

INF5110 – Compiler Construction

Semantic analysis

Spring 2016



1. Semantic analysis

Intro

Attribute grammars

Rest

1. Semantic analysis

Intro

Attribute grammars

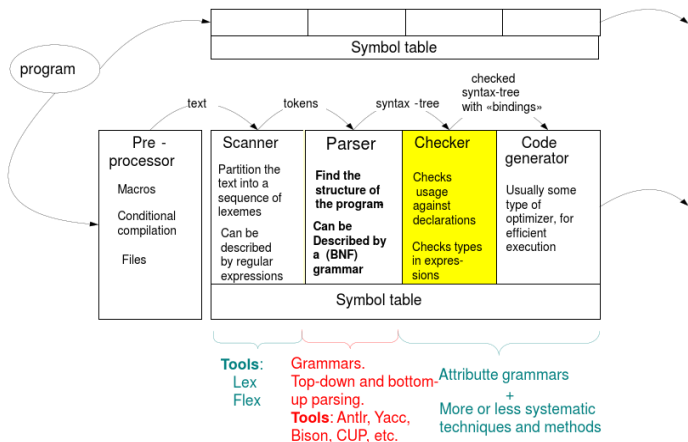
Rest

Overview over the chapter^a

^aSlides originally from Birger Møller-Pedersen

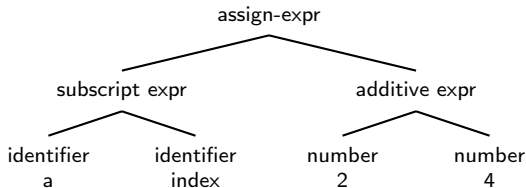
- semantics analysis in general
- attribute grammars
- symbol tables (not today)
- data types and type checking (not today)

Where are we now?



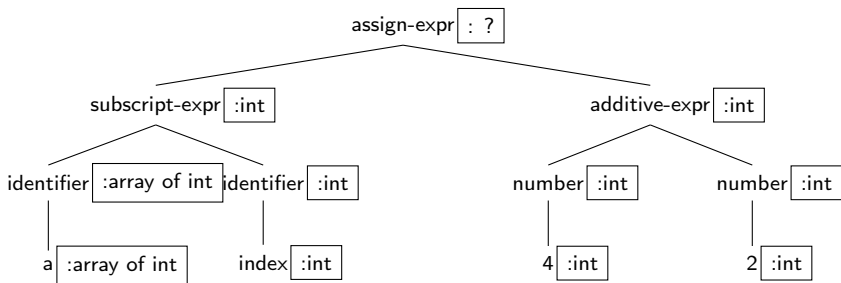
What do we get from the parser?

- output of the parser: (abstract) syntax tree
- often: in anticipation: nodes in the tree contain “space” to be filled out by SA
- examples:
 - for expression nodes: *types*
 - for identifier/name nodes: reference or pointer to the *declaration*



What do we get from the parser?

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 - for expression nodes: *types*
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Rule of thumb

Check everything which is possible *before* executing (run-time vs. compile-time), but cannot already done during lexing/parsing (syntactical vs. semantical analysis)

- Goal: fill out “semantic” info (typically in the AST)
- typically:
 - *names declared?* (somewhere/uniquely/before use)
 - *typing:*
 - declared type consistent with use
 - types of (sub)-expression consistent with used operations
- *border* between sematical vs. syntactic checking not always 100% clear
 - if a then ...: checked for syntax
 - if a + b then ...: semantical aspects as well?

SA is necessarily approximative

- note: not all can (precisely) be checked at compile-time¹
 - division by zero?
 - “array out of bounds”
 - “null pointer deref” (like `r.a`, if `r` is null)
- but note also: *exact* type cannot be determined statically either

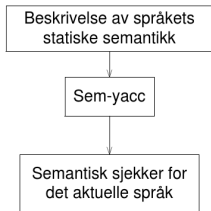
```
if x then 1 else "abc"
```

- statically: ill-typed^a
- dynamically (“run-time type”): string or int, or run-time type error, if `x` turns out not to be a boolean, or if it’s null

^aUnless some fancy behind-the-scenes type conversions are done by the language (the compiler). Perhaps `print(if x then 1 else "abc")` is accepted, and integer `1` is implicitly converted to `"1"`.

