

Chapter 10

Code generation

Course “Compiler Construction”

Martin Steffen

Spring 2022



Chapter 10

Learning Targets of Chapter “Code generation”.

1. 2AC
2. cost model
3. register allocation
4. control-flow graph
5. local liveness analysis (data flow analysis)
6. “global” liveness analysis



Chapter 10

Outline of Chapter “Code generation”.

Intro

2AC and costs of instructions

Basic blocks and control-flow graphs

Liveness analysis (general)

Local liveness: dead or alive

Local liveness^{++}: Dependence graph

Global analysis

Code generation algo



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Intro

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Code generation

- note: *code generation* so far: AST$^+$ to intermediate code
  - three address intermediate code (3AIC)
  - P-code
- $\Rightarrow$ *intermediate code generation*
- i.e., we are still not there ...
- material here: based on the (old) *dragon book* [2] (but principles still ok)
- there is also a new edition [1]



Intro: code generation

- goal: translate intermediate code (= 3AI-code) to machine language
- machine language/assembler:
  - even *more* restricted
  - here: 2 address code
- limited number of *registers*
- different *address modes* with different *costs* (registers vs. main memory)

Goals

- efficient code
- small code size also desirable
- but first of all: correct code



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Code "optimization"

- often conflicting goals
- code generation: *prime* arena for achieving *efficiency*
- optimal code: undecidable anyhow (and: don't forget there's trade-offs).
- even for many more clearly defined subproblems: *untractable*

"optimization"

interpreted as: *heuristics* to achieve "good code" (without hope for *optimal* code)

- due to importance of optimization at code generation
  - time to bring out the "heavy artillery"
  - so far: all techniques (parsing, lexing, even sometimes type checking) are computationally "easy"
  - at code generation/optimization: perhaps *invest* in aggressive, computationally complex and rather advanced techniques
  - many different techniques used



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2AC and costs of instructions

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2-address machine code used here

- "typical" op-codes, but not a instruction set of a *concrete* machine
- two address instructions
- Note: cf. 3-address-code intermediate representation vs. 2-address machine code
  - machine code is not lower-level/closer to HW because it has one argument less than 3AC
  - it's just one illustrative choice
  - the new Dragon book: uses 3-address-machine code
- translation task from IR to 3AC or 2AC: comparable challenge



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2-address instructions format

Format

OP source dest

- note: *order* of arguments here (esp. for minus)
- restrictions on *source* and *target*
  - register or memory cell
  - source: can additionally be a constant

```
ADD a b   // b := b + a
SUB a b   // b := b - a
MUL a b   // b := b * a
GOTO i    // unconditional jump
```

- further opcodes for conditional jumps, procedure calls ....



Side remarks: 3A machine code

Possible format

```
OP source1 source2 dest
```

- but: what's the *difference* to 3A *intermediate* code?
- apart from a more restricted instruction set:
- restriction on the operands, for example:
  - only *one* of the arguments allowed to be a memory access
  - *no fancy addressing* modes (indirect, indexed ... see later) for memory cells, only for registers
- not "too much" memory-register traffic back and forth per machine instruction
- example:

$$\&x = \&y + *z$$

may be 3A-intermediate code, but not 3A-machine code



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Cost model

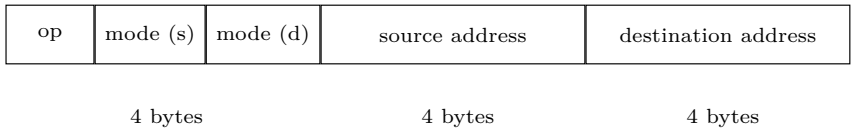
- "optimization": need some well-defined "measure" of the "quality" of the produced code
- interested here in *execution* time
- not all instructions take the same time
- estimation of execution
- factors outside our control/not part of the cost model: effect of *caching*

cost factors:

- *size* of instruction
  - it's here not about code size, but
  - instructions need to be *loaded*
  - longer instructions ⇒ perhaps longer load
- address modes (as *additional costs*: see later)
  - registers vs. main memory vs. constants
  - direct vs. indirect, or indexed access



Instruction modes and additional costs



mode	abbr.	address	**added cost**	
absolute	M	M	1	
register	R	R	0	
indexed	c(R)	$c + cont(R)$	1	
indirect register	*R	$cont(R)$	0	
indirect indexed	*c(R)	$cont(c + cont(R))$	1	
literal	$\#M$	the *value* M	1	only for source

