Course Script

INF 5110: Compiler construction

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Contents

Chapter 3 Grammars

What is it about?

Learning Targets of this Chapter

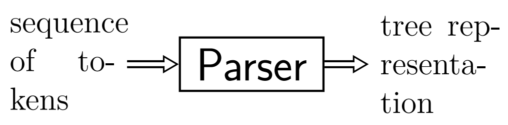
1. (context-free) grammars + BNF
2. ambiguity and other properties
3. terminology: tokens, lexemes,
4. different trees connected to grammars/parsing
5. derivations, sentential forms

The chapter corresponds to [2, Section 3.1–3.2] (or [3, Chapter 3]).

Contents

3.1 Introduction

Bird’s eye view of a parser



- *check* that the token sequence correspond to a *syntactically correct* program
  - if yes: yield *tree* as intermediate representation for subsequent phases
  - if not: give *understandable* error message(s)
- we will encounter various kinds of trees
  - derivation trees (derivation in a (context-free) grammar)
  - *parse* tree, *concrete syntax tree*
  - *abstract syntax trees*
- mentioned tree forms hang together, dividing line a bit fuzzy
- result of a parser: typically AST

(Context-free) grammars

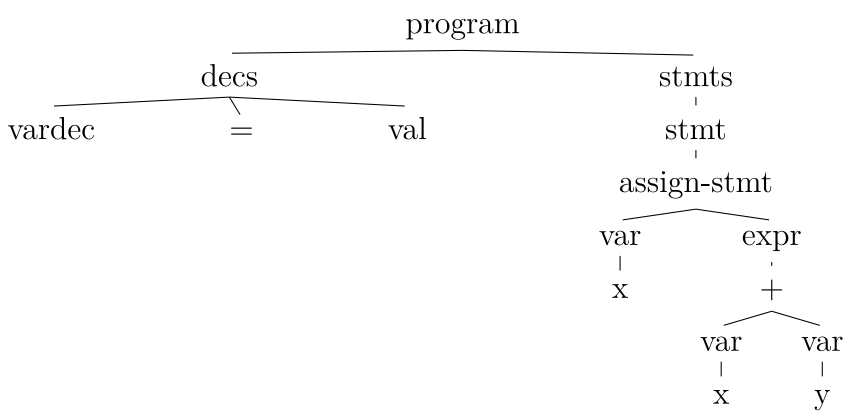
- specifies the *syntactic structure* of a language
- here: grammar means CFG
- G **derives** word w

Parsing

Given a stream of "symbols" w and a grammar G, find a *derivation* from G that prodices w

The slide talks about deriving "words". In general, words are finite sequences of symbols from a given alphabet (as was the case for regular languages). In the concrete picture of a parser, the words are sequences of *tokens*, which are the elements that come out of the scanner. A successful derivation leads to tree-like representations. There a various slightly different forms of trees connected with grammars and parsing, which we will later see in more detail; for now we just illustrated such tree-like structures, without distinguishing between (abstract) syntax trees and parse trees.

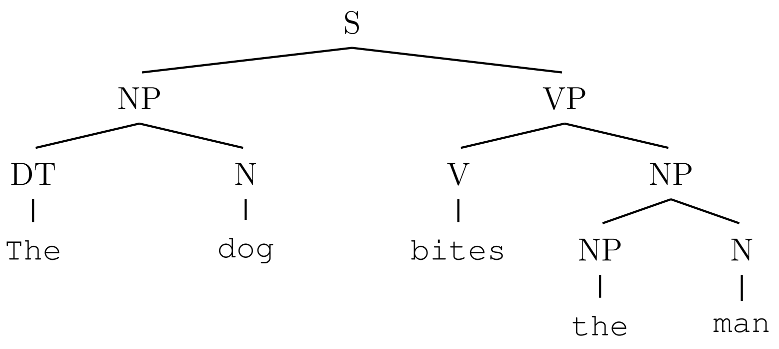
Sample syntax tree



Syntax tree

The displayed syntax tree is meant "impressionistic" rather then formal. Neither is it a sample syntax tree of a real programming language, nor do we want to illustrate for instance special features of an *abstract* syntax tree vs. a *concrete* syntax tree (or a parse tree). Those notions are closely related and corresponding trees might all looks similar to the tree shown. There might, however, be subtle conceptual and representational differences in the various classes of trees. Those are not relevant yet, at the beginning of the section.

Natural-language parse tree



"Interface" between scanner and parser

- remember: task of scanner = "chopping up" the input char stream (throw away white space etc) and *classify* the pieces (1 piece = *lexeme*)
- classified lexeme = **token**
- sometimes we use ⟨`integer`, "42"⟩
  - `integer`: "class" or "type" of the token, also called *token name*
  - "42" : *value of the token attribute* (or just value). Here: directly the *lexeme* (a string or sequence of chars)
- a note on (sloppyness/ease of) terminology: often: the token name is simply just called the token
- for (context-free) grammars: the *token (symbol)* corrresponds there to **terminal symbols** (or terminals, for short)

Token names and terminals

Remark 1 (Token (names) and terminals)**.** *We said, that sometimes one uses the name "token" just to mean token symbol, ignoring its value (like "42" from above). Especially, in the conceptual discussion and treatment of context-free grammars, which form the core of the specifications of a parser, the token value* is basically irrelevant. *Therefore, one simply identifies "tokens = terminals of the grammar" and silently ignores the presence of the values. In an* *implementation, and in lexer/parser generators, the value "42" of an integer-representing token must obviously* not *be forgotten, though ... The grammar maybe the core of the specification of the syntactical analysis, but the result of the scanner, which resulted in the lexeme "42" must nevertheless not be thrown away, it's only not really part of the parser's tasks.*

Notations

Remark 2. *Writing a compiler, especially a compiler front-end comprising a scanner and a parser, but to a lesser extent also for later phases, is about implementing representation of syntactic structures. The slides here don't* implement *a lexer or a parser or similar, but* describe *in a hopefully unambiguous way the principles of how a compiler front end works and is implemented. To describe that, one needs "language" as well, such as English language (mostly for intuitions) but also "mathematical" notations such as regular expressions, or in this section, context-free grammars. Those mathematical definitions have* themselves *a particular* syntax *one can see them as formal* domain-specific languages *to describe (other) languages. One faces therefore the (unavoidable) fact that one deals with two levels of languages: the language that is described (or at least whose* syntax *is described) and the language used to descibe that language. The situation is, of course, analogous when implementing a language: there is the language used to implement the compiler on the one hand, and the language for which the compiler is written for. For instance, one may choose to implement a C++-compiler in C. It may increase the confusion, if one chooses to write a C compiler in C .... Anyhow, the language for describing (or implementing) the language of interest is called the* meta-language*, and the other one described therefore just "the language".*

When writing texts or slides about such syntactic issues, typically one wants to make clear to the reader what is meant. One standard way are typographic *conventions, i.e., using specific typographic fonts. I am stressing "nowadays" because in classic texts in compiler construction, sometimes the typographic choices were limited.*

3.2 Context-free grammars and BNF notation

Grammars

- in this chapter(s): focus on **context-free grammars**
- thus here: grammar = CFG
- as in the context of regular expressions/languages: *language* = (typically infinite) set of words

- **grammar** = formalism to unambiguously specify a language
- intended language: all **syntactically correct** programs of a given progamming language

Slogan

A CFG describes the syntax of a programming language. [1]

Note: a compiler might reject some syntactically correct programs, whose violations *cannot* be captured by CFGs. That is done by *subsequent* phases. For instance, the type checker may reject syntactically correct programs that are ill-typed. The type checker is an important part from the *semantic phase* (or static analysis phase/. A typing discipline is *not* a syntactic property of a language (in that it cannot captured most commonly by a context-free grammar), it's therefore a "semantics" property.

Remarks on grammars

Sometimes, the word "grammar" is synonymously for context-free grammars, as CFGs are so central. However, context-sensitive and Turing-expressive grammars exists, both more expressive than CFGs. Also a restricted class of CFG correspond to regular expressions/languages. Seen as a grammar, regular expressions correspond so-called left-linear grammars (or alternativelty, right-linear grammars), which are a special form of context-free grammars.

