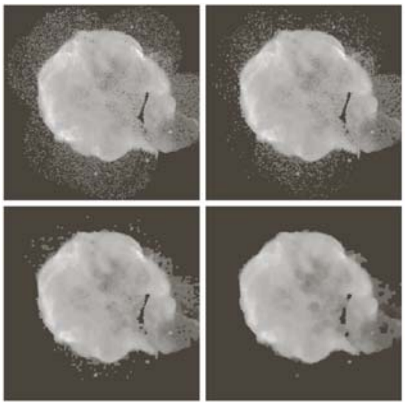


Morphological filtering

- Grey-level opening and closing with flat structure elements can be used to filter out noise.
- This is particularly useful for e.g. dark or bright noise.
- Remark: bright or dark is relative to surroundings, eg. local extremas are filtered.
- To remove bright noise, do first opening, then closing. This can be repeated.
- The size of the structure element should reflect the size of the noise objects.

Example – morphological filtering

Original



Opening and closing by disk of size 1

Opening and closing by disk of size 5

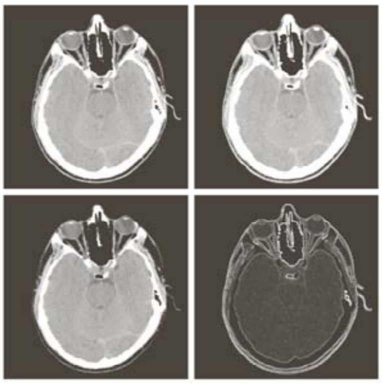
Opening and closing by disk of size 3

Morphological gradient

- Gray level dilation will (under some conditions) give an image with equal or brighter values
- Erosion will under the same conditions produce an image with equal or lower values
- This can be used for edge detection

Morphological gradient = $(f \oplus S) - (f \ominus S)$

Morphological gradient

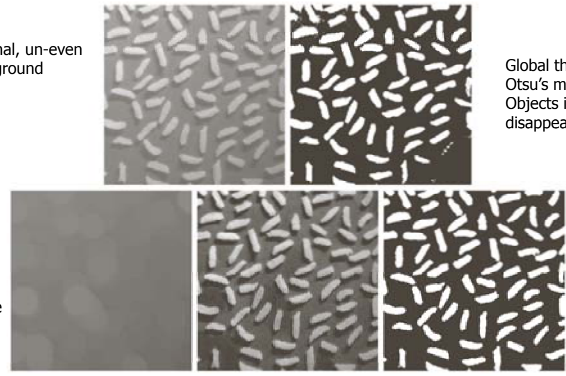


Top-hat transformation

- Purpose: detect (or remove) structures of a certain size.
- Top-hat: light objects on a dark background (also called white top-hat).
- Bottom-hat: dark objects on a bright background (also called black top-hat)
- Top-hat: $f - (f \circ b)$
- Bottom-hat: $(f \bullet b) - f$
- Very useful for correcting uneven illumination/objects on a varying background ☺

Example – top-hat

Original, un-even background



Global thresholding using Otsu's method
Objects in the lower left disappear

Opening with a 40x40 structure element removes objects and gives an estimate of the background

Top-hat transform (original – opening)

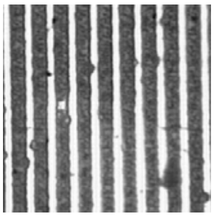
Top-hat, thresholded
Global threshold

Bottom-Hat transformation

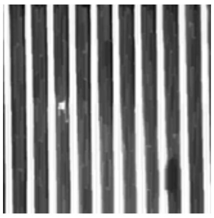
Bottom-Hat = Image – Closing of image

$$f - f \bullet S$$

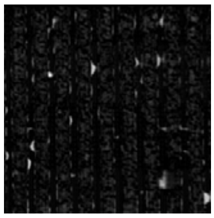
Fault detection using 'Bottom-hat'



Original

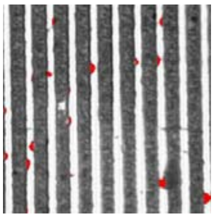


Closing' with vertical structure element



Bottom-Hat= Original – closing

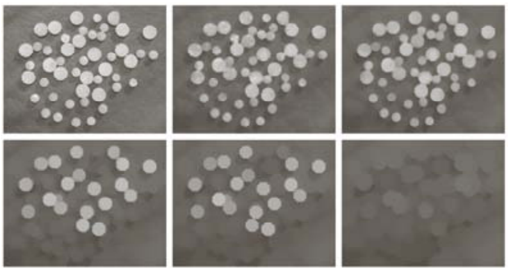
Threholded previous image



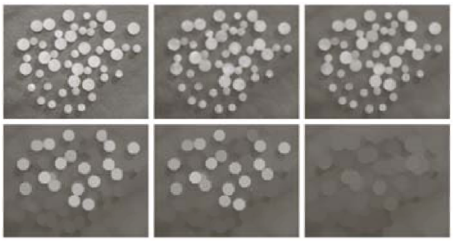
Error detection

Example application: granulometry

- Granulometry: determin the size distribuiton of particles in an image.
- Assumption: objects with regular shape on a background.
- Principle: perform a series of openings with disks with increasing radius r.
- Compute the sum of all pixels values after the opening.
- Compute the difference in this sum between radius r and r-1, and plot this as a function of radius.