Table: Cost model

- indirect: useful for elements in "records" with known off-set
- indirect: useful for slots in arrays

Examples a := b + c

Using registers

```
MOV b, R0   // R0 = b
ADD c, R0   // R0 = c + R0
MOV R0, a   // a = R0
```

Listing: Using registers, costs=6

Memory-memory ops

```
MOV b, a    // a = b
ADD c, a    // a = c + a
```

Listing: Memory-memory ops, costs=6

Addresses in registers

```
MOV *R1, *R0  // *R0 = *R1
ADD *R2, *R1  // *R1 = *R2 +
```

Listing: Addresses in registers, costs=2

Assume `R0`, `R1`, and `R2` contain *addresses* for `a`, `b`, and `c`

Storing back to memory

```
ADD R2, R1   // R1 = R2 + R1
MOV R1, a    // a = R1
```

Listing: Storing back to memory, costs=3

Assume `R1` and `R2` contain *values* for `b`, and `c`





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Basic blocks and control-flow graphs

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Basic blocks

- machine code level equivalent of straight-line code
- (a largest possible) sequence of instructions without
  - jump out
  - jump in
- elementary unit of code analysis/optimization[1]
- amenable to analysis techniques like
  - static simulation/symbolic evaluation
  - abstract interpretation
- basic unit of code generation

[1] Those techniques can also be used across basic blocks, but then they become more costly and challenging.



Control-flow graphs

CFG

basically: *graph* with

- nodes = basic blocks
- edges = (potential) jumps (and "fall-throughs")

- here (as often): CFG on 3AIC (linear intermediate code)
- also possible CFG on low-level code,
- or also:
  - CFG extracted from AST[2]
  - here: the opposite: synthesizing a CFG from the linear code
- explicit data structure (as another intermediate representation) or implicit only.

[2] See also the exam 2016.



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From 3AC to CFG: "partitioning algo"

- remember: 3AIC contains *labels* and (conditional) jumps

$\Rightarrow$ algo rather straightforward

- the only complication: some labels can be ignored
- we ignore procedure/method calls here
- concept: "leader" representing the nodes/basic blocks

Leader

- first line is a leader
- $\mathbf{GOTO}\, i$: line labelled i is a leader
- instruction *after* a $\mathbf{GOTO}$ is a leader

Basic block

instruction sequence from (and including) one leader to (but excluding) the next leader or to the end of code



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Partitioning algo

```
    ..............
    ..............
    ..............
    if ... goto L5
L1  ..............
L2  ..............
    ..............
    ..............
    goto L3
L5  ..............
    ..............
L3  ..............
    ..............
    if ... goto L1
    ..............
    goto L3
```

- note: no line jumps to L_2



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Partitioning algo

```
    ..............
    ..............
    ..............
    if ... goto L5
L1  ..............
L2  ..............
    ..............
    ..............
    goto L3
L5  ..............
    ..............
L3  ..............
    ..............
    if ... goto L1
    ..............
    goto L3
```

- note: no line jumps to L_2



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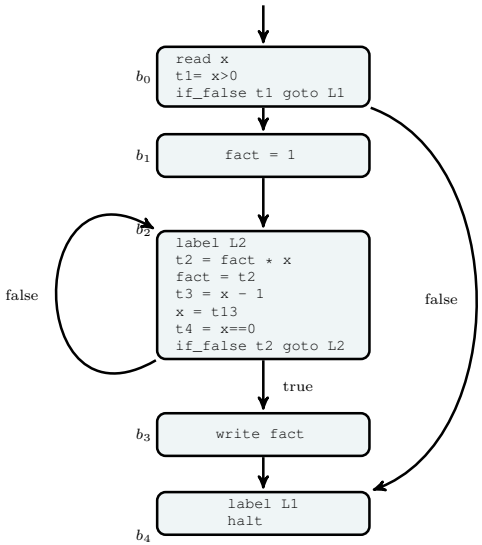
3AIC for faculty (from previous chapter)

```
1  read x
2  t1 = x > 0
3  if_false t1 goto L1
4  fact = 1
5  label L2
6  t2 = fact * x
7  fact = t2
8  t3 = x - 1
9  x = t3
10 t4 = x == 0
11 if_false t4 goto L2
12 write fact
13 label L1
14 halt
```



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Faculty: CFG



- goto/conditional goto: never *inside* block
- not every block
  - ends in a goto
  - starts with a label
- ignored here: function/method calls, i.e., focus on
- *intra-procedural* cfg

Levels of analysis

- here: *three* levels where to apply code analysis / optimizations
  1. local: per basic block (block-level)
  2. global: per function body/intra-procedural CFG
  3. inter-procedural: really global, whole-program analysis
- the "more global", the more *costly* the analysis and, especially the optimization (if done at all)



Loops in CFGs

- *loop optimization*: "loops" are thankful places for optimizations
- important for analysis to *detect* loops (in the cfg)
- importance of *loop discovery*: not too important any longer in modern languages.