Context-free grammar

Definition 3.2.1 (CFG)**.** A *context-free grammar* G is a 4-tuple $G = (\Sigma_T, \Sigma_N, S, P)$:

1. 2 disjoint finite alphabets of terminals Σ_T and
2. non-terminals Σ_N
3. 1 start-symbol $S \in \Sigma_N$ (a non-terminal)
4. productions P = finite subset of $\Sigma_N \times (\Sigma_N + \Sigma_T)^*$

- terminal symbols: corresponds to tokens in parser = basic building blocks of syntax
- non-terminals: (e.g. "expression", "while-loop", "method-definition" ...)
- grammar: generating (via "derivations") languages
- **parsing**: the *inverse* problem

$\Rightarrow$ CFG = specification

[1] And some say, regular expressions describe its microsyntax.

Further notions

- sentence and sentential form
- productions (or rules)
- derivation
- *language* of a grammar $\mathcal{L}(G)$
- parse tree

Those notions will be explained with the help of examples.

BNF notation

- popular & common format to write CFGs, i.e., describe context-free languages
- named after *pioneering* (seriously) work on Algol 60
- notation to write productions/rules + some extra meta-symbols for convenience and grouping

Slogan: Backus-Naur form

What regular expressions are for regular languages is BNF for context-free languages.

"Expressions" in BNF

$$\begin{array}{rcl} exp & \rightarrow & exp\ \ op\ \ exp \mid \mathbf{(}\ exp\ \mathbf{)} \mid \mathbf{number} \\ op & \rightarrow & \mathbf{+} \mid \mathbf{-} \mid \mathbf{*} \end{array} \tag{3.1}$$

- "$\rightarrow$" indicating productions and " $\mid$ " indicating alternatives [2]
- convention: terminals written **boldface**, non-terminals *italic*
- also simple math symbols like "+" and "(" are meant above as terminals
- start symbol here: *exp*
- remember: terminals like **number** correspond to tokens, resp. token classes. The attributes/token values are not relevant here.

[2]The grammar consists of 6 productions/rules, 3 for *expr* and 3 for *op*, the $\mid$ is just for convenience. Side remark: Often also ::= is used for $\rightarrow$.

Terminals

Conventions are not 100% followed, often bold fonts for symbols such as + or (are unavailable or not easily visible. The alternative using, for instance, boldface "identifiers" like **PLUS** and **LPAREN** looks ugly. Some books would write '+' and '('.

In a concrete parser implementation, in an object-oriented setting, one might choose to implement terminals as classes (resp. concrete terminals as instances of classes). in that case, a class name + is typically not available and the class might be named `Plus`. Later we will have a look at how to systematically implement terminals and non-terminals, and having a class `Plus` for a non-terminal '+' etc. is a systematic way of doing it (maybe not the most efficient one available though.)

Most texts don't follow conventions so slavishly and hope for an intuitive understanding by the educated reader, that + is a terminal in a grammar, as it's not a non-terminal, which are written here in *italics*.

Different notations

- BNF: notationally not 100% "standardized" across books/tools
- "classic" way (Algol 60):

```
<exp> ::=   <exp> <op> <exp>
        |   ( <exp> )
        | NUMBER
<op>  ::=  + | - | *
```

- Extended BNF (EBNF) and yet another style

$$\begin{array}{rcl} exp & \rightarrow & exp \;(\;"+"\;\mid\;"-"\;\mid\;"*"\;)\; exp \\ & \mid & "(" \; exp \; ")" \;\mid\; "number" \end{array} \tag{3.2}$$

- note: parentheses as terminals vs. as *metasymbols*

"Standard" BNF

Specific and unambiguous notation is important, in particular if you *implement* a concrete language on a computer. On the other hand: understanding the underlying concepts by *humans* is at least equally important. In that way, bureaucratically fixed notations may distract from the core, which is *understanding* the principles. XML, anyone? Most textbooks (and we) rely on simple typographic conventions (boldfaces, italics). For "implementations" of BNF specification (as in tools like yacc), the notations, based mostly on ASCII, cannot rely on such typographic conventions.

Syntax of BNF

BNF and its variations is a notation to describe "languages", more precisely the "syntax" of context-free languages. Of course, BNF notation, when exactly defined, is a language in itself, namely a domain-specific language to describe context-free languages. It may be instructive to write a grammar for BNF in BNF, i.e., using BNF as meta-language to describe BNF notation (or regular expressions). Is it possible to use regular expressions as meta-language to describe regular expression?

Different ways of writing the same grammar

- directly written as 6 pairs (6 rules, 6 productions) from $\Sigma_N \times (\Sigma_N \cup \Sigma_T)^*$, with "$\rightarrow$" as nice looking "separator":

$$\begin{array}{rcl} exp & \rightarrow & exp\ op\ exp \\ exp & \rightarrow & \textbf{(}\ exp\ \textbf{)} \\ exp & \rightarrow & \textbf{number} \\ op & \rightarrow & \textbf{+} \\ op & \rightarrow & \textbf{-} \\ op & \rightarrow & \textbf{*} \end{array} \tag{3.3}$$

- choice of non-terminals: irrelevant (except for human readability):

$$\begin{array}{rcl} E & \rightarrow & E\ O\ E \mid \textbf{(}\ E\ \textbf{)} \mid \textbf{number} \\ O & \rightarrow & \textbf{+} \mid \textbf{-} \mid \textbf{*} \end{array} \tag{3.4}$$

- still: we count 6 productions

Grammars as language generators

Deriving a word:

Start from start symbol. Pick a "matching" rule to rewrite the current word to a new one; repeat until *terminal* symbols, only.

- *non-deterministic* process
- rewrite relation for derivations:
  - one step rewriting: $w_1 \Rightarrow w_2$
  - one step using rule n: $w_1 \Rightarrow_n w_2$
  - many steps: $\Rightarrow^*$ etc.

By non-determinisic we mean the following. One can distinguish 2 forms of non-determinism here: 1) a sentential form contains (most often) more than one non-terminal. In that situation, one has the choice of expanding one non-terminal or the other. 2) Besides that, there may be more than one production or rule for a given non-terminal. Again, one has a choice.

As far as 1) is concerned. whether one expands one symbol or the other leads to different derivations, but won't lead to different *derivation trees* or *parse trees* in the end. Below, we impose a fixed discipline on *where* to expand. That leads to *left-most* or *right-most* derivations.

Language of grammar G

$$\mathcal{L}(G) = \{s \mid \mathit{start} \Rightarrow^* s \text{ and } s \in \Sigma_T^*\}$$

Example derivation for $(\mathbf{number}-\mathbf{number})*\mathbf{number}$

$$\begin{array}{rcl}
\underline{\mathit{exp}} & \Rightarrow & \underline{\mathit{exp}}\ \mathit{op}\ \mathit{exp} \\
 & \Rightarrow & (\underline{\mathit{exp}})\ \mathit{op}\ \mathit{exp} \\
 & \Rightarrow & (\underline{\mathit{exp}}\ \mathit{op}\ \mathit{exp})\ \mathit{op}\ \mathit{exp} \\
 & \Rightarrow & (\mathbf{n}\ \underline{\mathit{op}}\ \mathit{exp})\ \mathit{op}\ \mathit{exp} \\
 & \Rightarrow & (\mathbf{n}-\underline{\mathit{exp}})\ \mathit{op}\ \mathit{exp} \\
 & \Rightarrow & (\mathbf{n}-\mathbf{n})\underline{\mathit{op}}\ \mathit{exp} \\
 & \Rightarrow & (\mathbf{n}-\mathbf{n})*\underline{\mathit{exp}} \\
 & \Rightarrow & (\mathbf{n}-\mathbf{n})*\mathbf{n}
\end{array}$$

- underline the "place" were a rule is used, i.e., an *occurrence* of the non-terminal symbol is being rewritten/expanded
- here: *leftmost* derivation[3]

Rightmost derivation

$$\begin{array}{rcl}
\underline{\mathit{exp}} & \Rightarrow & \mathit{exp}\ \mathit{op}\ \underline{\mathit{exp}} \\
 & \Rightarrow & \mathit{exp}\ \underline{\mathit{op}}\ \mathbf{n} \\
 & \Rightarrow & \underline{\mathit{exp}}*\mathbf{n} \\
 & \Rightarrow & (\mathit{exp}\ \mathit{op}\ \underline{\mathit{exp}})*\mathbf{n} \\
 & \Rightarrow & (\mathit{exp}\ \underline{\mathit{op}}\ \mathbf{n})*\mathbf{n} \\
 & \Rightarrow & (\underline{\mathit{exp}}-\mathbf{n})*\mathbf{n} \\
 & \Rightarrow & (\mathbf{n}-\mathbf{n})*\mathbf{n}
\end{array}$$

[3]We'll come back to that later, it will be important.

