UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Tuesday December 13, 2011**
Exam hours:	**14.30-18.30 (4 hours)**
Number of pages:	**5 pages of text, plus 1 page enclosure**
Enclosures:	**1 sheet containing two sketches**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the “spirit” of the exercise. In such a case, you should make it clear what assumptions you have made.

- Please note that all parts of the exercises have equal weight. You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

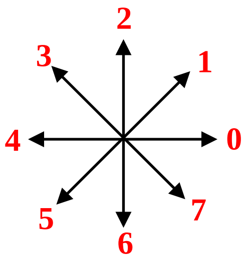
- Two of the questions are based on sketches enclosed as an extra sheet at the end of the exam text. Please give your solution on this sheet, mark it with your candidate number, and include it in your solution.

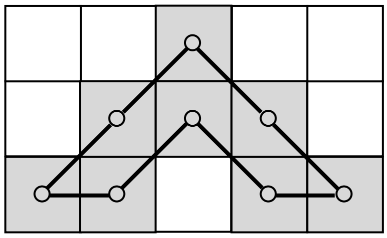
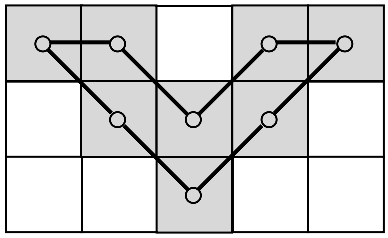
- Your answers should be **short**, typically 1-3 sentences and / or a sketch should be sufficient.

  Good luck!!

Exercise 1: Chain Codes

You are given the 8-directional chain code and the two objects below.



a) Chain code the boundary of the Λ-shaped object clockwise from the lower left pixel.

b) Which technique, based on the 8-directional <u>absolute</u> chain code, can be used to make a description of the Λ-shaped object that is independent of the start point? Demonstrate this by starting at the top pixel of the object, instead of the lower left.

c) The V- shaped object is a rotation of the Λ-shaped object.
Which technique, based on the clockwise <u>relative</u> chain code, will give you the same description of the two objects, independent of the start point?
Demonstrate this by starting at the upper left pixel of the V-shaped object.

Exercise 2: Classification

a) Given a classification problem with 2 classes.

$$\mu_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mu_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Sketch the mean values and the covariance matrices in a plot.

b) Sketch the decision boundary for a multivariate normal distribution when the classes have equal *a priori* probabilities.

c) What do we know about the value of the discriminant functions for the two classes at the decision boundary?

d) The discriminant function for a normal distribution with common covariance matrix is given by the expression

$$g_i(\mathbf{x}) = \mathbf{w}_i^t \mathbf{x} + w_{i0}$$

$$\text{where} \quad \mathbf{w}_i = \frac{1}{\sigma^2}\boldsymbol{\mu}_i \text{ and } w_{i0} = -\frac{1}{2\sigma^2}\boldsymbol{\mu}_i^t\boldsymbol{\mu}_i + \ln P(\omega_i)$$

Assume that the classes still have equal *a priori* probabilities and mean and covariance matrix as given in a). Given two features x_1 and x_2, give an expression for the decision boundary as a function of x_1 and x_2.

