

Chapter 7

Types and type checking

Course “Compiler Construction”

Martin Steffen

Spring 2022



Chapter 7

Learning Targets of Chapter "Types and type checking".

1. the concept of types
2. specific common types
3. type safety
4. type checking
5. polymorphism, subtyping and other complications



Chapter 7

Outline of Chapter “Types and type checking”.

Introduction

Various types and their representation

Equality of types

Type checking



Section

Introduction

Chapter 7 “Types and type checking”
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General remarks and overview

- Goal here:
  - what are *types*?
  - static vs. dynamic typing
  - how to describe types *syntactically?*
  - how to *represent* and use types in a compiler?
- coverage of various types
  - basic types (often predefined/built-in)
  - type constructors
  - values of a type
  - type operators
  - representation at run-time
  - run-time tests and special problems (array, union, record, pointers)
- specification and implementation of type systems/type checkers
- advanced concepts



Why types?

- crucial, user-visible abstraction describing program behavior
- one view: type describes a set of (mostly related) *values*
- static typing: checking/enforcing a type discipline at compile time
- dynamic typing: same at run-time, mixtures possible
- completely untyped languages: very rare to non-existant, types were part of PLs from the start.

Milner's dictum ("type safety")

Well-typed programs cannot go wrong!

- *strong* typing:[1] rigorously prevent "misuse" of data
- types useful for later phases and optimizations
- documentation and partial specification

[1] Terminology rather fuzzy, and perhaps changed a bit over time.



Types: in first approximation

Conceptually

- semantic view: set of values *plus* a set of corresponding operations
- syntactic view: notation to *construct* basic elements of the type (its values) *plus* "procedures" operating on them
- compiler implementor's view: data of the same type have same underlying memory representation

further classification:

- built-in/predefined vs. *user-defined* types
- basic/base/elementary/primitive types vs. compound types
- type constructors: building more compex types from simpler ones
- reference vs. value types





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Various types and their representation

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Some typical basic types

base types			
`int`	`0, 1, ...`	`+, -, *, /`	integers
`real`	`5.05E4 ...`	`+, -, *`	real numbers
`bool`	`true, false`	`and or (|) ...`	booleans
`char`	`'a'`		characters
⋮			

- often HW support for some of those (including some of the op's)
- mostly: elements of `int` are not exactly mathematical *integers*, same for `real`
- often variations offered: `int32`, `int64`
- often implicit *conversions* and relations between basic types
  - which the type system has to specify/check for legality
  - which the compiler has to implement



Some compound types

compound types		
`array[0..9] of real`		`a[i+1]`
`list`	`[], [1;2;3]`	concat
`string`	`"text"`	concat ...
`struct / record`		`r.x`
...		

- mostly reference types
- when built in, special "easy syntax" (same for basic built-in types)
  - `4 + 5` as opposed to `plus(4,5)`
  - `a[6]` as opposed to `array_access(a, 6)` ...
- parser/lexer aware of built-in types/operators (special precedences, associativity, etc.)
- cf. functionality "built-in/predefined" via libraries



Abstract data types

- unit of *data* together with *functions/procedures/operations* ... operating on them
- encapsulation + interface
- often: separation between exported and internal operations
  - for instance `public`, `private` ...
  - or via separate interfaces
- (static) classes in Java: may be used/seen as ADTs, methods are then the "operations"

```
ADT  begin
    integer i;
    real x;
    int proc total(int a) {
       return i * x + a  // or: ``total = i * x + a''
    }
end
```



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Type constructors: building new types

- array type
- record type (also known as struct-types)
- union type
- pair/tuple type
- pointer type
  - explict as in C
  - implict distinction between reference and value types, hidden from programmers (e.g. Java)
- *signatures* (specifying methods / procedures / subroutines / functions) as type
- function type constructor, incl. higher-order types (in functional languages)
- (names of) classes and subclasses
- ...