Loops in a CFG vs. graph cycles

- concept of loops in CFGs not identical with cycles in a graph
- all loops are graph cycles but not vice versa

- intuitively: loops are cycles originating from source-level looping constructs ("while")
- goto's may lead to non-loop cycles in the CFG
- importance of loops: loops are "well-behaved" when considering certain optimizations/code transformations (goto's can destroy that...)



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Loops in CFGs



Loop

Loop L in a CFG: set of nodes, including header node $h \in L$:

1. any node in L: a path in L to h
2. a path in L from h to any node in L
3. no edge in the graph goes into h from outside L

often additional assumption/condition: "root" node of a CFG (there's only one) is *not* itself an entry of a loop

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Loop

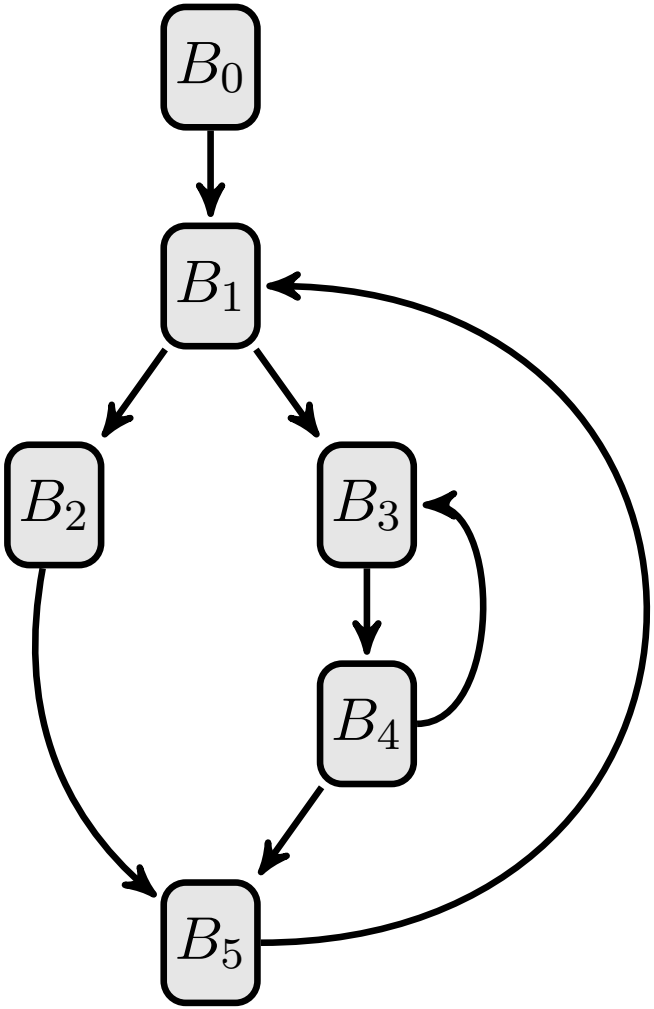
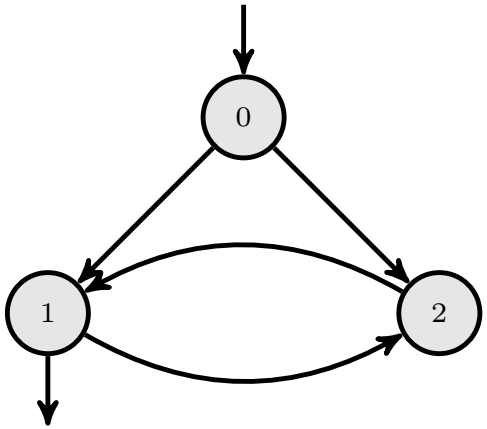


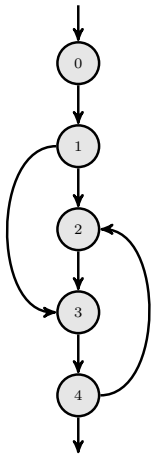
Figure: Loop example

- Loops:
  - $\{B_3, B_4\}$ (nested)
  - $\{B_4, B_3, B_1, B_5, B_2\}$
- Non-loop:
  - $\{B_1, B_2, B_5\}$
- unique entry marked red

