Oversampling Data Converters
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Last time – and today, Tuesday 15th of March:

Last time:
12.3 Switched Capacitor Amplifiers
12.4 Switched Capacitor Integrator

Today, from chapter 14 in "J. & M."
14.1 Oversampling without noise shaping
14.2 Oversampling with noise shaping
14.3 System Architectures
14.4 Digital Decimation Filters
14.5 Higher-Order Modulators
(14.6 Bandpass Oversampling Converters)
14.7 Practical Considerations
14.8 Multi-bit oversampling converters
2nd order sigma delta design example
Oversampling converters (chapter 14 in “J & M”)

- For high resolution, low-to-medium-speed applications like for example digital audio
- Relaxes requirements placed on analog circuitry, including matching tolerances and amplifier gains
- Simplify requirements placed on the analog anti-aliasing filters for A/D converters and smoothing filters for D/A converters.
- Sample-and-Hold is usually not required on the input
- Extra bits of resolution can be extracted from converters that samples much faster than the Nyquist-rate. Extra resolution can be obtained with lower oversampling rates by exploiting noise shaping

Resolution and clock cycles per sample

- Dependence of achievable resolution and required clock cycles per sample for various ADC systems.

A Gigasample/Second 5-b ADC with On-Chip Track and Hold Based on an Industrial 1-μm GaAs MESFET E/D Process

Richard Stephens, Howard, Ald, Frank Talley, Gainer Bultman, Joel Bauer, and David E. Stimson, Accent Microtech, ABG
Transfer function for simple discrete time integrator

\[ \frac{U_2(z)}{U_1(z)} = \frac{z^{-1}}{1 - z^{-1}} \]

Transfer function not dependent on \( C_{p1} \): (Circuit in Fig. 10.9)

At time \( t = (n-1)T \), in \( q_1 \), a "sample" of the input voltage is taken, and \( C \)
gets charged:

\[ q_{1n}[(n-1)T] = C_1 \text{vin}[(n-1)T] \]

At the same time, \( q_{2n}[(n-1)T] = C_2 \text{vout}[(n-1)T] \)

At time \( t = nT \), the charge on \( C_1 \) is balanced to \( C_2 \):

\[ q_{1n}[nT] = q_{2n}[nT] + q_{1n}[(n-1)T] \]
The quantization noise is the difference between the input and output values, \( e(n) \). The model is exact under the assumption that the quantization noise is strongly related to the input signal (see p. 518).

This model becomes approximate when quantization error \( e(n) \) is made small, i.e., when the statistical properties of \( e(n) \) such as \( E(e(n)) \) being an independent white-noise signal. This model leads to a simpler understanding of \( Y_a \) and with some exceptions is widely accepted.

- If \( e(n) \) is very active, \( e(n) \) can be approximated as an independent random variable uniformly distributed between \( \pm \frac{1}{2} \).

The quantization noise \( e(n) \) is independent of the sampling frequency \( f_s \).

The output spectrum of \( e(n) \), \( Y_e(f) \), is a white (constant) noise and is not present in \( X_e(f) \), as shown in the figure.

\[ Y_e(f) = \frac{A^2}{2} \frac{1}{f_s} \]

The spectral density of \( e(n) \) is given by, where \( A \) is the total noise power and is assumed to be a constant. The power in \( e(n) \) under \( Y_e(f) \) is:

\[ P_e = \int Y_e(f) \, df = \frac{A^2}{2} \frac{1}{f_s} \]

The power of the input signal \( Y_a(f) \) remains the same as before since we assume the input frequency content is below \( f_s \). The quantization noise power is reduced to:

\[ P_e = \frac{A^2}{2} \frac{1}{f_s} \]

Therefore, increasing \( f_s \) decreases the quantization noise power at one-half, or equivalently, 3 dB (or equivalent 6x).

\[ P_{max} = 10 \log \left( \frac{P_e}{P_{max}} \right) = 10 \log \left( \frac{1}{2} \right) = 4.8 \log \left( \frac{1}{2} \right) \]

\[ = 6.02 \text{ dBm} = 10 \log (50) \text{ [W]} \]

Hence, quantization noise due to oversampling.