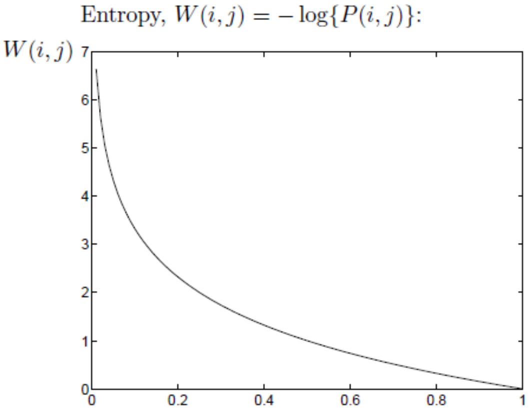
Exercise 3: GLCM

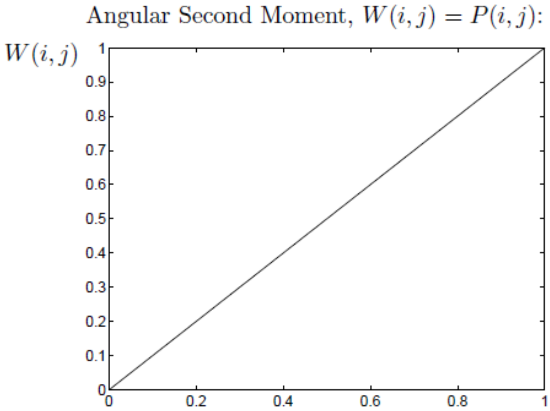
To the right you see a remote sensing image showing both steep slopes, deep valleys, flat grassland, numerous small streams, and some man-made constructions. Two subregions are incircled.

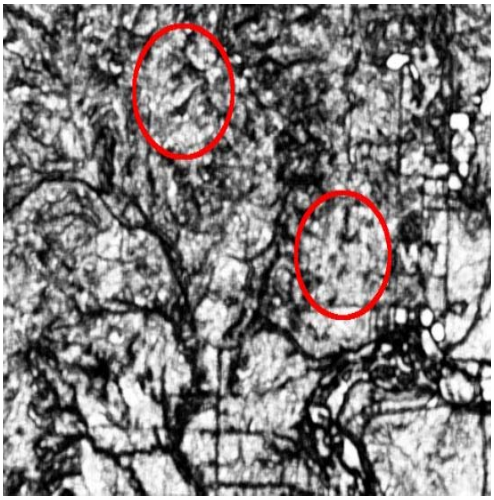
The two texture feature images at the bottom of the page were made using a moving window to compute a normalized GLCM and deriving two features that are using weight functions based on the value of the matrix elements, namely ENTROPY and ASM. The two equations are:

$$ENT = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} P(i,j)W_{ENT}(i,j), \quad P(i,j) > 0; \qquad ASM = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} P(i,j)W_{ASM}(i,j)$$

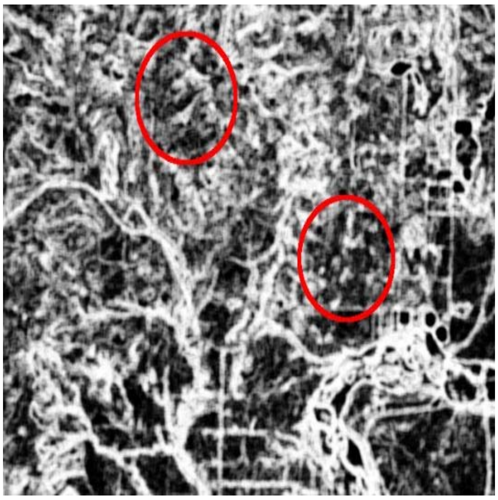
Where the weights are functions of the GLCM element probabilities, given by two curves:







Left feature image



Right feature image

a) A direction invariant normalized GLCM was computed with a pixel offset of 1. Explain the steps of this computation.

b) Based on what you observe in the circled regions and elsewhere in the two feature images, explain which is ENTROPY and which is ASM, and give some reasons.