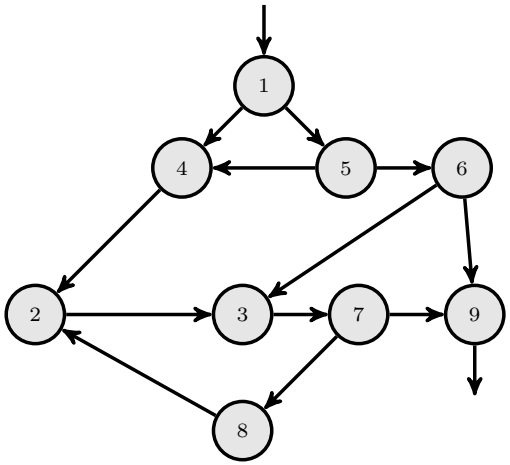
Loop non-examples



(a)



(b)



(c)

Figure: Non-loops

Loops as fertile ground for optimizations

```
while (i < n) {i++; A[i] = 3*k }
```

- possible optimizations
  - move `3*k` "out" of the loop
  - put frequently used variables into *registers* while in the loop (like i)
- when moving out computation from the loop:
- put it "right in front of the loop"

$\Rightarrow$ add extra node/basic block in front of the *entry* of the loop[3]

[3] That's one of the motivations for unique entry.



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Data flow analysis in general

- general *analysis technique* working on CFGs
- many concrete forms of analyses
- such analyses: basis for (many) *optimizations*
- *data*: info stored in memory/temporaries/registers etc.
- *control*:
  - movement of the instruction pointer
  - abstractly represented by the CFG
    - inside elementary blocks: increment of the instruction pointer
    - edges of the CFG: (conditional) jumps
    - jumps together with RTE and calling convention

Data flowing from (a) to (b)

Given the control flow (normally as CFG): is it *possible* or is it *guaranteed* ("may" vs. "must" analysis) that some "data" originating at one control-flow point (a) reaches control flow point (b).



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Data flow as abstraction

- data flow analysis DFA: fundamental and important *static* analysis technique
- it's impossible to decide statically if data from (a) *actually* "flows to" (b)

$\Rightarrow$ approximative (= abstraction)

- therefore: work on the CFG: if there are two options/outgoing edges: *consider both*
- Data-flow answers therefore approximatively
  - if it's *possible* that the data flows from (a) to (b)
  - it's *neccessary* or unavoidable that data flows from (a) to (b)
- for *basic blocks*: exact answers possible





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Liveness analysis (general)

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Data flow analysis: Liveness

- prototypical / important data flow analysis
- especially important for register allocation

Basic question

When (at which control-flow point) can I be *sure* that I don't need a specific variable (temporary, register) any more?

- optimization: if not needed for sure in the future: register can be used otherwise

Definition (Live)

A "variable" is live at a given control-flow point if there *exists* an execution starting from there (given the level of abstraction), where the variable is *used* in the future.



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Definitions and uses of variables

- talking about "variables": also temporary variables are meant.
- basic notions underlying most data-flow analyses (including liveness analysis)
- here: def's and uses of *variables* (or temporaries etc.)
- all data, including intermediate results, has to be stored somewhere, in variables, temporaries, etc.

Def's and uses

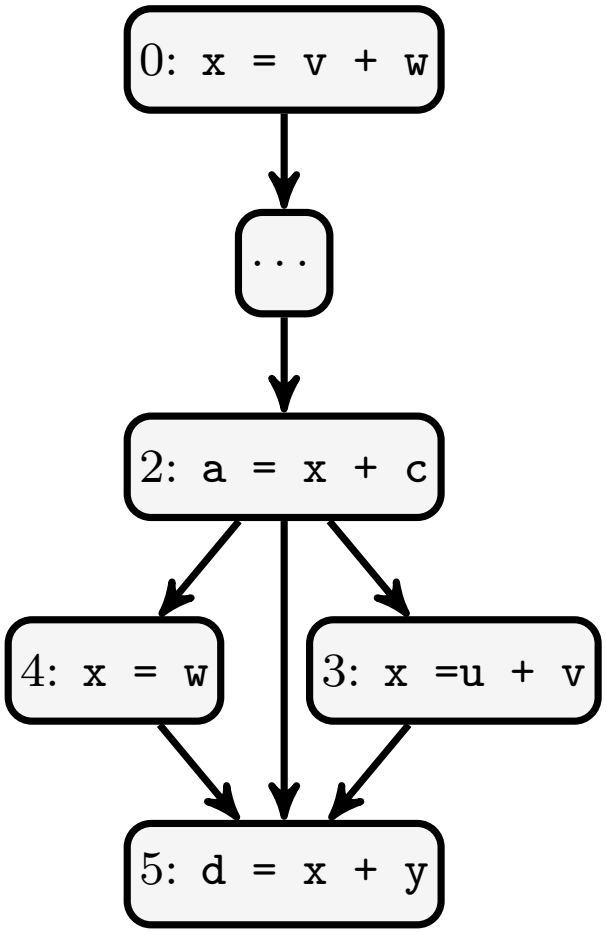
- a "definition" of x = assignment to x (store to x)
- a "use" of x: read content of x (load x)