¹For fundamental reasons (cf. also Rice’s theorem). Note that *approximative* checking is doable, resp. that’s what the SA is doing anyhow. ☰

A dream



However

- no standard description language
- no standard “theory” (apart from the too general “context sensitive languages”)
 - part of SA may seem ad-hoc, more “art” than “engineering”, complex
- *but*: well-established/well-founded (and decidedly non-ad-hoc) fields do exist
 - *type systems*, type checking
 - *data-flow* analysis
- in general
 - semantic “rules” must be individually specified and implemented per language
 - rules: defined based on trees (for AST): often straightforward to implement
 - clean language design includes *clean semantic rules*

1. Semantic analysis

Intro

Attribute grammars

Rest

Attribute

- a “property” or characteristic feature of something
- here: of language “constructs”. More specific in this chapter:
- of syntactic elements, i.e., for non-terminals/terminal nodes in syntax trees

Static vs. dynamic

- distinction between **static** and *dynamic attributes*
- association attribute \leftrightarrow element: *binding*
- *static* attributes: possible to determine at/determined at compile time
- dynamic attributes: the others ...

- data *type* of a variable : static/dynamic
- *value* of an expression: dynamic (but seldomly static as well)
- *location* of a variable in memory: typically dynamic (but in old FORTRAN: static)
- *object-code*: static (but also: dynamic loading possible)

Attribute grammar in a nutshell

- AG: general formalism to bind “attributes to trees” (where trees are given by a CFG)²
- two potential ways to calculate “properties” of nodes in a tree:

“Synthesize” properties

define/calculate prop's *bottom-up*

- allows both *at the same time*

“Inherit” properties

define/calculate prop's *top-down*

Attribute grammar

CFG + **attributes** one grammar symbols + **rules** specifying for each production, how to determine attributes

- *evaluation* of attributes: requires some thought, more complex if mixing bottom-up + top-down dependencies

²attributes in AG's: *static*, obviously.

Example: evaluation of numerical expressions

Expression grammar (similar as seen before)

$$\begin{aligned} \text{exp} &\rightarrow \text{exp} + \text{term} \mid \text{exp} - \text{term} \mid \text{term} \\ \text{term} &\rightarrow \text{term} * \text{factor} \mid \text{factor} \\ \text{factor} &\rightarrow (\text{exp}) \mid \text{number} \end{aligned}$$

- goal now: **evaluate** a given expression, i.e., the syntax tree of an expression, resp:

more concrete goal

Specify, in terms of the grammar, how expressions are evaluated

- grammar: describes the “format” or “shape” of (syntax) trees
- syntax-directedness
- value of (sub-)expressions: *attribute* here³

³stated earlier: values of syntactic entities are generally *dynamic* attributes and cannot therefore be treated by an AG. In this AG example it's statically doable (because no variables, no state-change etc).

Expression evaluation: how to do it on one's own?

- simple problem, easy solvable without having heard of AGs
- given an expression, in the form of a syntax tree
- evaluation:
 - simple bottom-up calculation of values
 - the value of a compound expression (parent node) **determined by the value of its subnodes**
 - realizable, for example by a simple recursive procedure⁴

Connection to AG's

- AGs: basically a formalism to specify things like that
- *however*: general AGs will allow *more complex* calculations:
 - not just **bottom up** calculations like here but also
 - **top-down**, including both at the same time^a

^atop-down calculation will not be needed for the simple expression evaluation example.

⁴resp. a number of mutually recursive procedures, one for factors, one for terms etc. See next slide

Pseudo code for evaluation

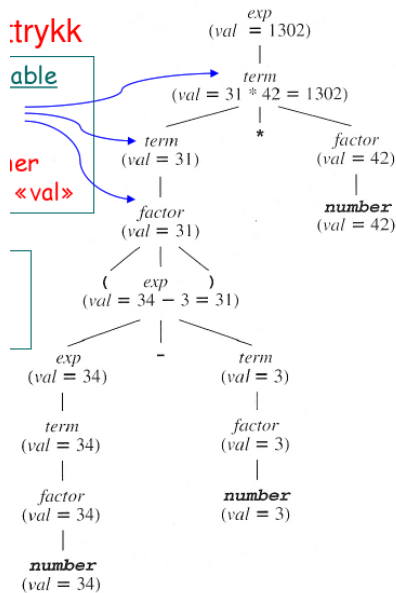
```
eval_exp(e) =  
  case  
  :: e equals PLUSnode →  
    return eval_exp(e.left) + eval_term(e.right)  
  :: e equals MINUSnode →  
    return eval_exp(e.left) - eval_term(e.right)  
  ...  
end case
```