- other ("mixed") derivations for the same word possible

Some easy requirements for reasonable grammars

- all symbols (terminals and non-terminals): should occur in a some word derivable from the start symbol
- words containing only non-terminals should be derivable
- an example of a silly grammar G (start-symbol A)

$$\begin{array}{rcl} A & \to & B\mathbf{x} \\ B & \to & A\mathbf{y} \\ C & \to & \mathbf{z} \end{array}$$

- $\mathcal{L}(G) = \emptyset$
- those "sanitary conditions": very minimal "common sense" requirements

Remark 3. *There can be further conditions one would like to impose on grammars besides the one sketched. A CFG that derives ultimately only 1 word of terminals (or a finite set of those) does not make much sense either. There are further conditions on grammar characterizing there usefulness for* parsing. *So far, we mentioned just some obvious conditions of "useless" grammars or "defects" in a grammer (like superfluous symbols). "Usefulness conditions" may refer to the use of ϵ-productions and other situations. Those conditions will be discussed when the lecture covers* parsing *(not just grammars).*

Remark 4 ("Easy" sanitary conditions for CFGs)**.** *We stated a few conditions to avoid grammars which technically qualify as CFGs but don't make much sense; there are easier ways to describe an empty set …*

There's a catch, though: it might not immediately be obvious that, for a given G, the question $\mathcal{L}(G) =^? \emptyset$ is decidable!

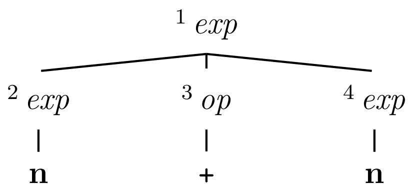
Whether a regular expression describes the empty language is trivially decidable immediately. Whether or not a finite state automaton descibes the empty language or not is, if not trivial, then at least a very easily decidable question. For context-sensitive *grammars (which are more expressive than CFG but not yet Turing complete), the emptyness question turns out to be undecidable. Also, other interesting questions concerning CFGs are, in fact, undecidable, like: given two CFGs, do they describe the same language? Or: given a CFG, does it actually describe a regular language? Most disturbingly perhaps: given a grammar, it's undecidable whether the grammar is ambiguous or not. So there are interesting and relevant properties concerning CFGs which are undecidable. Why that is, is not part of the pensum of this lecture (but we will at least have to deal with the important concept of grammatical ambiguity later). Coming back for the initial question: fortunately, the emptyness problem for CFGs* is *decidable.*

Questions concerning decidability may seem not too relevant at first sight. Even if some grammars can be constructed to demonstrate difficult questions, for instance related to decidability or worst-case complexity, the designer of a language will not intentionally try to achieve an obscure set of rules whose status is unclear, but hopefully strive to capture in a clear manner the syntactic principles of an equally hopefully clearly structured language. Nonetheless: grammars for real languages may become large and complex, and, even if conceptually clear, may contain unexpected bugs which makes them behave unexpectedly (for instance caused by a simple typo in one of the many rules).

In general, the implementor of a parser will often rely on automatic tools ("parser generators") which take as an input a CFG and turns it in into an implementation of a recognizer, which does the syntactic analysis. Such tools obviously can reliably and accurately help the implementor of the parser automatically only for problems which are decidable. *For undecidable problems, one could still achieve things automatically, provided one would compromise by not insisting that the parser always terminates (but that's generally is seen as unacceptable), or at the price of* approximative *answers. It should also be mentioned that parser generators typcially won't tackle CFGs in their* full generality *but are tailor-made for well-defined and well-understood subclasses thereof, where efficient recognizers are automaticlly generatable. In the part about parsing, we will cover some such classes.*

Parse tree

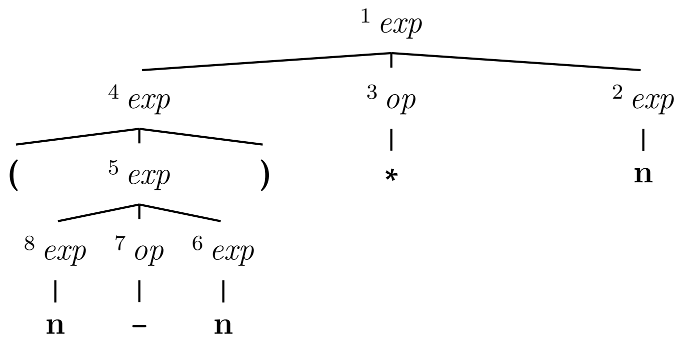
- derivation: if viewed as sequence of steps ⇒ linear "structure"
- order of individual steps: irrelevant
- ⇒ order not needed for subsequent steps
- **parse tree**: structure for the *essence* of derivation
- also called *concrete* syntax tree.[4]



- numbers in the tree
  - *not* part of the parse tree, indicate order of derivation, only
  - here: leftmost derivation

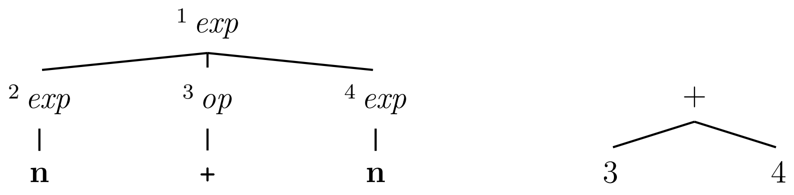
[4]There will be *abstract* syntax trees, as well.

Another parse tree (numbers for rightmost derivation)



Abstract syntax tree

- parse tree: contains still unnecessary details
- specifically: *parentheses* or similar, used for grouping
- tree-structure: can express the intended grouping already
- remember: tokens contain also attribute values (e.g.: full token for token class **n** may contain lexeme like ”`42`” . . .)

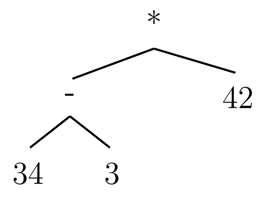


AST vs. CST

- **parse tree**
  - important *conceptual* structure, to talk about grammars and derivations. . . ,
  - most likely *not explicitly implemented* in a parser
- **AST** is a *concrete* data structure
  - important IR of the syntax (for the language being implemented)
  - written in the meta-language used in the implementation
  - therefore: nodes like + and 3 *are no longer (necessarily and directly) tokens or lexemes*
  - concrete data stuctures in the meta-language (C-structs, instances of Java classes, or what suits best)
  - the figure is meant schematic, only
  - produced by the parser, used by later phases
  - note also: we use 3 in the AST, where lexeme was `"3"`

⇒ at some point, the lexeme *string* (for numbers) is translated to a *number* in the meta-language (typically already by the lexer)

Plausible schematic AST (for the other parse tree)



- this AST: rather "simplified" version of the CST
- an AST closer to the CST (just dropping the parentheses): in principle nothing "wrong" with it either

