Outline

- Oversampling
- Noise shaping
- Circuit design issues
- Higher order noise shaping
Introduction

So far we have considered so called Nyquist data converters.

Quantization noise is a fundamental limit.

Improving the resolution of the converter, translates to increasing the number of quantization steps (bits). Requires better component matching, $A_{OL} > \beta^{-1} 2^{N+1}$, and $\text{GBW} > f_s \ln 2^{N+1} \pi^{-1} \beta^{-1}$.

Introduction

$\Delta\Sigma$ modulator based data converters relies on oversampling and noise shaping to improve the resolution.

Oversampling means that the data rate is increased to several times what is required by the Nyquist sampling theorem.

Noise shaping means that the quantization noise is moved away from the signal band that we are interested in.
Introduction

We can make a high resolution data converter with few quantization steps!

The most obvious trade-off is the increase in speed and more complex digital processing. However, this is a good fit for CMOS.

We can apply this to both DACs and ADCs.

Oversampling

The total quantization noise depends only on the number of steps. Not the bandwidth.

\[ P_Q = \frac{\Delta^2}{12}, \quad \Delta = \frac{V_{\text{ref}}}{2^n} \]

If we increase the sampling rate, the quantization noise will not increase and it will spread over a larger area. The power spectral density will decrease.
Oversampling

Take a regular ADC and run it at a much higher speed than twice the Nyquist frequency.

Quantization noise is reduced because only a fraction remains in the signal bandwidth, $f_b$.

Doubling the OSR improves SNR by 0.5 bit

Increasing the resolution by oversampling is not practical. We can do better! Oversampling is almost always used with noise shaping.
Noise shaping

The idea behind noise shaping is to suppress the noise in the signal band, at the expense of increasing noise at higher frequencies.

\[ x(t) \xrightarrow{+} \int \xrightarrow{\text{\ldots\ldots}} y(t) \]

The \( \Delta \Sigma \) modulator does noise shaping.

Noise shaping

Linear discrete time model

Two independent inputs, \( u \) and \( e \). We derive a transfer function for the signal and quantization noise separately.

\[ u(nT) \xrightarrow{+} H_a(z) \xrightarrow{+} y(nT) \xrightarrow{e(nT)} \]
Noise shaping

\[ u(nT) \xrightarrow{+} H_a(z) \xrightarrow{-} y(nT) \]

Signal transfer function (STF), \( H_s \)

\[ Y(z) = H_a(z) [U(z) - Y(z)] \]

\[ H_s(z) = \frac{Y(z)}{U(z)} = \frac{H_a(z)}{1 + H_a(z)} \]

Noise shaping

\[ e(nT) \xrightarrow{+} H_a(z) \xrightarrow{-} y(nT) \]

Noise transfer function (NTF), \( H_n \)

\[ Y(z) = E(z) - H_a(z) Y(z) \]

\[ H_n(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + H_a(z)} \]
Noise shaping

\[ \begin{align*}
H_s(z) &= \frac{Y(z)}{U(z)} = \frac{H_d(z)}{1 + H_d(z)} = z^{-1} \\
H_n(z) &= \frac{Y(z)}{E(z)} = \frac{1}{1 + H_d(z)} = 1 - z^{-1}
\end{align*} \]

Noise shaping

\[ H_n(e^{j\omega T}) = 1 - e^{-j\omega T} = 2 j e^{-j\omega T} \sin \frac{\omega T}{2} \]

\[ f = \frac{f_s}{4}, \quad \frac{f_s}{2} \]
Noise shaping

First order $\Delta\Sigma$ modulator based data converter

$$P_Q = \int_0^{f_b} \frac{2f}{f_s} \frac{\Delta^2}{12} \cdot 4\sin^2 \frac{\pi f}{f_s} \, df \approx \left(\frac{2f_b}{f_s}\right)^3 \frac{\pi^2 \Delta^2}{3 \cdot 12}$$

Assuming full-scale sine wave input (as before)

$$\text{SNR} = 6.02N + 1.76 - 5.17 + 30\log_{10}\text{OSR}$$

Doubling OSR improves SNR by 1.5 bits

Circuit example

First order $\Delta\Sigma$ ADC (sampled data single bit quantizer) implementation.
Single-bit quantization

Quantizer nonlinearity is shaped by the NTF, but still needs to be less than the inherent quantization noise.