AG for expression evaluation

	productions/grammar rules	semantic rules
1	$exp_1 \rightarrow exp_2 + term$	$exp_1.val \leftarrow exp_2.val + term.val$
2	$exp_1 \rightarrow exp_2 - term$	$exp_1.val \leftarrow exp_2.val - term.val$
3	$exp \rightarrow term$	$exp.val \leftarrow term.val$
4	$term_1 \rightarrow term_2 * factor$	$term_1.val \leftarrow term_2.val * factor.val$
5	$term \rightarrow factor$	$term.val \leftarrow factor.val$
6	$factor \rightarrow (exp)$	$factor.val \leftarrow exp.val$
7	$factor \rightarrow \mathbf{number}$	$factor.val \leftarrow \mathbf{number.val}$

- specific for this example
 - only one attribute (for all nodes), in general: different ones possible
 - (related to that): only one semantic rule per production
 - as mentioned: rules here define values of attributes
“bottom-up” only
- note: subscripts on the symbols for disambiguation (where needed)

Attributed parse tree



First observations concerning the example AG

- attributes
 - defined per grammar symbol (mainly non-terminals), but
 - get their values “per node”
 - notation $exp.val$
 - if one wants to be precise: val is an attribute of non-terminal exp (among others), val in an *expression-node* in the tree is an *instance* of that attribute
 - instance *not=* the *value*!

Semantic rules

- aka: attribution rule
- fix for each symbol X : **set of attributes**⁵
- attribute: intended as “fields” in the nodes of syntax trees
- notation: $X.a$: attribute a of symbol X
- but: attribute obtain values *not* per symbol, but per node in a tree (per instance)

Semantic rule for production $X_0 \rightarrow X_1 \dots X_n$

$$X_i.a_j \leftarrow f_{ij}(X_0.a_1, \dots, X_0.a_k, X_1.a_1, \dots, X_1.a_k, \dots, X_n.a_1, \dots, X_n.a_k)$$

- X_i on the left-hand side: **not** necessarily head symbol of the production X_0
- evaluation example: more restricted (making example simple)

⁵different symbols may share same attribute with the same name. Those may have different types but the type of an attribute per symbol is uniform. Cf. fields in classes (and objects).

Subtle point (forgotten by Louden): terminals

- terminals: can have attributes, yes,
- but looking carefully at the format of semantic rules: *not really* specified how terminals get values to their attribute (apart from *inheriting them*)
- dependencies for terminals
 - attributes of terminals: get value from the token, especially the *token value*
 - terminal nodes: commonly not allowed to depend on parents, siblings.
- i.e., commonly: only attributes “synthesized” from the corresponding token allowed.
- note: without allowing “importing” values from the **number** token to the **number.val**-attributes, the *evaluation* example would not work

Attribute dependencies and dependence graph

$$X_i.a_j \leftarrow f_{ij}(X_0.a_1, \dots, X_0.a_k, X_1.a_1, \dots, X_1.a_k, \dots, X_n.a_1, \dots, X_n.a_k)$$

- sem. rule: expresses **dependence** of attribute $X_i.a_j$ *on the left* on all attributes $Y.b$ *on the right*
- dependence of $X_i.a_j$
 - in principle, $X_i.a_j$: may depend on all attributes for all X_k of the production
 - but typically: *dependent* only on a subset

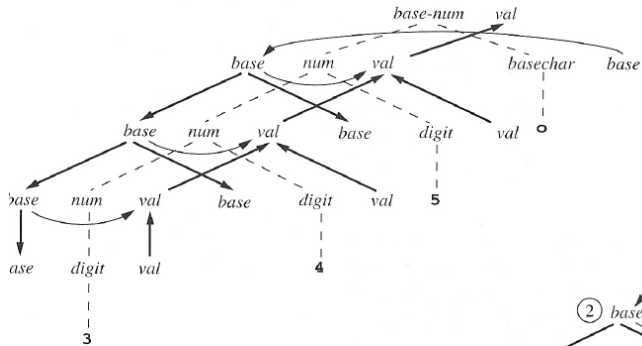
Possible dependencies (> 1 rule per production possible)

- parent attribute on children attributes
- attribute in a node dependent on other attribute of the node
- child attribute on parent attribute
- sibling attribute on sibling attribute
- mixture of all of the above at the same time
- but: no immediate dependence across generations