Example - granulometry



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

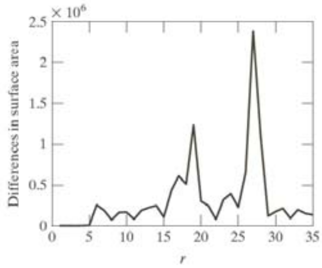


FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

Learning goals - morphology

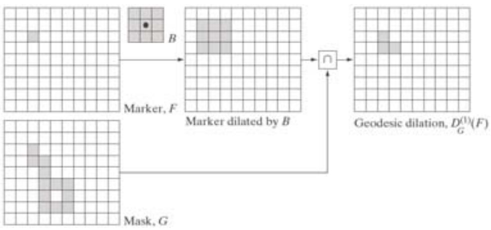
- Understand in detail binary morphological operations and selected applications:
  - Basic operators (erosion, dilation, opening, closing)
    - Understand the mathematical definition, perform them ”by hand” on new objects
  - Applications of morphology: edge detection, connected components, convex hull etc.
    - Verify the examples in the book
- Grey-level morphology:
  - Understand how grey-level erosion and dilation (and opening and closing) works.
  - Understand the effect these operations have on images.
  - Understand top-hat, bottom-hot and what they are useful for.

IF time only: morphological reconstruction

- Morphological reconstruction is a geodesic operator and involves a marker image, a mask and a structuring element.
- The mask restricts the result after erosion or dilation.
- With clever use of mask and marker, this operation can be used for several applications.
- The trick is to find a good choice of mask……..

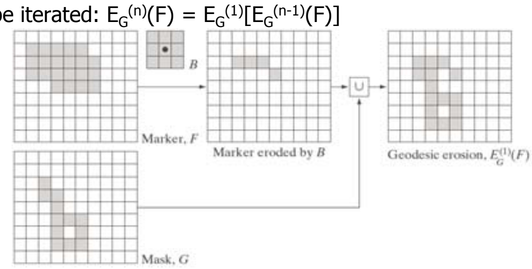
Geodesic dilation (9.5.9)

- Assume two binary images F and G and a structure element B.
- F is the marker image and G is the mask
- A geodesic dilation of F over the mask G is:
  - Dilate F with a structure element B
  - B is normally a 3x3 square element.
  - Perform a pixel wise logical AND (intersection) between the dilated image and the mask G
- $D_G^{(1)}(F) = (F \oplus B) \cap G$
- This can be iterated: $D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)]$



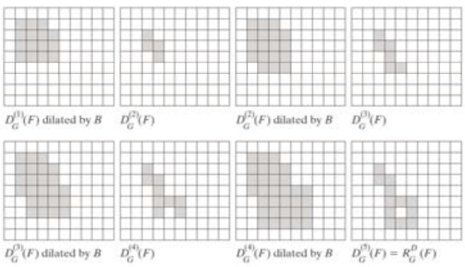
Geodesic erosion (9.5.9)

- Assume two binary images F and G and a structure element B.
- F is the marker image and G is the mask
- A geodesic erosion of F over the mask G is:
  - Erode F with a structure element B
  - B is normally a 3x3 square element.
  - Perform a pixel wise logical OR (union) between the dilated image and the mask G
- $E_G^{(1)}(F) = (F \theta B) \cup G$
- This can be iterated: $E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)]$



Morphological reconstruction

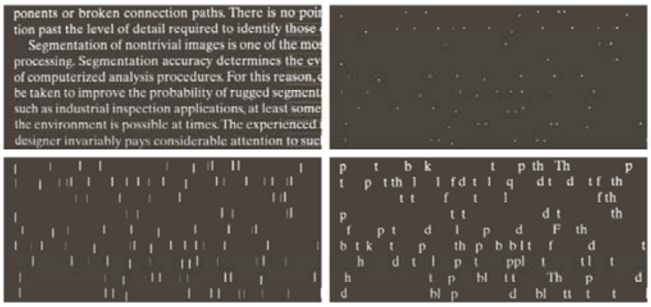
- Morphological reconstruction (based on dilation)
  - Defined as performing iterative geodesic dilation (iterated until no changes): $R_G^D(F)=D_G^{(k)}(F)$
  - The process is based on connectivity instead of using structure element
  - The peaks in the input image specify where the process starts
- Morphological reconstruction (based on erosion)
  - Defined as performing iterative geodesic erosion (iterated until no changes): $R_G^E(F)=E_G^{(k)}(F)$



Example – morphological reconstruction by opening
Find only characters with a vertical stroke

Original image

Erosion by a 51x1 structure element



For comparison: Regular opening by the same structuring element

Opening by reconstruction

Note: the opening by reconstruction will restore the original shape (in terms of connected component) of only the objects where the structure element fits.