Arrays

Array type

```
array [<indextype>] of <component type>
```

- elements (arrays) = (finite) functions from index-type to component type
- allowed index-types:
  - non-negative (unsigned) integers?, `from ... to ...`?
  - other types?: enumerated types, characters
- things to keep in mind:
  - indexing outside the array bounds?
  - are the array bounds (statically) known to the compiler?
  - *dynamic* arrays (extensible at run-time)?



One and more-dimensional arrays



- one-dimensional: efficiently implementable in standard hardware (relative memory addressing, known offset)
- two or more dimensions

```
array [1..4] of array [1..3] of real
array [1..4, 1..3] of real
```

- one can see it as "array of arrays" (Java), an array is typically a reference type
- conceptually "two-dimensional"- *linear layout* in memory (language dependent)

Records ("structs")

```
struct {
  real r;
  int  i;
}
```

- values: "labelled tuples" (`real` $\times$ `int`)
- constructing elements, e.g.

`struct point`

- access (read or update): *dot-notation* `x.i`
- implementation: linear memory layout given by the (types of the) attributes
- attributes accessible by statically fixed *offsets*
- *fast* access
- cf. objects as in Java



Tuple/product types

- $T_1 \times T_2$ (or in ascii `T_1 * T_2`)
- elements are *tuples*: for instance: `(1, "text")` is element of `int * string`
- generalization to n-tuples:

value	type
`(1, "text", true)`	`int * string * bool`
`(1, ("text", true))`	`int * (string * bool)`

- structs can be seen as "labeled tuples", resp. tuples as "anonymous structs"
- tuple types: common in functional languages,
- in C/Java-like languages: n-ary tuple types often only implicit as *input* types for procedures/methods (part of the "signature")



Union types (C-style again)

```
union {
  real r;
  int  i
}
```

- related to *sum types* (outside C)
- (more or less) represents *disjoint union* of values of "participating" types
- access in C (confusingly enough): dot-notation `u.i`



Union types in C and type safety

- union types is C: bad example for (safe) type disciplines, as it's simply type-unsafe, basically an *unsafe* hack ...

Union type (in C):

- nothing much more than a directive to allocate enough memory to hold largest member of the union.
- in the example: `real` takes more space than `int`

Explanation

- implementor's (= low level) focus and memory allocation, not "proper usage focus" or assuring strong typing

$\Rightarrow$ bad example of modern use of types

- better (type-safe) implementations known since

$\Rightarrow$ *variant record* ("tagged"/"discriminated" union) or even inductive data types



Variant records from Pascal

```
record case isReal: boolean of
  true:  (r:real);
  false: (i:integer);
```

- "variant record"
- non-overlapping memory layout[2]
- programmer responsible to set and check the "discriminator" self
- enforcing type-safety-wise: not really an improvement :-(

[2] Again, that's an implementor-centric view, not a user-centric one.



Inductive types in ML and similar

- *type-safe* and powerful
- allows *pattern matching*

```
IsReal of real | IsInteger of int
```

- allows *recursive* definitions ⇒ inductive data types:

```
type int_bintree =
    Node of int * int_bintree * int_bintree
|   Nil
```

- `Node`, `Leaf`, `IsReal`: *constructors* (cf. languages like Java)
- constructors used as discriminators in "union" types

```
type exp =
     Plus of exp * exp
   | Minus of exp * exp
   | Number of int
   | Var of string
```



Recursive data types in C

does not work

```
struct intBST  {
  int val;
  int isNull;
  struct intBST left, right;
}
```

"indirect" recursion

```
struct intBST {
  int val;
  struct intBST *left, *right;
};
```

In Java: references implicit

```
class BSTnode {
 int val;
 BSTnode left, right;
```

- note: *implementation* in ML: also uses "pointers" (but hidden from the user)
- no nil-pointers in ML (and `NIL` is not a nil-pointer, it's a constructor)



Pointer types

- *pointer* type: notation in C: `int*`
- " `*` ": can be seen as type constructor

```
int* p;
```

- random other languages: `^integer` in Pascal, `int ref` in ML
- value: *address* of (or reference/pointer to) values of the underlying type
- operations: *dereferencing* and determining the address of an data item (and C allows " *pointer arithmetic* ")

```
var a:  ^integer   (* pointer to an integer     *)
var b:   integer
...
a := &i            (* i an int var              *)
                   (* a :=  new integer ok too  *)
b:= ^a + b
```