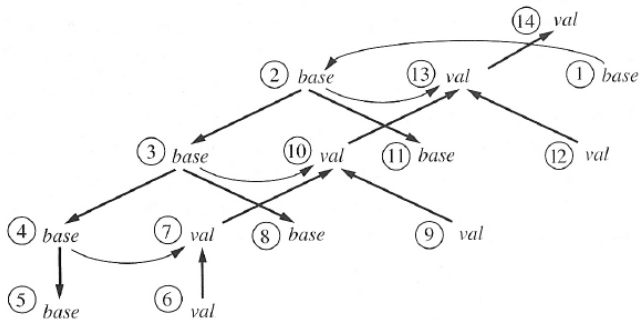
Attribute dependence graph

- dependencies ultimate between attributes in a syntax tree (instances) not between grammar symbols as such
- ⇒ attribute dependence graph (per syntax tree)
- complex dependencies possible:
 - evaluation complex
 - invalid dependencies possible, if not careful (especially **cyclic**)

Sample dependence graph (for later example)



Possible evaluation order



Restricting dependencies

- general GAs allow basically any kind of dependencies⁶
- complex/impossible to meaningfully evaluate (or understand)
- typically: restrictions, disallowing “mixtures” of dependencies
 - fine-grained: per attribute
 - or coarse-grained: for the whole attribute grammar

Synthesized attributes

bottom-up dependencies only
(same-node dependency allowed).

Inherited attributes

top-down dependencies only
(same-node and sibling dependencies allowed)

⁶apart from immediate cross-generation.

Synthesized attributes

Synthesized attribute

A **synthetic** attribute is defined wholly in terms of the node's *own* attributes, and those of its *children* (or constants).

Rule format for synth. attributes

For a **synthesized** attribute s of non-terminal A , *all* semantic rules with $A.s$ on the left-hand side must be of the form

$$A.s \leftarrow f(A.a, X_1.b_1, \dots, X_n.b_k)$$

and where the semantic rule belongs to production $A \rightarrow X_1 \dots X_n$

- Slight simplification in the formula: only 1 attribute per symbol. In general, instead depend on $A.a$ only, dependencies on $A.a_1, \dots, A.a_l$ possible. Similarly for the rest of the formula

S-attributed grammar:

all attributes are synthetic

Remarks on the definition of synthesized attributes

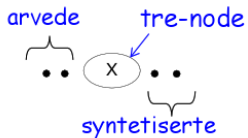
- Note the following aspects
 1. a synthesized attribute in a symbol: cannot *at the same time also* be “inherited”.
 2. a synthesized attribute:
 - depends on attributes of children (and other attributes of the same node) only. However:
 - those attributes need *not* themselves be *synthesized* (see also next slide)
- in Louden:
 - he does not allow “intra-node” dependencies
 - he assumes (in his wordings): attributes are “globally unique”

“Transitive” definition

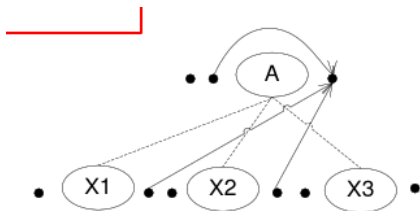
$$A.s \leftarrow f(A.i_1, \dots, A.i_m, X_1.s_1, \dots, X_n.s_k)$$

- in the rule: the $X_i.s_j$'s synthesized, the $A_i.i_j$'s inherited
- interpret the rule *carefully*: it says:
 - it's *allowed* to have synthesized & inherited attributes for A
 - it does **not** say: attributes in A *have to* be inherited (the X_i 's can be A as well)
 - it says: in A -node in the tree: a synthesized attribute
 - can depend on inherited att's in the same node and
 - on synthesized A -attributes of A -children-nodes

Conventional depiction



General synthesized attributes



Inherited attribute

An **inherited** attribute is defined wholly in terms of the node's *own* attributes, and those of its *siblings* or its *parent* node (or constants).