- variables can occur more than once, so
- a definition/use refers to *instances* or *occurrences* of variables ("use of x in line l " or "use of x in block b ")
- same for liveness: "x is live here, but not there"



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Defs, uses, and liveness



- x is "defined" (= assigned to) in 0, 3, and 4
- u is live "in" (= at the end of) block 2, as it *may* be *used* in 3
- a *non-live* variable at some point: "dead", which means: the corresponding memory can be reclaimed
- *note*: here, liveness across block-boundaries = "global" (but blocks contain only one instruction here)

Def-use or use-def analysis

- use-def: given a "use": determine all possible "definitions"
- def-use: given a "def": determine all possible "uses"
- for straight-line-code/inside one basic block
  - deterministic: each line has has exactly one place where a given variable has been assigned to last (or else not assigned to in the block). Equivalently for uses.
- for whole CFG:
  - approximative ("may be used in the future")
  - more advanced techiques (caused by presence of loops/cycles)
- def-use analysis:
  - closely connected to liveness analysis (basically the same)
  - *prototypical* data-flow question (same for use-def analysis), related to many data-flow analyses (but not all)



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Calculation of def/uses (or liveness . . .)

- three levels of complication
  1. inside basic block
  2. branching (but no loops)
  3. Loops
  4. [even more complex: inter-procedural analysis]

For SLC/inside basic block

- deterministic result
- simple "one-pass" treatment enough
- similar to "static simulation"
- [Remember also AG's]

For whole CFG

- *iterative* algo needed
- dealing with non-determinism: over-approximation
- "closure" algorithms, similar to the way e.g., dealing with *first* and *follow* sets
- = fix-point algorithms



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Local liveness: dead or alive

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Inside one block: optimizing use of temporaries

- simple setting: *intra*-block analysis & optimization, only
- temporaries:
  - symbolic representations to hold intermediate results
  - generated on request, assuming unbounded numbers
  - intention: use registers
- limited about of register available (platform dependent)

Assumption about temps (here)

- temp's *don't transfer* data across blocks ($\neq$ program var's)

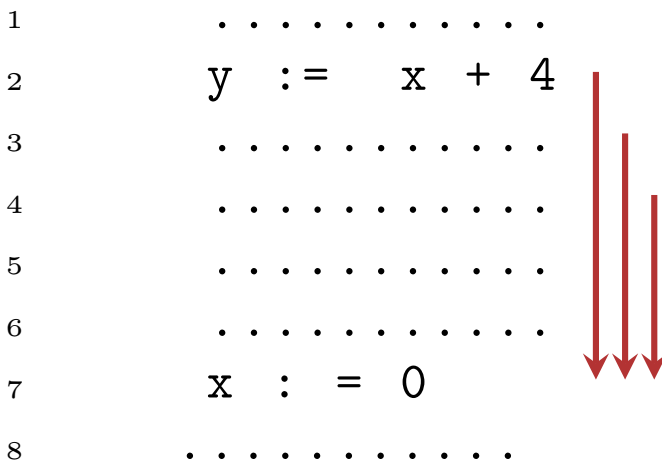
$\Rightarrow$ temp's *dead* at the beginning and at the end of a block