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Implicit dereferencing

- many languages: more or less hide existence of pointers
- cf. reference vs. value types often: automatic/implicit dereferencing

```
C r;
C r = new C();
```

- "sloppy" speaking: " `r` is an object (which is an instance of class `C` /which is of type `C`)",
- slightly more precise: variable " `r` contains an object..."
- precise: "variable `r` will contain a reference to an object"
- `r.field` corresponds to something like " `(*r).field`, similar in Simula



Programming with pointers

- "popular" source of errors
- test for non-null-ness often required
- explicit pointers: can lead to problems in block-structured language (when handled non-expertly)
- watch out for parameter passing
- aliasing
- null-pointers: "the billion-dollar-mistake"
- take care of concurrency



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Function variables

```
program Funcvar;
var pv : Procedure (x: integer);   (* procedur var      *)

    Procedure Q();
    var
        a : integer;
        Procedure P(i : integer);
        begin
            a:= a+i;    (* a def'ed outside             *)
        end;
    begin
        pv := @P;       (* ``return'' P (as side effect) *)
    end;                (* "@" dependent on dialect      *)
begin                   (* here: free Pascal             *)
    Q();
    pv(1);
end.
```



Function variables and nested scopes

- tricky part here: nested scope + function definition *escaping* surrounding function/scope.
- here: inner procedure "returned" via assignment to function variable
- stack discipline of dynamic memory management?
- related also: functions allowed as return value?
  - Pascal: not directly possible (unless one "returns" them via function-typed reference variables like here)
  - C: possible, but *nested* function definitions not allowed
- combination of nested function definitions and functions as official return values (and arguments): *higher-order functions*
- Note: functions as arguments less problematic than as return values.



Function signatures

- define the “header” (also “signature”) of a function[3]
- in the discussion: we don’t distinguish mostly: functions, procedures, methods, subroutines.
- functional type (independent of the name f): `int→int`

Modula-2

```
var f: procedure (integer): integer;
```

C

```
int (*f) (int)
```

- *values*: all functions (procedures . . .) with the given signature
- problems with block structure and free use of procedure variables.

[3] Actually, an identfier of the function is mentioned as well.

Escaping

```
program Funcvar;
var pv : Procedure (x: integer);  (* procedur var     *)

    Procedure Q();
    var
        a : integer;
        Procedure P(i : integer);
        begin
            a:= a+i;    (* a def'ed outside          *)
        end;
    begin
        pv := @P;       (* ``return'' P (as side effect) *)
    end;                (* "@" dependent on dialect      *)
begin                   (* here: free Pascal             *)
    Q();
    pv(1);
end.
```

- at the end of line 15: variable `a` no longer exists
- possible safe usage: only assign to such variables (here `pv`) a new value (= function) at the same blocklevel the variable is declared

Classes and subclasses

Parent class

```
class A {
  int i;
  void f() {...}
}
```

Subclass B

```
class B extends A {
  int i
  void f() {...}
}
```

Subclass C

```
class C extends A {
  int i
  void f() {...}
}
```

- classes resemble records, and subclasses variant types, but additionally
  - visibility: local methods possible (besides fields)
  - subclasses
  - objects mostly created dynamically, *no* references into the stack
  - subtyping and polymorphism (subtype polymorphism): a reference typed by `A` can also point to `B` or `C` objects
- special problems: not really many, nil-pointer still possible



Access to object members: late binding

- notation `rA.i` or `rA.f()`
- dynamic binding, late-binding, virtual access, dynamic dispatch . . . : all mean roughly the same
- central mechanism in many OO language, in connection with inheritance

Virtual access `rA.f()` (methods)

"deepest" `f` in the run-time class of the *object*, `rA` points to

- remember: "most-closely nested" access of variables in nested lexical block
- Java:
  - methods "in" objects are only dynamically bound (but there are class methods too)
  - instance variables not, neither static methods "in" classes.