Rule format for inh. attributes

For an **inherited** attribute of a symbol X , *all* semantic rules mentioning $X.i$ on the left-hand side must be of the form

$$X.i \leftarrow f(A.a, X_1.b_1, \dots, X, \dots X_n.b_k)$$

and where the semantic rule belongs to production

$$A \rightarrow X_1 \dots X, \dots X_n$$

Alternative definition

Rule format

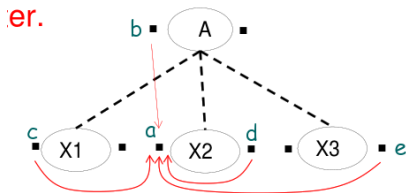
For an **inherited** attribute of a symbol X , *all* semantic rules mentioning $A.i$ on the left-hand side must be of the form

$$X.i \leftarrow f(A.i', X_1.b_1, \dots, X, \dots X_n.b_k)$$

and where the semantic rule belongs to production

$$A \rightarrow X_1 \dots X, \dots X_n$$

- additional requirement: $A.i'$ *inherited*
- rest of the attributes: inherited or synthesized



Simplistic example (normally done by the scanner)

- not only done by the scanner, but relying on built-in function of the implementing programming language...

CFG

$number \rightarrow numberdigit \mid digit$
 $digit \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \mid$

Attributes (just synthesized)

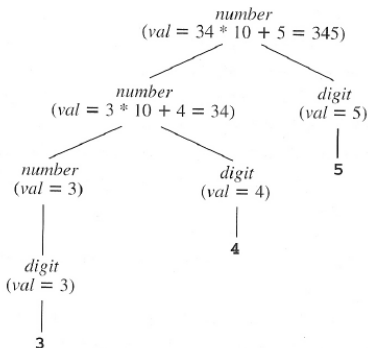
<i>number</i>	<i>val</i>
<i>digit</i>	<i>val</i>
terminals	<i>none</i>

Numbers: Attribute grammar and attributed tree

A-grammar

Grammar Rule	Semantic Rules
$number_1 \rightarrow number_2 digit$	$number_1.val = number_2.val * 10 + digit.val$
$number \rightarrow digit$	$number.val = digit.val$
$digit \rightarrow 0$	$digit.val = 0$
$digit \rightarrow 1$	$digit.val = 1$
$digit \rightarrow 2$	$digit.val = 2$
$digit \rightarrow 3$	$digit.val = 3$
$digit \rightarrow 4$	$digit.val = 4$
$digit \rightarrow 5$	$digit.val = 5$
$digit \rightarrow 6$	$digit.val = 6$
$digit \rightarrow 7$	$digit.val = 7$
$digit \rightarrow 8$	$digit.val = 8$
$digit \rightarrow 9$	$digit.val = 9$

A-tree



Attribute evaluation: works on trees

i.e.: works equally well for

- *abstract syntax trees*
- *ambiguous* grammars

Seriously ambiguous expression grammar^a

^aalternatively: grammar describing nice and cleans ASTs for an underlying, potentially less nice grammar used for parsing.

$$\text{exp} \rightarrow \text{exp} + \text{exp} \mid \text{exp} - \text{exp} \mid \text{exp} * \text{exp} \mid (\text{exp}) \mid \text{number}$$

Evaluation: Attribute grammar and attributed tree

A-grammar

Grammar Rule

Semantic Rules

$exp_1 \rightarrow exp_2 + exp_3$

$exp_1.val = exp_2.val + exp_3.val$

$exp_1 \rightarrow exp_2 - exp_3$

$exp_1.val = exp_2.val - exp_3.val$

$exp_1 \rightarrow exp_2 * exp_3$

$exp_1.val = exp_2.val * exp_3.val$

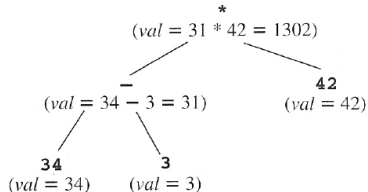
$exp_1 \rightarrow (exp_2)$

$exp_1.val = exp_2.val$

$exp \rightarrow \mathbf{number}$

$exp.val = \mathbf{number}.val$

A-tree



Expressions: generating ASTs

Expression grammar with precedences & assoc.