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**Oversampling Advantage:**

If the input signal is: (20 Hz, 200 Hz, 300 Hz, 400 Hz, 500 Hz), the frequency content is below the oversampling frequency. The power of the signal is: (100 mW) for each frequency. The quantization noise power is reduced to: (20 mW) for each frequency. Therefore, increasing the oversampling frequency decreases the quantization noise power at one-half, or equivalently, 3 dB (or equivalent 6x).
Nyquist Sampling and Oversampling

- Figure from [Kest05]
- Straight over-sampling gives an SNR improvement of 3 dB / octave
- $f_s > 2f_0$ ($2f_0$ = Nyquist Rate)
- OSR = $f_s/2f_0$
- $SNR_{\text{max}} = 6.02N + 1.76 + 10\log (\text{OSR})$
Oversampling (without noise shapin)

- Total støy er gitt av:
  
  $$P_e = \int_{-f_0}^{f_0} S_e(f) df = \frac{\Delta^2}{12} \cdot \frac{1}{OSR}$$

- Doubling of the sampling frequency increases the dynamic range by 3 dB = 0.5 bit.
- To get a high SNR a very high fs is needed \(\rightarrow\) high power consumption.
- Oversampling usually combined with noise shaping and higher order modulators, for higher increase in dynamic range per octave ("OSR")

\[ \text{SNR}_{\text{max}} = 6.02N + 1.76 + 10\log(\text{OSR}) \, [\text{dB}] \]

SNR improvement 0.5 bits / octave
Advantages of 1-bit A/D converters (p.537 in "J&M"): 

- Oversampling improves signal-to-noise ratio, but not linearity

- Ex.: 12-bit converter with oversampling needs component accuracy to match better than 16-bit accuracy if a 16-bit linear converter is desired

- Advantage of 1-bit D/A is that it is inherently linear. Two points define a straight line, so no laser trimming or calibration is required

- Many audio converters presently use 1-bit converters for realizing 16- to 18-bit linear converters (with noise shaping).
Oversampling with noise shaping (14.2)

• Oversampling combined with noise shaping can give much more dramatic improvement in dynamic range each time the sampling frequency is doubled.
• The sigma delta modulator converts the analog signal into a noise-shaped low-resolution digital signal.
• The decimator converts to a high resolution digital signal.

Multi-order sigma delta noise shapers

(Park, Motorola)

Note: Higher order Noise Shaper has less baseband noise.
Ex. 14.5 ”point”:

• 2 X increase in M → (6L+3)dB or (L+0.5) bit increase in DR.
• L: sigma-delta order
• 6 db Quantizer, for 96 dB SNR:
• Plain oversampling: \( f_s = 54 \) GHz
• 1st order : \( f_s = 75.48 \) MHz
• 2nd order: \( f_s = 5.81 \) MHz

Nyquist Sampling, Oversampling, Noise Shaping

• Figure from [Kest05]
• Straight oversampling gives an SNR improvement of 3 dB / octave
• \( f_s > 2f_0 \) (2\( f_0 \) = Nyquist Rate)
• OSR = \( f_s/2f_0 \)
• \( \text{SNR}_{\text{max}} = 6.02N+1.76+10\log \text{(OSR)} \)
OSR, modulator order and Dynamic Range

- 2X increase in $M \rightarrow (6L+3)\text{dB}$ or $(L+0.5)$ bit increase in DR.
- $L$: sigma-delta order
- Oversampling and noise shaping

14.2 Oversampling with noise shaping

- The anti aliasing filter bandlimits the input signals less than $f_s/2$.
- The continuous time signal $x_c(t)$ is sampled by a S/H (not necessary with separate S/H in Switched Capacitor impl.)
- The Delta Sigma modulator converts the analog signal to a noise shaped low resolution digital signal
- The decimator converts the oversampled low resolution digital signal into a high resolution digital signal at a lower sampling rate usually equal to twice the desired bandwidth of the desired input signal (conceptually a low-pass filter followed by a downsampler).
Noise shaped Delta Sigma Modulator

First-Order Noise Shaping (Figures from Schreier & Temes '05)

- $S_{TF}(z) = \frac{H(z)}{1+H(z)}$ (eq. 14.15) $N_{TF}(z) = \frac{1}{1+H(z)}$
- $Y(z) = S_{TF}(z) U(z) + N_{TF}(z) E(z)$
- $H(z) = \frac{1}{z-1}$ (discrete time integrator) gives 1st order noise shaping
- $S_{TF}(z) = \frac{H(z)}{1+H(z)} = \frac{1}{(z-1)(1+1/(z-1))} = z^{-1}$
- $N_{TF}(z) = \frac{1}{1+H(z)} = \frac{1}{(1+1/(z-1))} = (1-z^{-1})$
- The signal transfer function is simply a delay, while the noise transfer function is a discrete-time differentiator (i.e. a high-pass filter)
14.2 Oversampling with noise shaping