Example: fields and methods

```
1  public class Shadow {
2      public static void main(String[] args){
3          C2 c2 = new C2();
4          c2.n();
5      }
6  }
7
8  class C1 {
9      String s = "C1";
10     void m () {System.out.print(this.s);}
11 }
12
13
14 class C2 extends C1 {
15     String s = "C2";
16     void n () {this.m();}
17 }
```



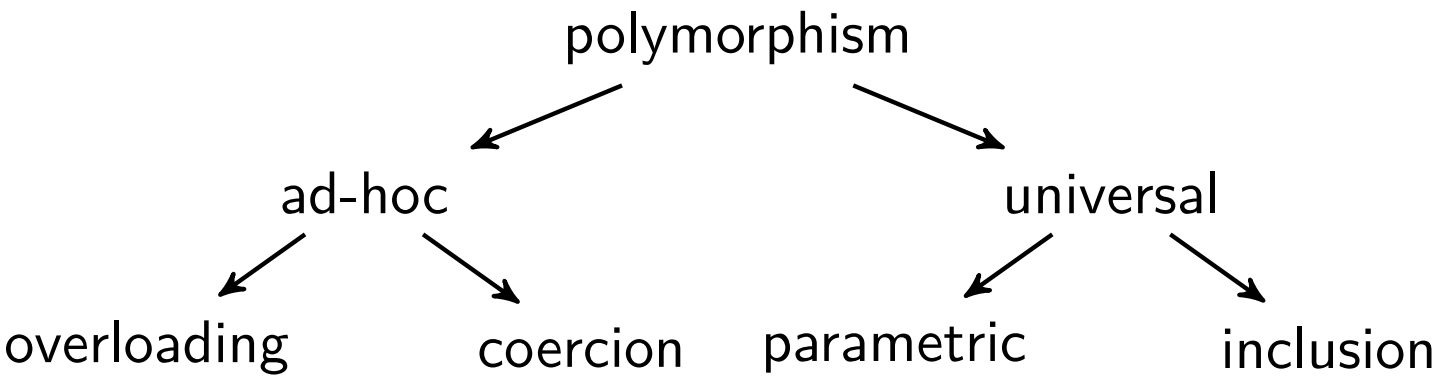
Diverse notions

- *Overloading*
  - common for (at least) standard, built-in operations
  - also possible for user defined functions/methods ...
  - disambiguation via (static) types of arguments
  - "ad-hoc" polymorphism
  - implementation:
    - put types of parameters as "part" of the name
    - look-up gives back a set of alternatives
- (generic) polymporphism

```
swap(var x,y:  anytype)
```



Polymorphism







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Classes as types

- classes = types? Not so fast
- more precise view:
  - design decision in Java and similar languages (but not all/even not all class-based OOLs): that class *names* are used in the role of (names of) types.
- other roles of classes (in class-based OOLs)
  - generator of objects (via constructor, again with the same name)[4]
  - containing *code* that implements the instances

[4] Not for Java's *static* classes etc, obviously.



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When are 2 types "equal"?

- type equivalence
- surprisingly many different answers possible
- implementor's focus (deprecated): type int and short are equal, because they "are" both 2 bytes
- type checker must often decide such equivalences
- related to a more fundamental question: what's a type?

Example: pairs of integers

```
1  type pair_of_ints = int * int;;
2  let x : pair_of_int = 1,4);;
```

- Is "the" type of (values of) x `pair_of_ints`, or
- the product type `int * int`, or
- both, as they are equal, i.e., `pair_of_int` is an abbreviation of the product type (type synonym)?