$exp \rightarrow exp + term \mid exp - term \mid term$
 $term \rightarrow term * factor \mid factor$
 $factor \rightarrow (exp) \mid number$

Attributes (just synthesized)

$exp, term, factor$	tree
number	lexval

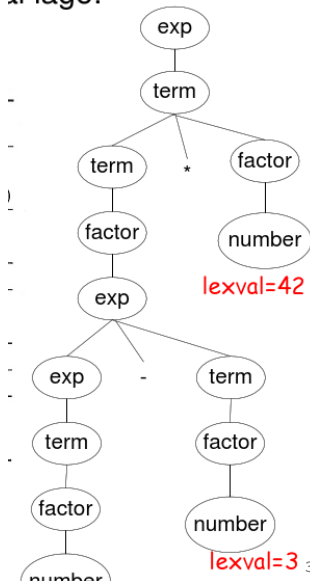
Expressions: Attribute grammar and attributed tree

A-grammar

Grammar Rule	Semantic Rules
$exp_1 \rightarrow exp_2 + term$	$exp_1.tree =$ $mkOpNode(+, exp_2.tree, term.tree)$
$exp_1 \rightarrow exp_2 - term$	$exp_1.tree =$ $mkOpNode(-, exp_2.tree, term.tree)$
$exp \rightarrow term$	$exp.tree = term.tree$
$term_1 \rightarrow term_2 * factor$	$term_1.tree =$ $mkOpNode(*, term_2.tree, factor.tree)$
$term \rightarrow factor$	$term.tree = factor.tree$
$factor \rightarrow (exp)$	$factor.tree = exp.tree$
$factor \rightarrow \mathbf{number}$	$factor.tree =$ $mkNumNode(\mathbf{number}.lexval)$

A-tree

all nodes.



Example: type declarations for variable lists

CFG

```
decl  →  type var-list
type  →  int
type  →  float
var-list1 → id , var-list2
var-list  →  id
```

- Goal: attribute type information to the syntax tree
- *attribute*: dtype (with values *integer* and *real*)⁷
- complication: “top-down” information flow: type declared for a list of vars \Rightarrow **inherited** to the elements of the list

⁷There are thus 2 different values. We don't mean “the attribute dtype has integer values”, like 0, 1, 2, ...

Types and variable lists: inherited attributes

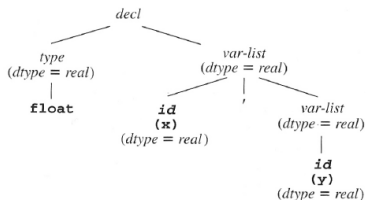
grammar productions		semantic rules	
<i>decl</i>	→ <i>type var-list</i>	<i>var-list</i> .dtype	← <i>type</i> .dtype
<i>type</i>	→ int	<i>type</i> .dtype	← <i>integer</i>
<i>type</i>	→ float	<i>type</i> .dtype	← <i>real</i>
<i>var-list</i> ₁	→ id , <i>var-list</i> ₂	id .dtype	← <i>var-list</i> ₁ .dtype
		<i>var-list</i> ₂ .dtype	← <i>var-list</i> ₁ .dtype
<i>var-list</i>	→ id	id .dtype	← <i>var-list</i> .dtype

- **inherited**: attribute for **id** and *var-list*
- but also *synthesized* use of attribute dtype: for *type* .dtype⁸

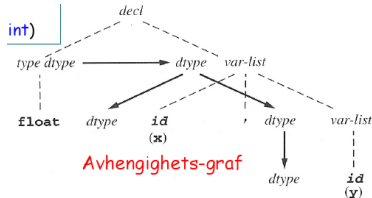
⁸Actually, it's conceptually better not to think of it as "the attribute dtype", it's better as "the attribute dtype of non-terminal *type*" (written *type* .dtype) etc. Note further: *type* .dtype is *not* yet what we called *instance* of an attribute.

float id(x), id(y)

Attributed parse tree



Dependence graph



Example: Based numbers (octal & decimal)

- remember: grammar for numbers (in decimal notation)
- evaluation: synthesized attributes
- now: *generalization* to numbers with decimal and octal notation

CFG

based-num → *num base-char*

base-char → **o**

base-char → **d**

num → *num digit*

num → *digit*

digit → **0**

digit → **1**

...

digit → **7**

digit → **8**

digit → **9**

Attributes

- *based-num*.val: synthesized
- *base-char*.base: synthesized
- for *num*:
 - *num*.val: synthesized
 - *num*.base: **inherited**
- *digit*.val: synthesized