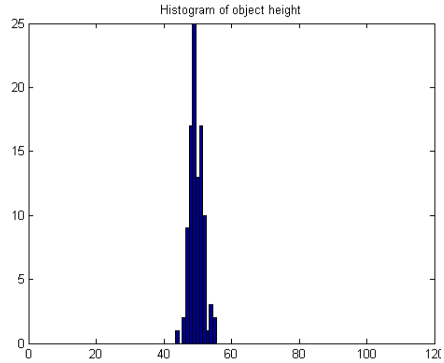
c) There are also GLCM features where the weighting is a function of the element position. Two examples of this are Inertia (INT) and Inverse Difference Moment (IDM), given by

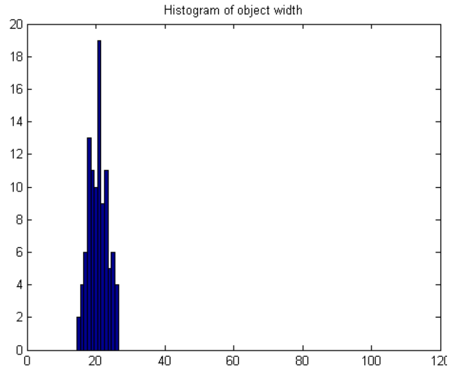
$$INT = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1}(i-j)^2 P(i,j) \;; \qquad IDM = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1}\frac{1}{1+(i-j)^2}P(i,j)$$

Discuss the two features in terms of local contrast and homogeneity.

Exercise 4: Morphology

A gray level image ***f (x,y)*** contains 100 dark objects on a lighter background. The background has varying intensity. The dark objects have a size distribution as indicated in the figures below, showing the histograms of object width and height.





a) Which morphological operation would you use in order to estimate the varying intensity in the image?

b) What should the structuring element look like?

c) Let ***g(x,y)*** be the image after a correction of the background intensity, but before segmentation by thresholding. Give an expression for ***g(y,x)*** given the structuring element and the operation in a).

d) Let ***h(x,y)*** be the image after thresholding. Which morphological operation would you use to split connected objects in ***h(x,y)***?

e) Which morphological operation would you use to connect fragmented objects?

f) Given a line in a gray level image. The line has pixel values

$f_1(x,y)$ = [1 2 3 4 5 4 3 2 1 2 3 3 3 3 3 3 3 3 3 3]

Compute $g_1(x,y)$, the result after a morphological opening by a the structuring element [1 1 1]. You can either give a graph, or give the values of $g_1(x,y)$.

Exercise 5: Moments and Hough Transform

Assume that you have thresholded a gray level image into a binary image ***b(x,y)*** containing a solid object (pixel value = 1) and background pixels (pixel value 0). We are going to use general discrete moments m_{pq} defined as

$$m_{pq} = \sum_x \sum_y x^p y^q b(x,y)$$

a) Describe a moment-based approach to find the centre of mass of the object.

b) Assume that the object is rectangular and rotated, located somewhere in the image. Describe a moment-based approach to estimate the orientation of the object.

c) The normal representation of a line segment in the (x,y)-plane is given as

$$\rho = x\cos\theta + y\sin\theta$$

Indicate the parameters $\boldsymbol{\rho}$ and $\boldsymbol{\theta}$ in the first sketch of the enclosed sheet.

d) If we translate the origin to the centre of mass of the rectangle, apply a gradient detector to the binary image, and use the "normal representation" Hough Transform, we get peaks in the Hough space illustrated by ***red dots*** in the second sketch of the enclosure.

Assume that the rectangle has sides ***a*** and ***b*** (***a***>***b***), and has an orientation $\boldsymbol{\theta}$ relative to the x-axis in the image, where $-\pi/2 \le \theta \le \pi/2$.
Please fill in the proper parameters into the seven empty ovals in the second sketch in the enclosure, and give reasons for your choises.
Please remember to mark this enclosure with your candidate number and include it in your solution.

e) What happens to the height of the peaks in the (ρ,θ)-plane as the rectangle becomes more like a square?

The following two questions are intended for the PhD-students:

f) What is the (approximate) orientation of the rectangle relative to the x-axis of the image, given the positions of the peaks in the Hough space in the sketch? Explain your reasoning, and make a simple drawing of the rectangle in the (x,y)-coordinate system.

g) Explain what happens to the positions of the peaks in the $(\boldsymbol{\rho},\boldsymbol{\theta})$-plane
   a. If the rectangle is turned into a square of sides ***s***.
   b. As the square is rotated through 360 degrees in the (x,y)-plane.