Example with interfaces

```
interface I1 { int m (int x); }
interface I2 { int m (int x); }
class C1 implements I1 {
    public int m(int y) {return y++;  }
}
class C2 implements I2 {
    public int m(int y) {return y++;  }
}

public class Noduck1 {
    public static void main(String[] arg) {
        I1 x1 = new C1();          // I2 not possible
        I2 x2 = new C2();
        x1 = x2;                   // ???
    }
}
```

Analogous when using classes in their roles as types



Structural vs. nominal equality

a, b

```
var a, b: record
    int i;
    double d
 end
```

c

```
var c: record
    int i;
    double d
 end
```

typedef

```
typedef idRecord: record
    int i;
    double d
 end
```

```
var d: idRecord;
var e: idRecord;;
```

what's possible?

```
a := c;
a := d;

a := b;
d := e;
```



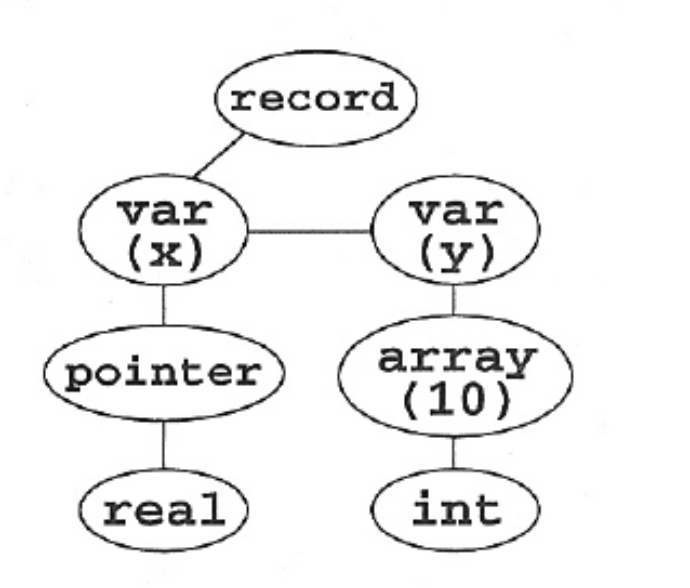
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Types in the AST

- types are part of the syntax, as well
- represent: either in a separate symbol table, or part of the AST

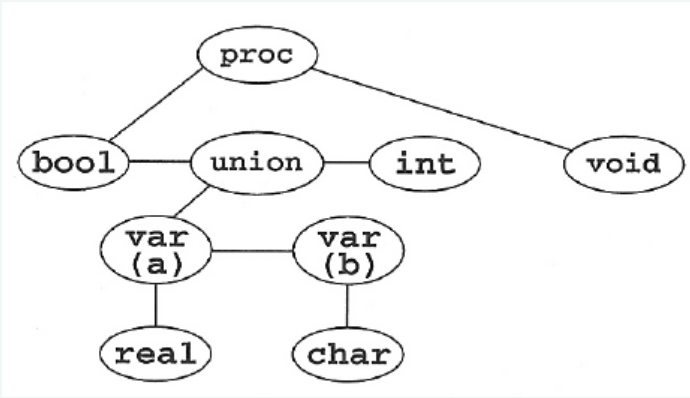
Record type

```
record
  x: pointer to real;
  y: array [10] of int
end
```



Procedure header

```
proc(bool,
     union a: real; b:char end,
     int):void
end
```



Structured types without names

$$
\begin{array}{rcl}
\textit{var-decls} & \rightarrow & \textit{var-decls}\ \mathbf{;}\ \textit{var-decl} \mid \textit{var-decl} \\
\textit{var-decl} & \rightarrow & \mathbf{id}\ \mathbf{:}\ \textit{type-exp} \\
\textit{type-exp} & \rightarrow & \textit{simple-type} \mid \textit{structured-type} \\
\textit{simple-type} & \rightarrow & \mathbf{int} \mid \mathbf{bool} \mid \mathbf{real} \mid \mathbf{char} \mid \mathbf{void} \\
\textit{structured-type} & \rightarrow & \mathbf{array}\ [\ \textit{num}\ ]\ :\ \textit{type-exp} \\
 & \mid & \mathbf{record}\ \textit{var-decls}\ \mathbf{end} \\
 & \mid & \mathbf{union}\ \textit{var-decls}\ \mathbf{end} \\
 & \mid & \mathbf{pointerto}\ \textit{type-exp} \\
 & \mid & \mathbf{proc}\ (\ \textit{type-exps}\ )\ \textit{type-exp} \\
\textit{type-exps} & \rightarrow & \textit{type-exps}\ \mathbf{,}\ \textit{type-exp} \mid \textit{type-exp}
\end{array}
$$