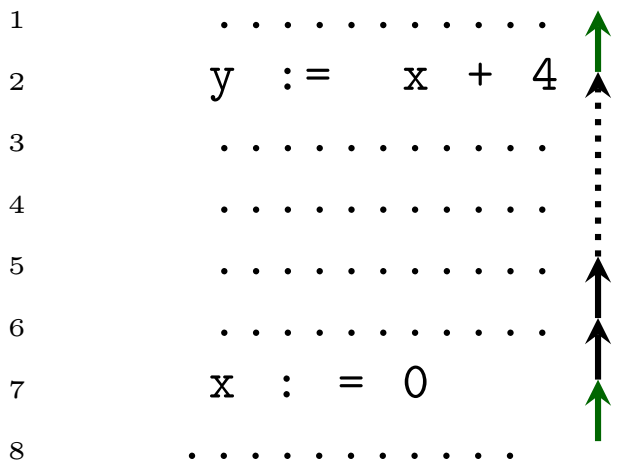
- but: variables have to be *assumed* live at the end of a block (block-local analysis, only)



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Forward vs backward





Intra-block liveness

```
t1 := a - b
t2 := t1 * a
a  := t1 * t2
t1 := t1 - c
a  := t1 * a
```

- neither temp's nor vars in the example are "single assignment",
- but first occurrence of a temp in a block: a definition (but for temps it would often be the case, anyhow)
- let's call *operand:* variables or temp's
- uses of operands: on the rhs's, definitions on the lhs's
- not good enough to say "t_1 is live in line 4" (why?)



Single step per line: transfer function

- liveness-status of an operand: *different* from lhs vs. rhs in a given instruction
- informal definition: an operand is live at some occurrence, if it's used some place in the future

Definition (consider statement $x_1 := x_2 \ op \ x_3$)

- A variable x is live at the *beginning* of $x_1 := x_2 \ op \ x_3$, if
  1. if x is x_2 or x_3, or
  2. if x live at its *end*, if x and x_1 are different variables
- A variable x is live at the *end* of an instruction,
  - if it's live at *beginning of the next* instruction
  - if no next instruction
    - temp's are dead
    - user-level variables are (assumed) live



Algo: dead or alive (binary info)

```
1  // ------ initialise T  ------------------------------------------
2    for all entries: T[i,x] := D
3    except: for all variables a // but not temps
4                T[n,a] := L,
5  //---------- backward pass ------------------------------------------
6  for instruction i =  n-1 down to 0
7     let current instruction at i+1: x := y op z;
8        T[i,o] := T[i+1,o] (for all other vars o)
9        T[i,x] := D // note order; x can ``equal'' y or z
10       T[i,y] := L
11       T[i,z] := L
12 end
```

- Data structure T: table, mapping for each line/instruction i and variable: boolean status of "live"/"dead"
- represents liveness status per variable *at the end (i.e. rhs)* of that line
- basic block: n instructions, from 1 until n, where "line 0" represents the "sentry" imaginary line "before" the first line (no instruction in line 0)
- *backward scan* through instructions/lines from n to 0



Run of of the algo

line	a	b	c	t_1	t_2
[0]	L	L	L	D	D
1	L	L	L	L	D
2	D	L	L	L	L
3	L	L	L	L	D
4	L	L	L	L	D
5	L	L	L	D	D

Table: Liveness analysis example: result of the analysis



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Local liveness^{++}: Dependence graph

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Adding information

- more refined information
- not just binary dead-or-alive but next-use info

$\Rightarrow$ three kinds of information

1. Dead: D
2. Live:
   - with *local* line number of *next use*: $L(n)$
   - *potential* use of outside local basic block $L(\perp)$

- otherwise: basically the same algo



Algo: alive with next use

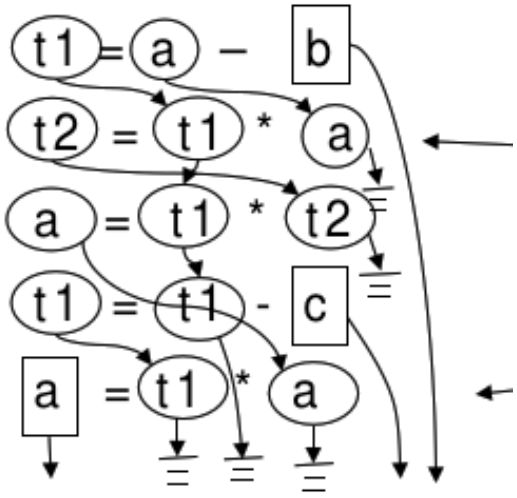
INCLUDE: "../code/codegen/liveness-slc.code" src pseudoe

```
1  // ------ initialise T  ------------------------------------------
2    for all entries: T[i,x] := D
3    except: for all variables a // but not temps
4                T[n,a] := L(⊥),
5  //---------- backward pass ------------------------------------------
6  for instruction i = n-1 down to 0
7     let current instruction at i+1: x := y op z;
8        T[i,o] := T[i+1,o] (for all other vars o)
9        T[i,x] := D // note order; x can ``equal'' y or z
10       T[i,y] := L(i+1)
11       T[i,z] := L(i+1)
12 end
```