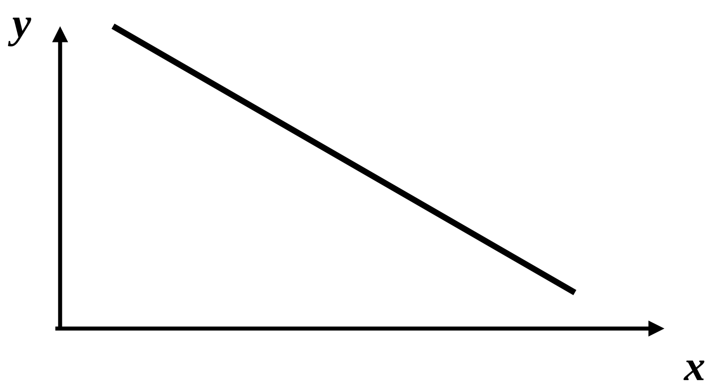
Good Luck !

Enclosure to exam paper, INF4300, Wednesday December 13, 2011

Candidate number:

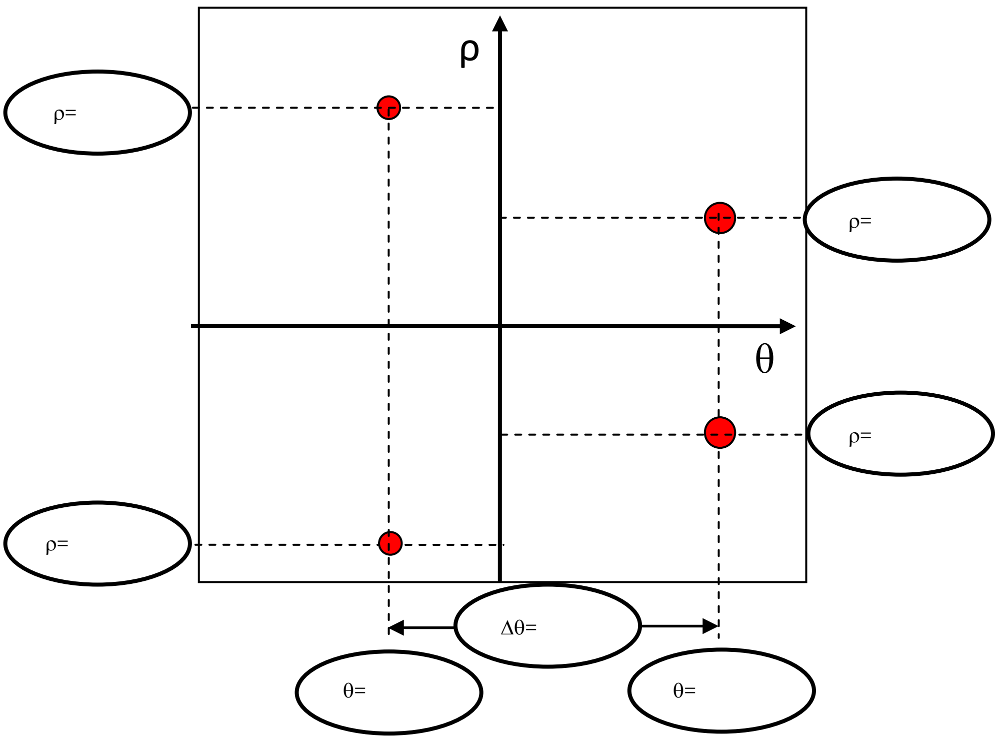
This sketch is part of Exercise 5 c.

Indicate the parameters ρ and θ of the normal representation of the line segment..



This sketch is part of Exercise 5 d.

Please fill in the proper parameters /values into the seven empty ovals



Thank you !