- **9** is not an octal character

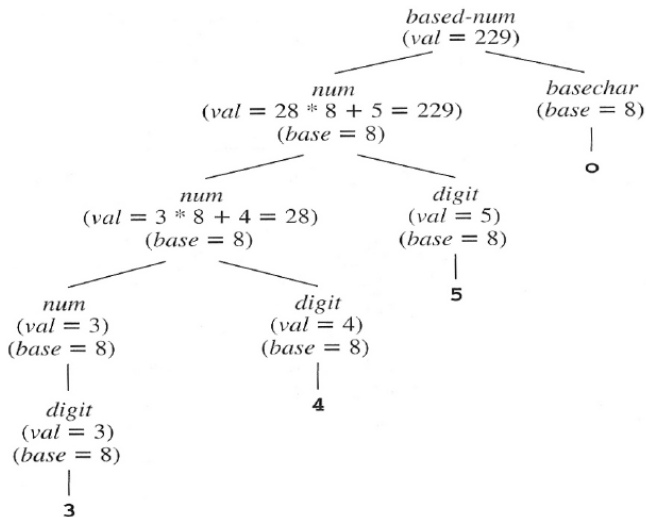
⇒ attribute val may get value “error”!

Based numbers: a-grammar

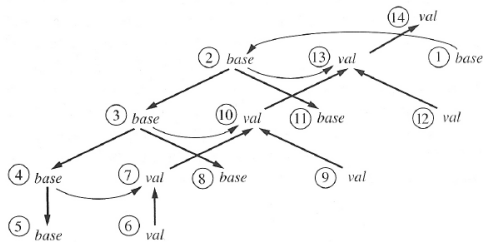
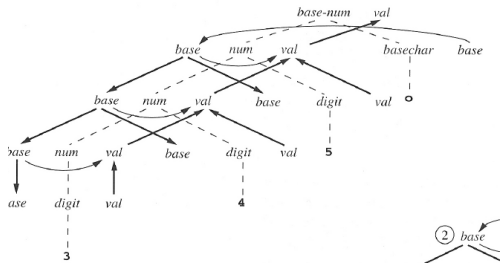
Grammar Rule	Semantic Rules
$based_num \rightarrow num\ basechar$	$based_num.val = num.val$ $num.base = basechar.base$
$basechar \rightarrow 0$	$basechar.base = 8$
$basechar \rightarrow d$	$basechar.base = 10$
$num_1 \rightarrow num_2\ digit$	$num_1.val =$ if $digit.val = error$ or $num_2.val = error$ then $error$ else $num_2.val * num_1.base + digit.val$ $num_2.base = num_1.base$ $digit.base = num_1.base$
$num \rightarrow digit$	$num.val = digit.val$ $digit.base = num.base$
$digit \rightarrow 0$	$digit.val = 0$
$digit \rightarrow 1$	$digit.val = 1$
...	...
$digit \rightarrow 7$	$digit.val = 7$
$digit \rightarrow 8$	$digit.val =$ if $digit.base = 8$ then $error$ else 8
$digit \rightarrow 9$	$digit.val =$ if $digit.base = 8$ then $error$ else 9

Based numbers: after eval of the semantic rules

Attributed syntax tree



Based nums: Dependence graph & possible evaluation order



Dependence graph & evaluation

- **evaluation order** must respect the edges in the *dependence graph*
- *cycles* must be avoided!
- directed acyclic graph (DAG)⁹
- dependence graph \sim partial order
- *topological sorting*: turning a partial order to a total/linear order (which is consistent with the PO)
- *roots* in the dependence graph (**not** *the* root of the syntax tree): they value must come “from outside” (or constant)
- often (and sometimes required): terminals in the syntax tree:
 - terminals *synthesized* / *not inherited*
 - ⇒ terminals: *roots* of dependence graph
 - ⇒ get their value from the parser (token value)

⁹it's not a tree. It may have more than one “root” (like a forest). Also: “shared descendents” are allowed. But no cycles.

Evaluation: parse tree method

For acyclic dependence graphs: possible “naive” approach

Parse tree method

Linearize the given partial order into a total order (topological sorting), and then simply evaluate the equations following that.

- works only if *all* dependence graphs of the AG are acyclic
- acyclicity of the dependence graphs?
 - decidable for given AG, but computationally expensive¹⁰
 - don't use general AGs but: restrict yourself to subclasses
- disadvantage of parse tree method: also not very efficient check per parse tree

¹⁰On the other hand: needs to be one only once.

Observation on the example: Is evaluation (uniquely) possible?