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Run of the algo

line	a	b	c	t_1	
[0]	$L(1)$	$L(1)$	$L(4)$	D	
1	$L(2)$	$L(\perp)$	$L(4)$	$L(2)$	
2	D	$L(\perp)$	$L(4)$	$L(3)$	
3	$L(5)$	$L(\perp)$	$L(4)$	$L(4)$	
4	$L(5)$	$L(\perp)$	$L(\perp)$	$L(5)$	
5	$L(\perp)$	$L(\perp)$	$L(\perp)$	D	D



INCLUDE: "../code/codegen/liveinblock1.tac" src tac

Dependency graph and def-use

- small step from next-use of all-future-uses

Def-use analysis

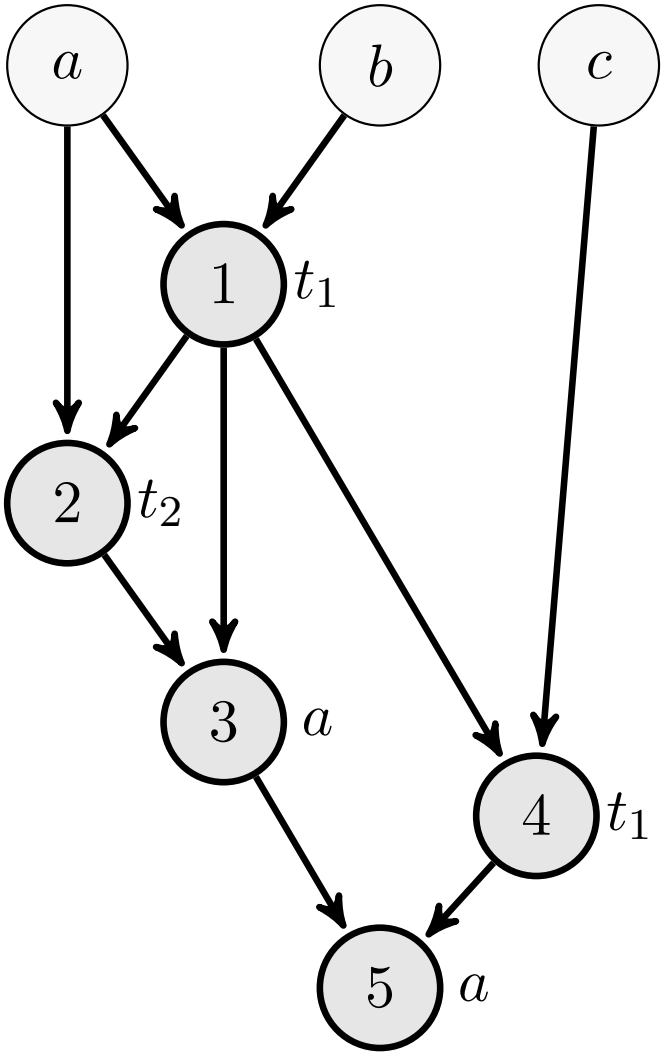
Connect definitions with all their uses ⇒ dependency graph

- straight-line code
- acyclic graph ⇒ DAG (or partial order)
- nodes: (lines of) instructions (or variable instnce



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DAG of the block



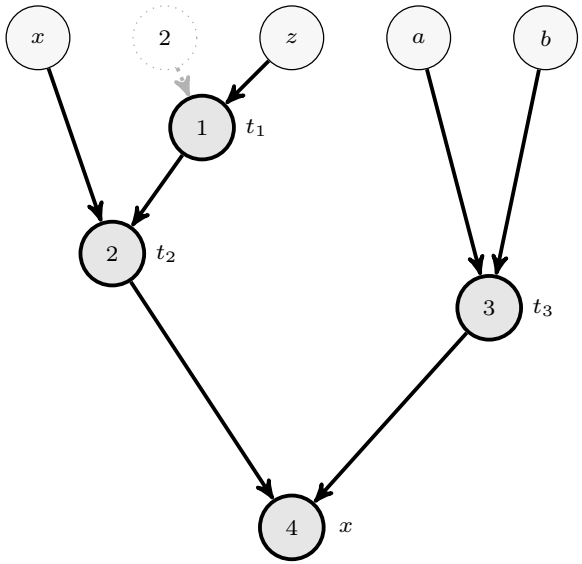
INCLUDE: "../code/code-gen/liveinblock1.tac" src tac