Feedback signal does not undergo shaping and adds directly to the input. Needs linearity better than the equivalent resolution of the ADC.

Single bit quantizer: Only two levels, inherent linearity. (Second order effects: switching, etc.)

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Single-bit quantization

Linear analysis assumed quantization noise is white, however input signal may give rise to patterns in the quantization noise. Quantization noise energy will be clustered at some frequencies. Tones in the output signal.

*Idle tones or pattern noise* for DC input.

Intentionally add noise to decorrelate the quantization noise pattern, *dithering.*
Multi-bit quantization

Single bit quantization will introduce significant (out of band) quantization noise which must be attenuated by a filter. We have assumed a brick-wall filter in our analysis.

Multi-bit quantization will reduce the inherent quantization noise, and performance is better predicted by the linear analysis.

Linearity is challenging (no shaping).

Several techniques for linearizing the DAC (not discussed further, see Schreier, 2005):

- Dual quantization
- Mismatch shaping
- Digital correction
**Integrator**

In the analysis so far, we have assumed an ideal integrator. Real integrators can only approximate the ideal integrator, because the amplifier has finite gain, bandwidth, offset, etc.

![Integrator Diagram]

**Finite gain**

Finite gain will shift the pole of the integrator from DC \((z = 1)\), to inside the unit circle (approximately \(z = 1 - 1 / A_0\)).

![Finite gain Diagram]
Finite bandwidth

Assuming the amplifier has one dominant pole and negligible non-dominant poles.

Must allow sufficient time for settling, the settling error is proportional to

\[ \varepsilon_b \propto e^{-\frac{\beta T}{2}\tau_d}, \beta = \frac{C_2}{C_1 + C_2} \]

Gain error due to bandwidth and passives introduce poles in both STF and NTF

2. order noise shaping

Introduce one more integrator to achieve better noise suppression (at low frequencies).

NTF is now a second order differentiator.
2. order noise shaping

Doubling the OSR improves the SNR by 2.5 bits. Compared to 1.5 bits for 1. order.

\[ \text{SNR} = 6.02N + 1.76 - 12.9 + 50 \log_{10} \text{OSR} \]

Higher order noise shaping

Noise shaping can be improved even further by using a 3. order (or higher) modulator.

Possible to design the gain of each integrator to shape the NTF.

Difficult to guarantee stability. Instead we can build a higher order modulator from a cascade of lower order modulators: Multi-stage noise shaping (MASH).
Multi-stage noise shaping

\[ Y_1(z) = H_{s1}(z)U(z) + H_{n1}(z)E_1(z) \]
\[ Y_2(z) = H_{s2}(z)E_1(z) + H_{n2}(z)E_2(z) \]
\[ Y(z) = H_1(z)Y_1(z) - H_2(z)Y_2(z) \]

Choose \( H_1(z) \) and \( H_2(z) \) such that \( E_1(z) \) is canceled.
\[ H_1(z)H_{n1}(z) - H_2(z)H_{s2}(z) = 0 \]
E.g. \( H_1(z) = kH_{s2}(z) \) and \( H_2(z) = kH_{n1}(z) \).
Multi-stage noise shaping

Cascading an $L$-th order and an $M$-th order modulator results in an overall $L + M$ order modulator, but prone to instability.

Non-ideal effects because $H_{s2}(z)$ and $H_{n1}(z)$ are in analog, while $H_{1}(z)$ and $H_{2}(z)$ are in digital. Imperfections in the analog circuitry (offset, gain, etc.) will deteriorate the noise suppression.

Oversampling DAC

- Interpolation increases the sampling rate
- $\Delta\Sigma$ modulator quantizes and shapes noise (digital integrator)
Resources

Schreier and Temes, *Understanding Delta-Sigma Data Converters*, IEEE Wiley, 2005