Structural equality (1)

```
1  function typeEqual(t1, t2: TypeExp) : Boolean;
2  var temp: Boolean;
3      p1, p2: TypeExp;
4  begin
5    if    t1 and t2 are of simple type
6    then return t1 = t2
7    else if t1.kind = array and t2.kind = array
8    then return t1.size = t2.size and typeEqual(t1.child, t2.child)
9    else if      t1.kind = record and t2.kind = record
10          or    t1.kind = union and t2.kind = union
11       then begin
12              p1 := t1.child;
13              p2 := t2.child;
14              temp := true;
15              while temp and p1 ≠ nil and p2 ≠ nil
16              do
17                 if    p1.name ≠ p2.name
18                 then temp := false
19                 else
20                 begin
21                    p1 := p1.sibling;
22                    p2 := p2.sibling;
23                 end;
24               return temp and p1 = nil and p2 = nil;
```

Structural equality (2)

```
1    else if t1.kind = pointer and t2.kind = pointer
2    then    return typeEqual(t1.child, t2.child)
3    else if t1.kind = proc and t2.kind = proc
4    then    begin
5                p1 := t1.child;
6                p2 := t2.child;
7                temp := true;
8                while temp and p1 ≠ nil and p2 ≠ nil
9                do
10                   if   not typeEqual(p1.child, p2.child)
11               then temp := false
12               else
13                 begin
14                   p1 := p1.sibling;
15                   p2 := p2.sibling;
16                 end;
17               return temp and p1 = nil and p2 = nil
18                              and typeEqual(t1.child,t2.child=
19               end
20   else if t1 and t2 are type names  (* if also names are checked *)
21   then    return typeEqual(getTypeExp(t1), getTypeExp(t2)
22   else    return false
23 end; (* typeEqual)
```

Types with names

$$
\begin{array}{rcl}
\textit{var-decls} & \rightarrow & \textit{var-decls}\ \mathbf{;}\ \textit{var-decl} \mid \textit{var-decl} \\
\textit{var-decl} & \rightarrow & \mathbf{id}\ \mathbf{:}\ \textit{simple-type-exp} \\
\textit{type-decls} & \rightarrow & \textit{type-decls}\ \mathbf{;}\ \textit{type-decl} \mid \textit{type-decl} \\
\textit{type-decl} & \rightarrow & \mathbf{id} = \textit{type-exp} \\
\textit{type-exp} & \rightarrow & \textit{simple-type-exp} \mid \textit{structured-type} \\
\textit{simple-type-exp} & \rightarrow & \textit{simple-type} \mid \mathbf{id} \\
\textit{simple-type} & \rightarrow & \mathbf{int} \mid \mathbf{bool} \mid \mathbf{real} \mid \mathbf{char} \mid \mathbf{void} \\
\textit{structured-type} & \rightarrow & \mathbf{array}\ [\ \textit{num}\ ]\ :\ \textit{simple-type-exp} \\
 & \mid & \mathbf{record}\ \textit{var-decls}\ \mathbf{end} \\
 & \mid & \mathbf{union}\ \textit{var-decls}\ \mathbf{end} \\
 & \mid & \mathbf{pointerto}\ \textit{simple-type-exp} \\
 & \mid & \mathbf{proc}\ (\ \textit{type-exps}\ )\ \textit{simple-type-exp} \\
\textit{type-exps} & \rightarrow & \textit{type-exps}\ \mathbf{,}\ \textit{simple-type-exp} \\
 & \mid & \textit{simple-type-exp}
\end{array}
$$



Name equality

- all types have "names", and two types are equal iff their names are equal
- type equality checking: obviously simpler
- of course: type names may have *scopes*...

```
1  function typeEqual(t1, t2: TypeExp): Boolean;
2  var temp: boolean
3      p1, p2: TypeExp;
4  begin
5    if      t1 and t2 are of simple type
6    then    return t1 = t2
7    else if t1 and t2 are type names
8    then    return t1 = t2
9    else    return false;
10 end
```