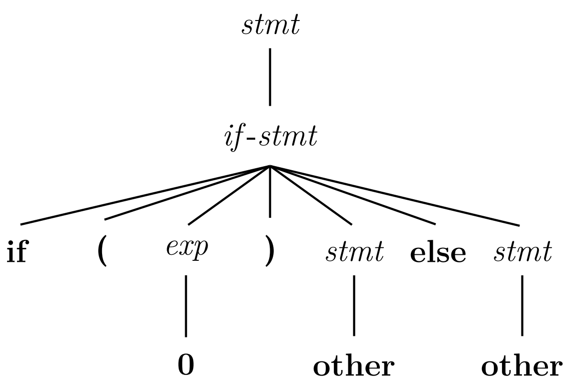
Conditionals

Conditionals G_1

$$\begin{array}{rcl} \mathit{stmt} & \rightarrow & \mathit{if\text{-}stmt} \mid \textbf{other} \\ \mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ \textbf{(}\ \mathit{exp}\ \textbf{)}\ \mathit{stmt} \\ & \mid & \textbf{if}\ \textbf{(}\ \mathit{exp}\ \textbf{)}\ \mathit{stmt}\ \textbf{else}\ \mathit{stmt} \\ \mathit{exp} & \rightarrow & \textbf{0} \mid \textbf{1} \end{array} \tag{3.5}$$

Parse tree

if (0) other else other



Another grammar for conditionals

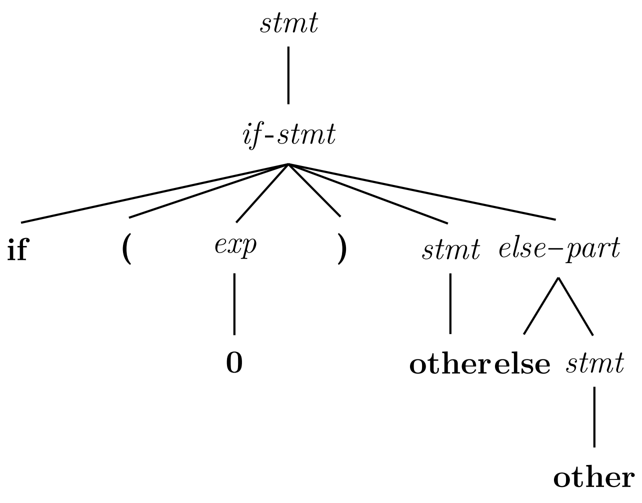
Conditionals G_2

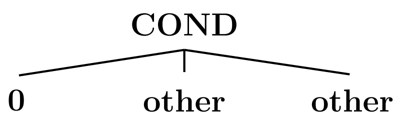
$$\begin{aligned} \mathit{stmt} &\to \mathit{if\text{-}stmt} \mid \textbf{other} \\ \mathit{if\text{-}stmt} &\to \textbf{if}\,\textbf{(}\,\mathit{exp}\,\textbf{)}\,\mathit{stmt}\;\mathit{else{-}part} \\ \mathit{else{-}part} &\to \textbf{else}\,\mathit{stmt} \mid \epsilon \\ \mathit{exp} &\to \textbf{0} \mid \textbf{1} \end{aligned} \tag{3.6}$$

Abbreviation

ϵ = empty word

A further parse tree + an AST





A potentially missing else part may be represented by null-“pointers” in languages like Java

3.3 Ambiguity

Before we mentioned some "easy" conditions to avoid "silly" grammars, without going into it. *Ambiguity* is more important and complex. Roughly speaking, a grammar is ambiguous, if there exist *sentences for which there are two different parse trees.* That's in general highly undesirable, as it means there are sentences with different syntactic interpretations (which therefore may ultimately interpreted differently). That is generally a no-no, but even *if* one would accept such a language definition, parsing would be problematic, as it would involve *backtracking* trying out different possible interpretations during parsing (which would also be a no-no for reasons of efficiency) In fact, later, when dealing with actual concrete parsing procedures, they cover certain *specific* forms of CFG (with names like LL(1), LR(1), etc.), which are in particular non-ambiguous. To say it differently: the fact that a grammar is parseable by some, say, LL(1) top-down parser (which does not do backtracking) implies directly that the grammar is unambiguous. Similar for the other classes we'll cover.

Note also: given an ambiguous grammar, it is often possible to find a *different* "equivalent" grammar that *is* unambiguous. Even if such reformulations are often possible, it's not guaranteed: there are context-free languages which do have an ambiguous grammar, but not unambigous one. In that case, one speaks of an ambiguous context-free language. We concentrate on *ambiguity* of grammars.

Tempus fugit ...

picture source: wikipedia

Ambiguous grammar

Definition 3.3.1 (Ambiguous grammar)**.** A grammar is *ambiguous* if there exists a word with *two different* parse trees.

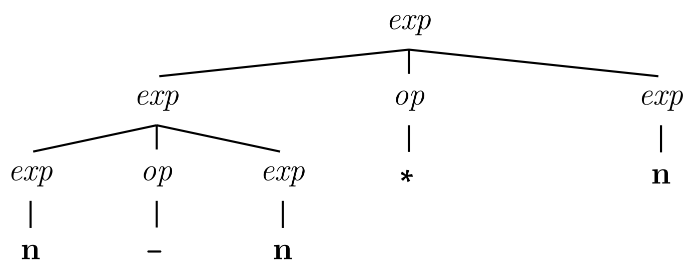
Remember grammar from equation (3.1):

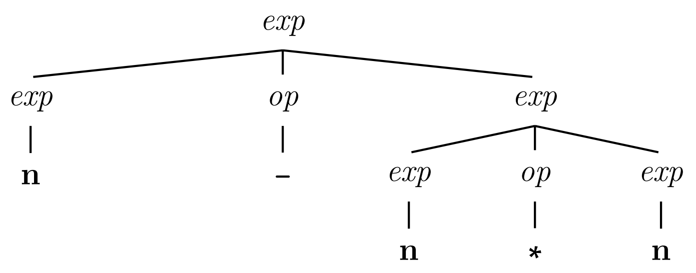
$$\begin{array}{rcl} \mathit{exp} & \rightarrow & \mathit{exp}\ \ \mathit{op}\ \ \mathit{exp} \mid \textbf{(}\, \mathit{exp}\, \textbf{)} \mid \textbf{number} \\ \mathit{op} & \rightarrow & \texttt{+} \mid \texttt{-} \mid \texttt{*} \end{array}$$

Consider:

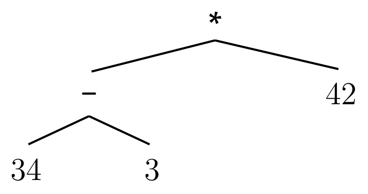
$$\mathbf{n} - \mathbf{n} * \mathbf{n}$$

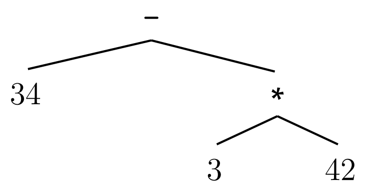
2 CTS's





2 resulting ASTs





different parse trees $\Rightarrow$ different[5] ASTs $\Rightarrow$ different[5] meaning

[5] At least in many cases.

Side remark: different meaning

The issue of "different meaning" may in practice be subtle: is $(x + y) - z$ the same as $x + (y - z)$? In principle yes, but what about MAXINT ?

