Outline

Barkhausen criterion

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Introduction

Oscillators are used for synchronizing computation in a digital system, timing the sampling in a data converter, carrier synthesis and LO in RF systems, etc ...

Introduction

Different applications have very different requirements on accuracy and stability (e.g. jitter in data converters, timing violations, BER, etc.)

Crystal oscillators are used for demanding applications. Excellent stability and frequency accuracy. Speed limitation and cost issues.
Feedback system

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{H(s)}{1 + H(s)}
\]

Usually, we want the feedback system (amplifier) to be stable (difficult to guarantee stability). Now we want to ensure sustained oscillation at a fixed frequency (also difficult).

Phase shift of 180 degrees at some frequency, \(\omega_0\), gives positive feedback. Each time the signal "goes around the loop". Amplifier input, \(V_x\), grows indefinitely if \(|H(j\omega_0)| > 1\)

\[
V_x = V_0 + |H(j\omega_0)|V_0 + |H(j\omega_0)|^2V_0 + |H(j\omega_0)|^3V_0 + \ldots
\]
Barkhausen criterion

The Barkhausen stability criterion is necessary but not sufficient for oscillation.

\[ |H(j\omega_0)| \geq 1 \]
\[ \angle H(j\omega_0) = \pi \]

The criteria for oscillation is not well understood, there is no known sufficient criteria for oscillation.

Oscillators

LC oscillator, inductor, \( L \), and capacitor, \( C \), to generate resonance

\[ \omega_0 \propto \frac{1}{\sqrt{LC}} \]

Used mostly for RF (inductors are expensive and impractical).

Relaxation oscillators typically relies on charging and discharging a capacitor. Some active circuit will monitor and switch charging at a threshold.
Ring oscillator

Ring oscillators are made from gain stages, or delay stages, in feedback.

A single CS stage in feedback will not oscillate, because it does not fulfill the Barkhausen criteria.

The CS stage is inverting (180°) and has one pole (90°), 270° phase shift in total.
Ring oscillator

Using two CS stages gives the required phase shift, but it is stable at either rail.

Still no sustained oscillation because the gain is much less than one when phase is inverted.
Ring oscillator

Three CS stages are enough for sustained oscillation provided the gain of each stage is sufficient (in this case, $A_0 \geq 2$).

Ring oscillator

If the gain of each stage is larger than necessary, $A_0 > 2$, the output will saturate and linear analysis becomes difficult.
**Ring oscillator**

The frequency of oscillation becomes $1 / (2^n \tau)$, where $n$ is the number of elements, and $\tau$ is the delay due to each element (inverter in this case).

**Fully differential oscillator**

Single ended oscillators are power efficient and capable of rail-to-rail output. However, as we now know, in mixed signal circuits there is supply and substrate noise which couples directly into the oscillator, or modulates its supply voltage. Causing undesirable fluctuations in the period time of the output signal.

Fully differential circuits have CMRR and PSRR to combat this!
Fully differential oscillator

The trip point for each stage is now the crossing of the inputs rather than a fraction of $V_{dd}$. Ideally, coupled noise will only affect the common mode. However, swing is not rail-to-rail.

In addition to rejecting coupling noise, the fully differential oscillator allows the number of stages to be even, which is a significant advantage if we need to generate a number of output phases.

Fully differential oscillator

Constant bias current.

In most cases, the resistors will be implemented by MOS transistors, requiring a bias circuit.
Symmetric load delay cell

Popular choice for implementing the fully differential delay cell.

The symmetric load approximates a voltage controlled resistor

Maneatis, JSSC, 1996

Next Generation Intel® Core™ Micro-Architecture (Nehalem) Clocking

Nasser Kurt, Member, IEEE, Praveen Mosalikanti, Mark Neidengard, Member, IEEE, Jonathan Douglas, and Rajesh Kumar

Low-Jitter Process-Independent DLL and PLL Based on Self-Biased Techniques

John G. Maneatis
Pseudo differential elements are common in many applications. Rail-to-rail swing. Trip point defined by $V_{dd}$ (worse CMRR).

Tuning output frequency

So far, the oscillators have a "fixed" output frequency. Deviation from the ideal output frequency is undesirable (modulated by the PVT condition, and perturbed by external and internal noise sources). VCOs have an input terminal that allows external control of the frequency.

Oscillator

Voltage controlled oscillator

\[ \omega_0 \rightarrow V_{ctl} \rightarrow \omega_0 + K_{VCO} V_{ctl} \]
Different schemes for controlling the output frequency.

- Modulating the driving strength
- Modulating the load

Control signal is usually a voltage (VCO) or a current (CCO). Sometimes a V/I converter is used to interface a CCO with a voltage signal.
Ring oscillator VCO

Several possibilities for implementing the delay stages and tuning circuit.

Starved inverter delay element

Starved inverter bias circuit
Ring oscillator VCO

Several specifications to consider

- Tuning range
- Linearity ($\omega_{\text{out}}$ vs. $V_{\text{ctl}}$)
- Amplitude
- Power
- CMRR, PSRR
- Jitter (phase noise)
- ...

Mathematical model

![Graph showing the relationship between time, $\phi$, and $\sin(\phi)$]
Mathematical model

Phase is not directly observable in a real oscillator. However, from observing the zero crossings of the output, we see when the phase has increased by $\pi$.

The rate of change of the phase, $\phi$, is the frequency, $\omega$.

Phase is the integral of frequency. Conversely, frequency is the derivative of the phase.

\[
V_{\text{out}}(t) = V_m \cos \left( \int \omega_{\text{out}}(t) \, dt \right) = V_m \cos \left( \omega_0 t + K_{VCO} \int V_{\text{ctl}}(t) \, dt \right)
\]

\[
\phi_e = K_{VCO} \int V_{\text{ctl}}(t) \, dt
\]

\[
H_{VCO}(s) = \frac{\phi_e}{V_{\text{ctl}}} = \frac{K_{VCO}}{s}
\]