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Type aliases

- languages with type aliases (type synonyms): C, Pascal, ML ....
- often very convenient (`type Coordinate = float * float`)
- light-weight mechanism

type alias; make `t1` known also under name `t2`

```
t2  = t1    // t2 is the ``same type''.
```

Rest

- also here: different choices wrt. *type equality*



Type aliases: different choices



Alias, for simple types

```
t1 = int;
t2 = int;
```

- often: `t1` and `t2` are the "same" type

Alias of structured types

```
t1 = array [10] of int;
t2 = array [10] of int;
t3 = t2
```

- mostly $t3 \neq t1 \neq t2$

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Type checking of expressions (and statements)

- types of subexpressions must "fit" to the expected types the contructs can operate on
- type checking: top-down and *bottom-up* task

$\Rightarrow$ *synthesized* attributes, when using AGs

- Here: using an attribute grammar specification of the type checker
  - type checking conceptually done *while parsing* (as actions of the parser)
  - more common: type checker operates on the AST *after* the parser has done its job
- type system vs. type checker
  - type system: specification of the rules governing the use of types in a language, type discipline
  - type checker: algorithmic formulation of the type system (resp. implementation thereof)



Grammar for statements and expressions

$$
\begin{array}{rcl}
\mathit{program} & \rightarrow & \mathit{var\text{-}decls}\ \mathbf{;}\ \mathit{stmts} \\
\mathit{var\text{-}decls} & \rightarrow & \mathit{var\text{-}decls}\ \mathbf{;}\ \mathit{var\text{-}decl} \mid \mathit{var\text{-}decl} \\
\mathit{var\text{-}decl} & \rightarrow & \mathbf{id}\ \mathbf{:}\ \mathit{type\text{-}exp} \\
\mathit{type\text{-}exp} & \rightarrow & \mathbf{int} \mid \mathbf{bool} \mid \mathbf{array}\ [\ \mathit{num}\ ]\ \mathbf{:}\ \mathit{type\text{-}exp} \\
\mathit{stmts} & \rightarrow & \mathit{stmts}\ \mathbf{;}\ \mathit{stmt} \mid \mathit{stmt} \\
\mathit{stmt} & \rightarrow & \mathbf{if}\ \ \mathit{exp}\ \ \mathbf{then}\ \ \mathit{stmt} \mid \mathbf{id}\ \mathbf{:=}\ \mathit{exp} \\
\mathit{exp} & \rightarrow & \mathit{exp} + \mathit{exp} \mid \mathit{exp}\ \mathbf{or}\ \mathit{exp} \mid \mathit{exp}\ [\ \mathit{exp}\ ]
\end{array}
$$



Type checking as semantic rules

Grammar Rule	Semantic Rules
var-decl → **id** : *type-exp*	*insert*(**id** .*name*, *type-exp.type*)
type-exp → **int**	*type-exp.type* := *integer*
type-exp → **bool**	*type-exp.type* := *boolean*
$type\text{-}exp_1$ → **array** [**num**] **of** $type\text{-}exp_2$	$type\text{-}exp_1$.*type* := *makeTypeNode*(*array*, **num** .*size*, $type\text{-}exp_2$.*type*)
stmt → **if** *exp* **then** *stmt*	**if not** *typeEqual*(*exp.type*, *boolean*) **then** *type-error*(*stmt*)
stmt → **id** := *exp*	**if not** *typeEqual*(*lookup*(**id** .*name*), *exp.type*) **then** *type-error*(*stmt*)
exp_1 → exp_2 + exp_3	**if not** (*typeEqual*(exp_2 .*type*, *integer*) **and** *typeEqual*(exp_3 .*type*, *integer*)) **then** *type-error*(exp_1) ; exp_1 .*type* := *integer*
exp_1 → exp_2 **or** exp_3	**if not** (*typeEqual*(exp_2 .*type*, *boolean*) **and** *typeEqual*(exp_3 .*type*, *boolean*)) **then** *type-error*(exp_1) ; exp_1 .*type* := *boolean*
exp_1 → exp_2 [exp_3]	**if** *isArrayType*(exp_2 .*type*) **and** *typeEqual*(exp_3 .*type*, *integer*) **then** exp_1 .*type* := exp_2 .*type.child1* **else** *type-error*(exp_1)
exp → **num**	*exp.type* := *integer*
exp → **true**	*exp.type* := *boolean*
exp → **false**	*exp.type* := *boolean*
exp → **id**	*exp.type* := *lookup*(**id** .*name*)