Precendence & associativity

- one way to make a grammar unambiguous (or less ambiguous)
- for instance:

binary op's	precedence	associativity
$+, -$	low	left
$\times, /$	higher	left
$\uparrow$	highest	right

- $a \uparrow b$ written in standard math as a^b:

$$\begin{array}{ll} 5 + 3/5 \times 2 + 4 \uparrow 2 \uparrow 3 & = \\ 5 + 3/5 \times 2 + 4^{2^3} & = \\ (5 + ((3/5 \times 2)) + (4^{(2^3)})) \ . & \end{array}$$

- mostly fine for *binary* ops, but usually also for unary ones (postfix or prefix)

Unambiguity without imposing explicit associativity and precedence

- removing ambiguity by reformulating the grammar
- **precedence** for op's: *precedence cascade*
  - some bind stronger than others (`*` more than `+`)
  - introduce separate *non-terminal* for each precedence level (here: terms and factors)

Expressions, revisited

- *associativity*
  - *left*-assoc: write the corresponding rules in *left-recursive* manner, e.g.:

$$exp \to exp\ addop\ term \ \mid\ term$$

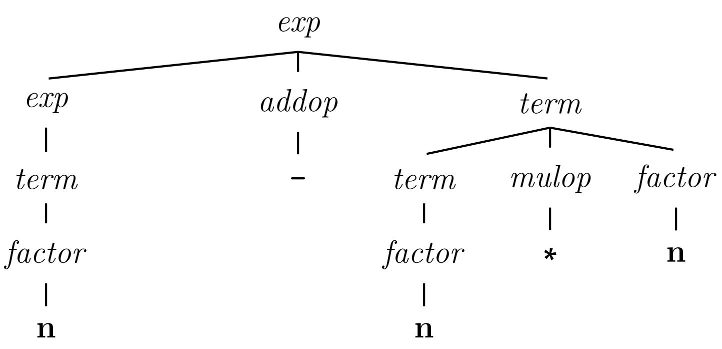
  - *right*-assoc: analogous, but right-recursive
  - *non*-assoc:

$$exp \to term\ addop\ term \ \mid\ term$$

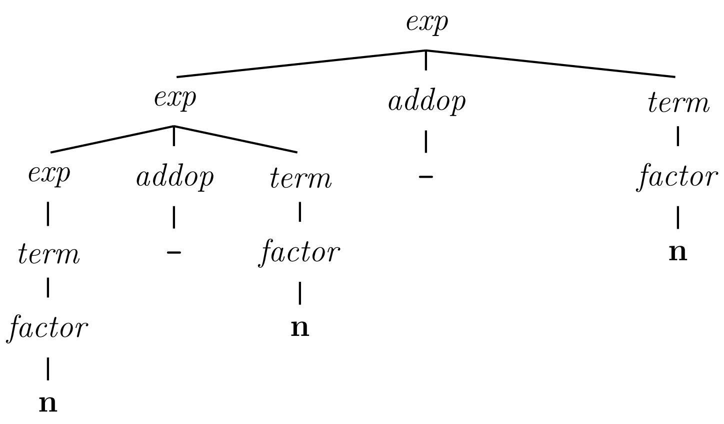
factors and terms

$$
\begin{aligned}
exp &\rightarrow exp\ \ addop\ \ term \mid term \\
addop &\rightarrow \texttt{+} \mid \texttt{-} \\
term &\rightarrow term\ \ mulop\ \ factor \mid factor \\
mulop &\rightarrow \texttt{*} \\
factor &\rightarrow \textbf{(}\ exp\ \textbf{)} \mid \textbf{number}
\end{aligned}
\tag{3.7}
$$

$34 - 3 * 42$



$34 - 3 - 42$



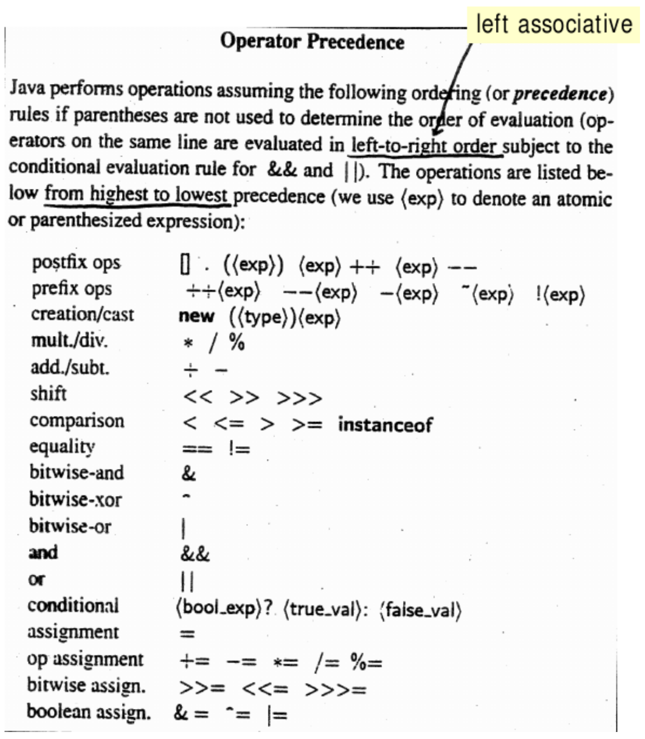
Ambiguity

As mentioned, the question whether a given CFG is ambiguous or not is *undecidable.* Note also: if one uses a parser generator, such as yacc or bison (which cover a practically usefull subset of CFGs), the resulting recognizer is *always* deterministic. In case the construction encounters ambiguous situations, they are "resolved" by making a specific choice. Nonetheless, such ambiguities indicate often that the formulation of the grammar (or even the language it defines) has problematic aspects. Most programmers as "users" of a programming language may not read the full BNF definition, most will try to grasp the language looking at sample code pieces mentioned in the manual, etc. And even if they bother studying the exact specification of the system, i.e., the full grammar, ambiguities are *not* obvious (after all, it's undecidable, at least the problem in general). Hidden ambiguities, "resolved" by the generated parser, may lead misconceptions as to what a program actually means. It's similar to the situation, when one tries to study a book with arithmetic being unaware that multiplication binds stronger than addition. Without being aware of that , not much will make sense. A parser implementing such grammars may make consistent choices, but the programmer using the compiler may not be aware of them. At least the compiler writer, responsible for designing the language, will be informed about "*conflicts*" in the grammar and a careful designer will try to get rid of them. This may be done by adding associativities and precedences (when appropriate) or reformulating the grammar, or even reconsider the syntax of the language. While ambiguities and conflicts are generally a bad sign, arbitrarily adding a complicated "precedence order" and "associativities" on all kinds of symbols or complicate the grammar adding ever more separate classes of nonterminals just to make the conflicts go away is not a real solution either. Chances are, that those parser-internal "tricks" will be lost on the programmer as user of the language, as well. Sometimes, making the *language* simpler (as opposed to complicate the grammar for the same language) might be the better choice. That can typically be done by making the language more verbose and reducing "overloading" of syntax. Of course, going overboard by making groupings etc. of all constructs crystal clear to the parser, may also lead to non-elegant designs. Lisp is a standard example, notoriously known for its extensive use of parentheses. Basically, the programmer directly writes down *syntax trees*, which certainly removes all ambiguities, but still, mountains of parentheses are also not the easiest syntax for human consumption. So it's a balance.

But in general: if it's enormously complex to come up with a reasonably unambigous grammar for an intended language, chances are, that reading programs in that language and intutively grasping what is intended may be hard for humans, too.

Note also: since already the question, whether a given CFG is ambiguous or not is undecidable, it should be clear, that the following question is undecidable as well: given a grammar, can I reformulate it, still accepting the same language, that it becomes unambiguous?