Quantization noise power over the frequency band 0 to $f_s$ is now given by

$$P_c = \int_{-f_s}^{f_s} S_c(f) \cdot |N_{AQ}(f)|^2 df = \int_{-f_s}^{f_s} \left( \frac{\alpha^2}{\bar{N}} \right) \frac{1}{4 \pi} \left[ 2 \sin \left( \frac{2 \pi f}{f_s} \right) \right]^2 df \quad (14.21)$$

Making the approximation that $f_s < f_c$ ($\text{OSR} \gg 1$) so that $\sin \left( \frac{2 \pi f}{f_s} \right) \approx \frac{2 \pi f}{f_s}$:

$$P_c = \frac{\alpha^2}{\bar{N}} \left( \frac{2}{3} \text{ OSR}^2 \right) \frac{1}{4 \pi} \left( \frac{2 \pi f_c}{f_s} \right)^3$$

It is assumed that the maximum signal power is the same as attained before, i.e. equation (14.11) ($P_c = \text{OSR}^2$), making maximum SNR:

$$\text{SNR}_{\text{max}} = 10 \log \left( \frac{P_c}{\text{OSR}^2} \right) = 10 \log \left( \frac{2}{3} \text{ OSR}^3 \right) + 10 \log \left( \frac{2}{3} \text{ OSR}^3 \right) = 0$$

Doubling the OSR gives an SNR improvement by a 4.1 Gb/s channel at 12 Gb/s.

Quantization noise power for linearized model of a general $\Delta \Sigma$ modulator:

$$P_c = \int_{-f_s}^{f_s} S_c(f) \cdot |N_{AQ}(f)|^2 df = \int_{-f_s}^{f_s} \left( \frac{\alpha^2}{\bar{N}} \right) \frac{1}{4 \pi} \left[ 2 \sin \left( \frac{2 \pi f}{f_s} \right) \right]^2 df \quad (14.21)$$

Using the approximation that $f_s < f_c$ ($\text{OSR} \gg 1$) so that we may approximate $\sin \left( \frac{2 \pi f}{f_s} \right)$ to be $\frac{2 \pi f}{f_s}$:

$$P_c = \int_{-f_s}^{f_s} \frac{\alpha^2}{\bar{N}} \frac{1}{4 \pi} \left[ 2 \frac{4 \pi f}{f_s} \right]^2 df = \int_{-f_s}^{f_s} \frac{\alpha^2}{\bar{N}} \frac{1}{4 \pi} \frac{4 \pi^2 f^2}{f_s^2} \cdot f^2 df$$

Letting $K = \frac{\alpha^2}{\bar{N}} \frac{1}{4 \pi} \frac{4 \pi^2}{f_s^2}$

$$P_c = K \int_{-f_s}^{f_s} f^2 df = K \left( \frac{f_s^3}{3} - \frac{f_s^3}{3} \right) = K \frac{f_s^3}{3}$$

Using $\text{OSR} = \frac{2 \pi f_c}{f_s}$:

$$P_c = \frac{\alpha^2}{\bar{N}} \frac{1}{4 \pi} \left( \frac{2 \pi f_c}{f_s} \right)^3 = \frac{\alpha^2}{\bar{N}} \frac{1}{4 \pi} \left( \frac{2 \pi \text{ OSR}}{f_s} \right)^3$$

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Ex. 14.5

- Given that a 1-bit A/D converter has a 6 dB SNR, which sample rate is required to obtain a 96-dB SNR (or 16 bits) if \( f_0 = 25 \) kHz for straight oversampling as well as first- and second-order noise shaping?

- Oversampling with no noise shaping: From ex. 14.3 we know that straight oversampling requires a sampling rate of 54 THz.