More “modern” presentation

- representation as derivation rules
- Γ: notation for symbol table
  - $\Gamma(x)$: look-up
  - $\Gamma, x : T$: insert
- more compact representation
- one reason: “errors” left implicit.



Type checking (expressions)

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T}\ \textsc{TE-Id} \qquad \frac{}{\Gamma \vdash \textbf{true} : \texttt{bool}}\ \textsc{TE-True} \qquad \frac{}{\Gamma \vdash \textbf{false} : \texttt{bool}}\ \textsc{T-Fals}$$

$$\frac{}{\Gamma \vdash n : \texttt{int}}\ \textsc{TE-Num}$$

$$\frac{\Gamma \vdash exp_2 : \texttt{array_of}\ T \qquad \Gamma \vdash exp_3 : \texttt{int}}{\Gamma \vdash exp_2\,[\,exp_3\,] : T}\ \textsc{TE-Array}$$

$$\frac{\Gamma \vdash exp_1 : \texttt{bool} \qquad \Gamma \vdash exp_2 : \texttt{bool}}{\Gamma \vdash exp_1\ \textbf{or}\ exp_2 : \texttt{bool}}\ \textsc{Te-Or}$$

$$\frac{\Gamma \vdash exp_1 : \texttt{int} \qquad \Gamma \vdash exp_2 : \texttt{int}}{\Gamma \vdash exp_1 + exp_2 : \texttt{int}}\ \textsc{TE-Plus}$$

Declarations and statements

$$\frac{\Gamma, x : \texttt{int} \vdash \mathit{rest} : \texttt{ok}}{\Gamma \vdash x \,\textbf{:int;}\, \mathit{rest} : \texttt{ok}}\ \text{TD-Int} \qquad \frac{\Gamma, x : \texttt{bool} \vdash \mathit{rest} : \texttt{ok}}{\Gamma \vdash x \,\textbf{:bool;}\, \mathit{rest} : \texttt{ok}}\ \text{TD-Bool}$$

$$\frac{\Gamma \vdash \mathit{num} : \texttt{int} \qquad \Gamma(\mathit{type\text{-}exp}) = T \qquad \Gamma, x : \texttt{array}\ \mathit{num}\ \texttt{of}\ T \vdash \mathit{rest} : \texttt{ok}}{\Gamma \vdash x \,\textbf{:array}\, [\, \mathit{num}\,] : \mathit{type\text{-}exp}\, \textbf{;}\, \mathit{rest} : \texttt{ok}}\ \text{TD-Array}$$

$$\frac{\Gamma \vdash x : T \qquad \Gamma \vdash \mathit{exp} : T}{\Gamma \vdash x \,\textbf{:=}\, \mathit{exp} : \texttt{ok}}\ \text{TS-Assign} \qquad \frac{\Gamma \vdash \mathit{exp} : \texttt{bool} \qquad \Gamma \vdash \mathit{stmt} : \texttt{ok}}{\Gamma \vdash \textbf{if}\ \ \mathit{exp}\ \ \textbf{then}\ \ \mathit{stmt} : \texttt{ok}}$$

$$\frac{\Gamma \vdash \mathit{stmt}_1 : \texttt{ok} \qquad \Gamma \vdash \mathit{stmt}_2 : \texttt{ok}}{\Gamma \vdash \mathit{stmt}_1\, \textbf{;}\, \mathit{stmt}_2 : \texttt{ok}}\ \text{TS-Seq}$$