Real life example

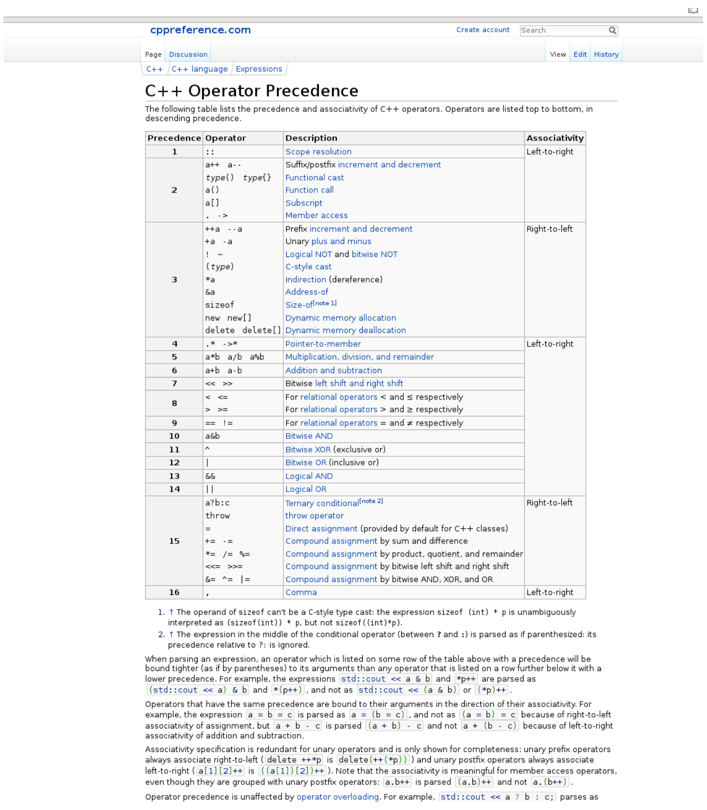

left associative

Operator Precedence

Java performs operations assuming the following ordering (or ***precedence***) rules if parentheses are not used to determine the order of evaluation (operators on the same line are evaluated in left-to-right order subject to the conditional evaluation rule for && and ||). The operations are listed below from highest to lowest precedence (we use ⟨exp⟩ to denote an atomic or parenthesized expression):

postfix ops	[] . (⟨exp⟩) ⟨exp⟩++ ⟨exp⟩−−
prefix ops	++⟨exp⟩ −−⟨exp⟩ −⟨exp⟩ ˜⟨exp⟩ !⟨exp⟩
creation/cast	**new** (⟨type⟩)⟨exp⟩
mult./div.	* / %
add./subt.	+ −
shift	<< >> >>>
comparison	< <= > >= **instanceof**
equality	== !=
bitwise-and	&
bitwise-xor	ˆ
bitwise-or	|
and	&&
or	||
conditional	⟨bool_exp⟩? ⟨true_val⟩: ⟨false_val⟩
assignment	=
op assignment	+= −= *= /= %=
bitwise assign.	>>= <<= >>>=
boolean assign.	&= ˆ= |=

Another example



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Page Discussion

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C++ / C++ language / Expressions

C++ Operator Precedence

The following table lists the precedence and associativity of C++ operators. Operators are listed top to bottom, in descending precedence.

Precedence	Operator	Description	Associativity
1	`::`	Scope resolution	Left-to-right
2	`a++ a--`	Suffix/postfix increment and decrement	
	`type() type{}`	Functional cast	
	`a()`	Function call	
	`a[]`	Subscript	
	`. ->`	Member access	
3	`++a --a`	Prefix increment and decrement	Right-to-left
	`+a -a`	Unary plus and minus	
	`! ~`	Logical NOT and bitwise NOT	
	`(type)`	C-style cast	
	`*a`	Indirection (dereference)	
	`&a`	Address-of	
	`sizeof`	Size-of[note 1]	
	`new new[]`	Dynamic memory allocation	
	`delete delete[]`	Dynamic memory deallocation	
4	`.* ->*`	Pointer-to-member	Left-to-right
5	`a*b a/b a%b`	Multiplication, division, and remainder	
6	`a+b a-b`	Addition and subtraction	
7	`<< >>`	Bitwise left shift and right shift	
8	`< <=`	For relational operators < and ≤ respectively	
	`> >=`	For relational operators > and ≥ respectively	
9	`== !=`	For relational operators = and ≠ respectively	
10	`a&b`	Bitwise AND	
11	`^`	Bitwise XOR (exclusive or)	
12	`|`	Bitwise OR (inclusive or)	
13	`&&`	Logical AND	
14	`||`	Logical OR	
15	`a?b:c`	Ternary conditional[note 2]	Right-to-left
	`throw`	throw operator	
	`=`	Direct assignment (provided by default for C++ classes)	
	`+= -=`	Compound assignment by sum and difference	
	`*= /= %=`	Compound assignment by product, quotient, and remainder	
	`<<= >>=`	Compound assignment by bitwise left shift and right shift	
	`&= ^= |=`	Compound assignment by bitwise AND, XOR, and OR	
16	`,`	Comma	Left-to-right

1. ↑ The operand of `sizeof` can't be a C-style type cast: the expression `sizeof (int) * p` is unambiguously interpreted as `(sizeof(int)) * p`, but not `sizeof((int)*p)`.
2. ↑ The expression in the middle of the conditional operator (between `?` and `:`) is parsed as if parenthesized: its precedence relative to `?:` is ignored.

When parsing an expression, an operator which is listed on some row of the table above with a precedence will be bound tighter (as if by parentheses) to its arguments than any operator that is listed on a row further below it with a lower precedence. For example, the expressions `std::cout << a & b` and `*p++` are parsed as `(std::cout << a) & b` and `*(p++)`, and not as `std::cout << (a & b)` or `(*p)++`.

Operators that have the same precedence are bound to their arguments in the direction of their associativity. For example, the expression `a = b = c` is parsed as `a = (b = c)`, and not as `(a = b) = c` because of right-to-left associativity of assignment, but `a + b - c` is parsed `(a + b) - c` and not `a + (b - c)` because of left-to-right associativity of addition and subtraction.

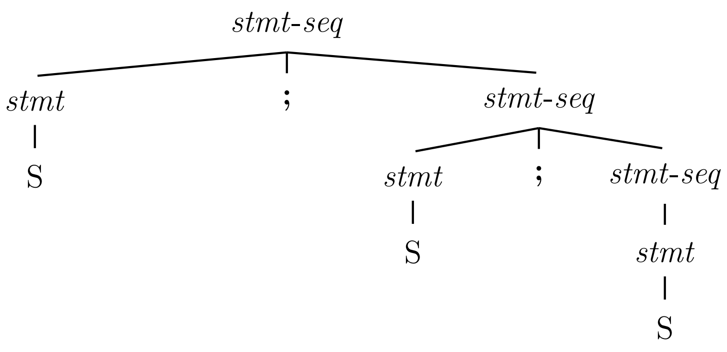
Associativity specification is redundant for unary operators and is only shown for completeness: unary prefix operators always associate right-to-left (`delete ++*p` is `delete(++(*p))`) and unary postfix operators always associate left-to-right (`a[1][2]++` is `((a[1])[2])++`). Note that the associativity is meaningful for member access operators, even though they are grouped with unary postfix operators: `a.b++` is parsed `(a.b)++` and not `a.(b++)`.

Operator precedence is unaffected by operator overloading. For example, `std::cout << a ? b : c;` parses as

Non-essential ambiguity

left-assoc

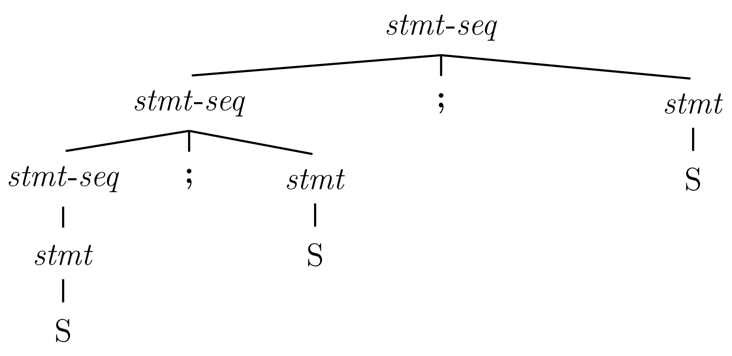
$$
\begin{array}{rcl}
\textit{stmt-seq} & \to & \textit{stmt-seq}\,;\textit{stmt} \mid \textit{stmt} \\
\textit{stmt} & \to & S
\end{array}
$$



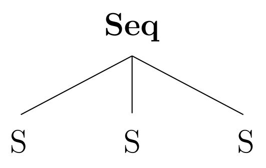
Non-essential ambiguity (2)

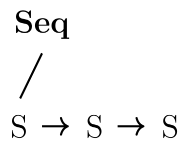
right-assoc representation instead

$$
\begin{array}{rcl}
\textit{stmt-seq} & \rightarrow & \textit{stmt}\,;\textit{stmt-seq} \mid \textit{stmt} \\
\textit{stmt} & \rightarrow & S
\end{array}
$$