- all attributes: *either* inherited *or* synthesized¹¹
- all attribute: must actually be defined (by some rule)
- guaranteed in that for every production:
 - all *synthesized* attributes (on the left) are defined
 - all *inherited* attributes (on the right) are defined
 - local loops forbidden
- since all attributes are either inherited or synthesized: each attribute in any parse tree: defined, and defined only *one* time (i.e., *uniquely defined*)

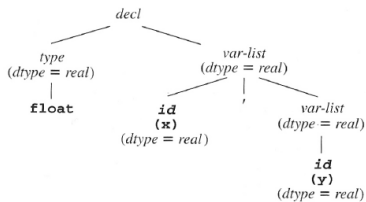
¹¹*base-char*.base (synthesized) considered different from *num*.base (inherited)

- a-grammars: allow to specify grammars where (some) parse-trees have cycles.
- however: loops intolerable for *evaluation*
- difficult to check (exponential complexity).¹²

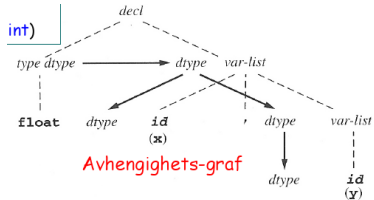
¹²acyclicity checking for a *given* dependence graph: not so hard (e.g., using topological sorting). Here: for *all* syntax trees.

Variable lists (repeated)

Attributed parse tree



Dependence graph



- code assume: tree given¹³

```
procedure EvalType ( T: treenode ); var-list → id
begin
  case nodekind of T of
    decl:
      EvalType ( type child of T );
      Assign dtype of type child of T to var-list child of T;
      EvalType ( var-list child of T );
    type:
      if child of T = int then T.dtype := integer
      else T.dtype := real;
    var-list:
      assign T.dtype to first child of T;
      if third child of T is not nil then
        assign T.dtype to third child;
        EvalType ( third child of T );
      end case;
end EvalType;
```

Dette er
også
skrevet ut
som et
program i
boka!

¹³reasonable, if AST. For parse-tree, the attribution of types must deal with the fact that the parse tree is being built during parsing. It also means: “blur” border between context-free and context-sensitive analysis

L-attributed grammars

- goal: attribute grammar suitable for “on-the-fly” attribution

Definition (L-attributed grammar)

An attribute grammar for attributes a_1, \dots, a_k is *L-attributed*, if for each inherited attribute a_j and each grammar rule

$$X_0 \rightarrow X_1 X_2 \dots X_n,$$

the associated equations for a_j are all of the form

$$X_i.a_j \leftarrow f_{ij}(X_0.\vec{a}, X_1.\vec{a} \dots X_{i-1}.\vec{a}).$$

where additionally for $X_0.\vec{a}$, only *inherited* attributes are allowed.

- $X.\vec{a}$: short-hand for $X.a_1 \dots X.a_k$
- Note S-attributed grammar \Rightarrow L-attributed grammar

Discussion

“Attribution” and LR-parsing

- easy (and typical) case: synthesized attributes
- for inherited attributes
 - not quite so easy
 - perhaps better: not “on-the-fly”
 - i.e., better postponed for later phase, when AST available.
- implementation: additional *value stack* for synthesized attributes, maintained “besides” the parse stack

Example of a value stack for synthesized attributes

	Parsing Stack	Input	Parsing Action	Value Stack	Semantic Action
1	\$	3*4+5 \$	shift	\$	
2	\$ n	*4+5 \$	reduce $E \rightarrow n$	\$ n	$E.val = n.val$
3	\$ E	*4+5 \$	shift	\$ 3	
4	\$ E *	4+5 \$	shift	\$ 3 *	
5	\$ E * n	+5 \$	reduce $E \rightarrow n$	\$ 3 * n	$E.val = n.val$
6	\$ E * E	+5 \$	reduce $E \rightarrow E * E$	\$ 3 * 4	$E_1.val =$ $E_2.val * E_3.val$
7	\$ E	+5 \$	shift	\$ 12	
8	\$ E +	5 \$	shift	\$ 12 +	
9	\$ E + n	\$	reduce $E \rightarrow n$	\$ 12 + n	$E.val = n.val$
10	\$ E + E	\$	reduce $E \rightarrow E + E$	\$ 12 + 5	$E_1.val =$ $E_2.val + E_3.val$
11	\$ E	\$		\$ 17	

Sample action

```
E : E + E { $$ = $1 + $3; }
```

in (classic) yacc notation

Value stack manipulation

```
pop t3 { get  $E_3.val$  from the value stack }
pop { discard the + token }
pop t2 { get  $E_2.val$  from the value stack }
t1 = t2 + t3 { add }
push t1 { push the result back onto the value stack }
```