- (\(6.02N+1.76+10 \log (\text{OSR})=96\))

- \(6 + 10 \log \text{OSR} = 96\)

- \(10 \log \text{OSR} = 90\)
Ex. 14.5

**Oversampling with 1st order noise shaping:**

- \[ 6 - 5.17 + 30 \log(\text{OSR}) = 96 \]
- \[ \text{OSR} = \frac{\text{fs}}{2f_0} \]
- \[ 30 \log (\text{OSR}) = 96 - 6 + 5.17 = 95.17 \]

A doubling of the OSR gives an SNR improvement of 9 dB / octave for a 1st order modulator;

\[ \frac{95.17}{9} = 10.57 \quad 2^{10.57} \times 2 \times 25 \text{kHz} = 75.48 \text{ MHz} \]

**OR:** \[ \log(\text{OSR}) = \frac{95.17}{30} = 3.17 \]

\[ \text{OSR} = 1509.6 \]

\[ 1509.6 \times (2 \times 25 \text{kHz}) = 75.48 \text{ MHz} \]

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Ex. 14.5

**Oversampling with 2nd order noise shaping:**

- \[ 6 - 12.9 + 50 \log(\text{OSR}) = 96 \]
- \[ \text{OSR} = \frac{\text{fs}}{2f_0} \]
- \[ 50 \log (\text{OSR}) = 96 - 6 + 12.9 = 102.9 \]

A doubling of the OSR gives an SNR improvement of 15 dB / octave for a 2nd order modulator;

\[ \frac{102.9}{15} = 6.86 \quad 2^{6.86} \times 2 \times 25 \text{kHz} = 5.81 \text{ MHz} \]
14.3 System Architectures (A/D)

- $X_c(t)$ is sampled and held, resulting in $x_{sh}(t)$.
- $x_{sh}(t)$ is applied to an A/D Sigma Delta modulator which has a 1-bit output, $x_{ds}(n)$. The 1-bit signal is assumed to be linearly related to the input $X_c(t)$ (accurate to many orders of resolution), although it includes a large amount of out-of-band quantization noise (seen to the right).
- A digital LP filter removes any high frequency content, including out of band quantization noise, resulting in $X_{lp}(n)$.
- Next, $X_{lp}(n)$ is resampled at $2f_0$ to obtain $X_s(n)$ by keeping samples at a submultiple of the OSR.
14.4 Digital decimation filters

- Many techniques
- a) FIR filter removes much of the quantization noise, so that the output can be downsamped by a 2nd stage filter which may be either IIR type (as in a), uppermost) or a cascade of FIR filters (as in b), below)
- In b) a few halfband FIR filters in combination with a sinc compensation FIR-filter are used.
In some applications, these halfband and sinc compensation filters can be realized using no general multi-bit multipliers [Saramaki, 1990]
14.5 Higher-Order Modulators — Interpolative structure

- $L$th order noise shaping modulators improve SNR by $6L+3$ dB/octave.
- Typically a single high-order structure with feedback from the quantized signal.
- In figure 14.20 a single-bit D/A is used for feedback, providing excellent linearity.
- Unfortunately, modulators of order two or more can go unstable, especially when large input signals are present (and may not return to stability) Guaranteed stability for an interpolative modulator is nontrivial.

Multi-Stage Noise Shaping architecture (“MASH”)

- Overall higher order modulators are constructed using lower-order, more stable, ones → more stable overall system.
- Fig. 14.21: 2nd order using two first-order modulators.
- Higher order noise filtering can be achieved using lower-order modulators.
- Unfortunately sensitive to finite opamp gain and mismatch
14.7 Practical considerations

- Stability
- Linearity of two-level converters
- Idle tones
- Dithering
- Opamp gain

Design example, 14b 2nd order Sigma-Delta mod

- 16 bit, 24 kHz, OSR as powers of two, and allowing for increased baseband noise due to nonidealities: OSR 512 was chosen
Design example, 14b 2nd order Sigma-Delta mod

- Among most relevant nonidealities:
  - Finite DC gain
  - Bandwidth
  - Slew rate
  - Swing limitation
  - Offset voltage
  - Gain nonlinearity
  - Flicker noise
  - Sampling jitter
  - Voltage dependent capacitors
  - Switch on-resistance
  - Offset voltage and settling time for comparators

Noninverting parasitic insensitive integrator (fig 10.9) was used (fully differential implementation)
2nd order modulator; top level schematics

- Two-phase clock generator, switches, chopper stabilized OTA (1st int.), OTA (2nd int.- fully differential folded cascode), comparator, latch, two-level DAC. Biasing circuit. Functional after test.

Sigma Delta converters, ISSCC 2011

- ISSCC- Foremost global forum
- "CT": continuous time
litterature

- Richard Lyons, Randy Yates: "Reducing ADC Quantization Noise", MicroWaves & RF, 2005
- Sangil Park: ”Principles of sigma-delta modulators for analog to digital converters”, Motorola

Next week, 22/3-11

- Ch. 13; Nonlinearity and Mismatch plus beginning of chapter 14; Oscillators (?)
- Messages are given on the INF4420 homepage.

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