Possible AST representations





Dangling else

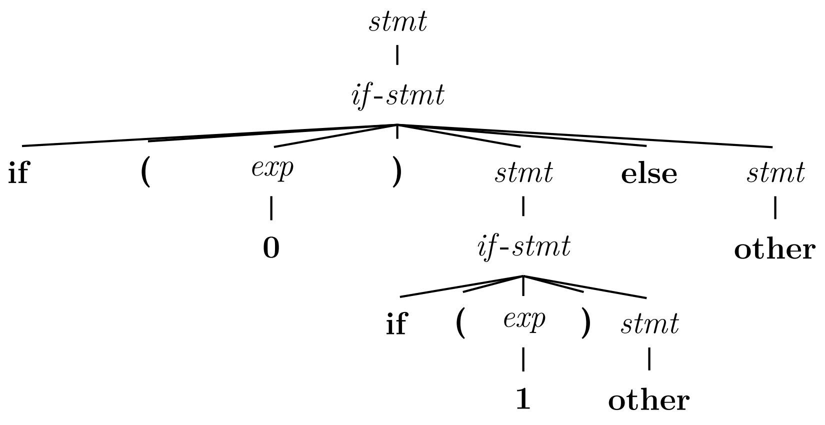
Nested if's

$$\textbf{if}\,(\,0\,)\,\textbf{if}\,(\,1\,)\,\textbf{other}\ \textbf{else}\ \textbf{other}$$

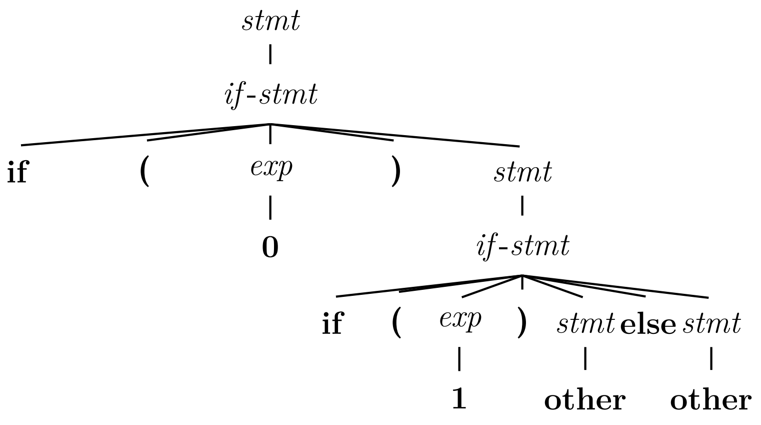
Remember grammar from equation (3.5):

$$
\begin{array}{rcl}
\textit{stmt} & \rightarrow & \textit{if-stmt} \mid \textbf{other} \\
\textit{if-stmt} & \rightarrow & \textbf{if}\,(\,\textit{exp}\,)\,\textit{stmt} \\
 & \mid & \textbf{if}\,(\,\textit{exp}\,)\,\textit{stmt}\,\textbf{else}\,\textit{stmt} \\
\textit{exp} & \rightarrow & \mathbf{0} \mid \mathbf{1}
\end{array}
$$

Should it be like this . . .



. . . or like this



- common convention: connect **else** to closest "free" (= dangling) occurrence

Unambiguous grammar

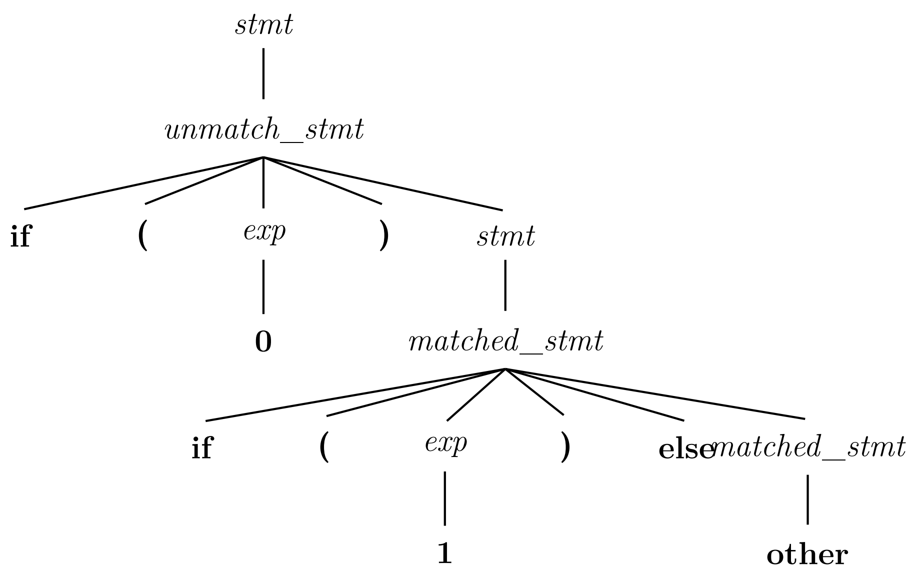
Grammar

$$
\begin{array}{rcl}
\textit{stmt} & \to & \textit{matched\_stmt} \mid \textit{unmatch\_stmt} \\
\textit{matched\_stmt} & \to & \textbf{if}\ \textbf{(}\ \textit{exp}\ \textbf{)}\ \textit{matched\_stmt}\ \textbf{else}\ \textit{matched\_stmt} \\
 & \mid & \textbf{other} \\
\textit{unmatch\_stmt} & \to & \textbf{if}\ \textbf{(}\ \textit{exp}\ \textbf{)}\ \textit{stmt} \\
 & \mid & \textbf{if}\ \textbf{(}\ \textit{exp}\ \textbf{)}\ \textit{matched\_stmt}\ \textbf{else}\ \textit{unmatch\_stmt} \\
\textit{exp} & \to & \textbf{0} \mid \textbf{1}
\end{array}
$$

- never have an unmatched statement inside a matched

- complex grammar, seldomly used
- instead: ambiguous one, with extra "rule": connect each **else** to closest free **if**
- alternative: *different* syntax, e.g.,
  - *mandatory* **else**,
  - or require **endif**

CST



Adding sugar: extended BNF

- make CFG-notation more "convenient" (but without more theoretical expressiveness)
- syntactic sugar

EBNF

Main additional notational freedom: use regular expressions on the rhs of productions. They can contain terminals and non-terminals

- EBNF: officially standardized, but often: all "sugared" BNFs are called EBNF
- in the standard:
  - α^* written as $\{\alpha\}$
  - $\alpha?$ written as $[\alpha]$

- supported (in the standardized form or other) by some parser tools, but not in all
- remember equation (3.2)

EBNF examples

$$
\begin{array}{rcl@{\qquad\qquad}l}
A & \to & \beta\{\alpha\} & \text{for} \quad A \to A\alpha \mid \beta \\[1em]
A & \to & \{\alpha\}\beta & \text{for} \quad A \to \alpha A \mid \beta
\end{array}
$$

$$
\begin{array}{rcl}
\textit{stmt-seq} & \to & \textit{stmt}\ \{\mathbf{;}\ \textit{stmt}\} \\
\textit{stmt-seq} & \to & \{\textit{stmt}\ \mathbf{;}\}\ \ \textit{stmt} \\
\textit{if-stmt} & \to & \mathbf{if}\ \mathbf{(}\ \textit{exp}\ \mathbf{)}\ \textit{stmt}[\mathbf{else}\ \textit{stmt}]
\end{array}
$$

greek letters: for non-terminals or terminals.

3.4 Syntax of a “Tiny” language

BNF-grammar for TINY

$$
\begin{array}{rcl}
\textit{program} & \to & \textit{stmt-seq} \\
\textit{stmt-seq} & \to & \textit{stmt-seq}\ \mathbf{;}\ \textit{stmt} \mid \textit{stmt} \\
\textit{stmt} & \to & \textit{if-stmt} \mid \textit{repeat-stmt} \mid \textit{assign-stmt} \\
 & \mid & \textit{read-stmt} \mid \textit{write-stmt} \\
\textit{if-stmt} & \to & \mathbf{if}\ \textit{expr}\ \mathbf{then}\ \textit{stmt}\ \mathbf{end} \\
 & \mid & \mathbf{if}\ \textit{expr}\ \mathbf{then}\ \textit{stmt}\ \mathbf{else}\ \textit{stmt}\ \mathbf{end} \\
\textit{repeat-stmt} & \to & \mathbf{repeat}\ \textit{stmt-seq}\ \mathbf{until}\ \textit{expr} \\
\textit{assign-stmt} & \to & \mathbf{identifier}\ \mathbf{:=}\ \textit{expr} \\
\textit{read-stmt} & \to & \mathbf{read}\ \mathbf{identifier} \\
\textit{write-stmt} & \to & \mathbf{write}\ \ \textit{expr} \\
\textit{expr} & \to & \textit{simple-expr}\ \textit{comparison-op}\ \textit{simple-expr} \mid \textit{simple-expr} \\
\textit{comparison-op} & \to & \mathtt{<} \mid \mathtt{=} \\
\textit{simple-expr} & \to & \textit{simple-expr}\ \textit{addop}\ \textit{term} \mid \textit{term} \\
\textit{addop} & \to & \mathtt{+} \mid \mathtt{-} \\
\textit{term} & \to & \textit{term}\ \textit{mulop}\ \textit{factor} \mid \textit{factor} \\
\textit{mulop} & \to & \mathtt{*} \mid \mathtt{/} \\
\textit{factor} & \to & \mathbf{(}\ \textit{expr}\ \mathbf{)} \mid \mathbf{number} \mid \mathbf{identifier}
\end{array}
$$

Syntax tree nodes

```
typedef enum {StmtK,ExpK} NodeKind;
typedef enum {IfK,RepeatK,AssignK,ReadK,WriteK} StmtKind;
typedef enum {OpK,ConstK,IdK} ExpKind;

/* ExpType is used for type checking */
typedef enum {Void,Integer,Boolean} ExpType;

#define MAXCHILDREN 3

typedef struct treeNode
   { struct treeNode * child[MAXCHILDREN];
     struct treeNode * sibling;
     int lineno;
     NodeKind nodekind;
     union { StmtKind stmt; ExpKind exp;} kind;
     union { TokenType op;
             int val;
             char * name; } attr;
     ExpType type; /* for type checking of exps */
```

Comments on C-representation

- typical use of `enum` type for that (in C)
- `enum`'s in C can be very efficient
- `treeNode` struct (records) is a bit “unstructured”
- newer languages/higher-level than C: better structuring advisable, especially for languages larger than Tiny.
- in Java-kind of languages: inheritance/subtyping and abstract classes/interfaces often used for better structuring

Sample Tiny program

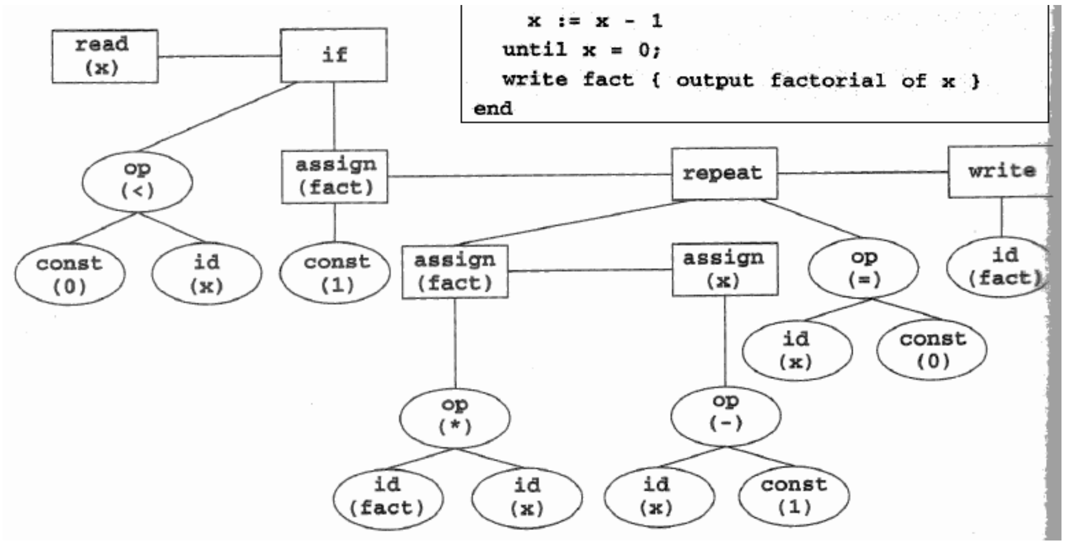
```
read x; { input as integer }
if 0 < x then { don't compute if x <= 0 }
  fact := 1;
  repeat
    fact := fact * x;
    x := x -1
  until x = 0;
  write fact   { output factorial of x }
end
```

Same Tiny program again

```
read x; { input as integer }
if 0 < x then { don't compute if x <= 0 }
  fact := 1;
  repeat
    fact := fact * x;
    x := x -1
  until x = 0;
  write fact   { output factorial of x }
end
```

- *keywords* / *reserved words* highlighted by bold-face type setting
- reserved syntax like `0`, `:=`, ... is not bold-faced
- comments are italicized

Abstract syntax tree for a tiny program



```
   x := x - 1
  until x = 0;
  write fact { output factorial of x }
end
```

Some questions about the Tiny grammy

- is the grammar unambiguous?
- How can we change it so that the Tiny allows empty statements?
- What if we want semicolons *in between* statements and not *after*?
- What is the precedence and associativity of the different operators?

3.5 Chomsky hierarchy

The Chomsky hierarchy

- linguist Noam Chomsky [1]
- **important** classification of (formal) languages (sometimes Chomsky-Sch\"utzenberger)
- 4 levels: type 0 languages – type 3 languages
- levels related to machine models that generate/recognize them
- so far: regular languages and CF languages

Overview

	rule format	languages	machines	closed
3	$A \to aB$, $A \to a$	regular	NFA, DFA	all
2	$A \to \alpha_1 \beta \alpha_2$	CF	pushdown automata	$\cup$, $*$, $\circ$
1	$\alpha_1 A \alpha_2 \to \alpha_1 \beta \alpha_2$	context-sensitive	(linearly restricted automata)	all
0	$\alpha \to \beta$, $\alpha \neq \epsilon$	recursively enumerable	Turing machines	all, except complement

Conventions

- terminals $a, b, \ldots \in \Sigma_T$,
- non-terminals $A, B, \ldots \in \Sigma_N$
- general words $\alpha, \beta \ldots \in (\Sigma_T \cup \Sigma_N)^*$

Remark: Chomsky hierarchy

The rule format for type 3 languages (= regular languages) is also called right-linear. Alternatively, one can use *right-linear* rules. If one mixes right- and left-linear rules, one leaves the class of regular languages. The rule-format above allows only *one* terminal symbol. In principle, if one had sequences of terminal symbols in a right-linear (or else left-linear) rule, that would be ok too.

Phases of a compiler & hierarchy

"Simplified" design?

1 big grammar for the whole compiler? Or at least a CSG for the front-end, or a CFG combining parsing and scanning?

Remarks

theoretically possible, but bad idea:

- efficiency
- bad design
- especially combining scanner + parser in one BNF:
  - grammar would be needlessly large
  - separation of concerns: much clearer/ more efficient design
- for scanner/parsers: regular expressions + (E)BNF: simply **the formalisms of choice!**
  - front-end needs to do more than checking syntax, CFGs not expressive enough
  - for level-2 and higher: situation gets less clear-cut, plain CSG not too useful for compilers

Chapter 4
References

Bibliography

[1] Chomsky, N. (1956). : Three models for the description of language. *IRE Transactions on Information Theory*, 2(113–124).

[2] Cooper, K. D. and Torczon, L. (2004). *Engineering a Compiler*. Elsevier.

[3] Louden, K. (1997). *Compiler Construction, Principles and Practice*. PWS Publishing.

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