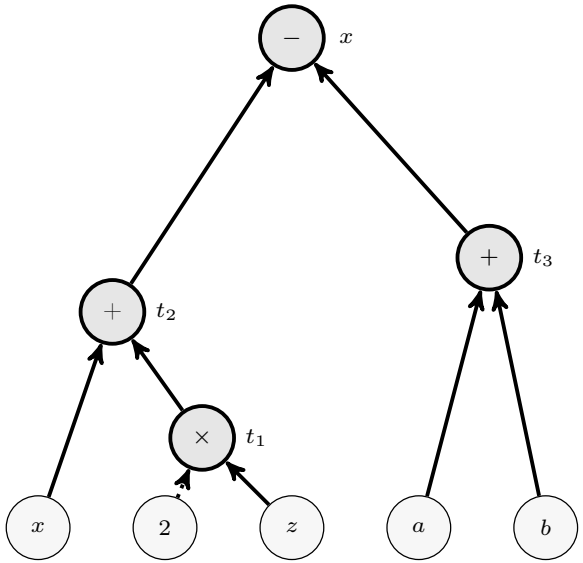
- no linear order
code), only *pa*
- *the* next use:
- but: *all "next*
visible
- node $\equiv$ occur
variable
- e.g.: the "low
"defining" t_1
uses
- different "vers
(instances) of

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Dependence graphs and ASTs

+include: "../code/codegen/dag-ofexpression-pure.tac" src tac





(S)SA format

INCLUDE:
"../code/codegen/liveinblock1-sa.tac" src tac

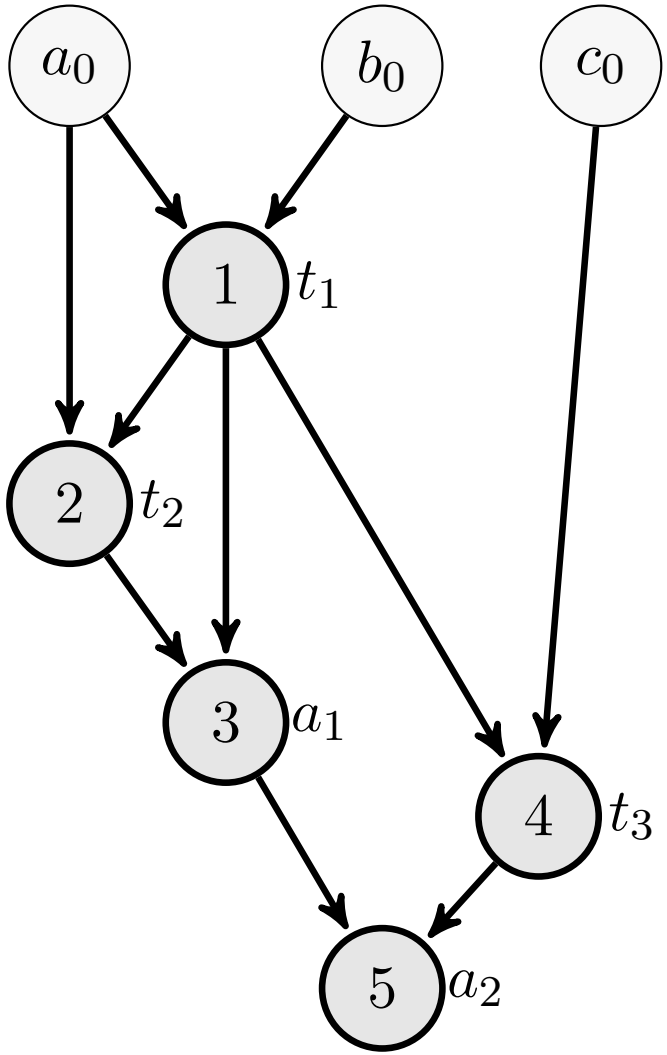


Figure: DAG for the 3AIC code block

Liveness

Previous "inductive" definition

expresses liveness status of variables *before* a statement dependent on the liveness status of variables *after* a statement (and the variables used in the statement)

- *core* of a straightforward iterative algo
- simple backward scan
- the algo we sketch:
  - not just boolean info (live = yes/no), instead:
  - operand live?
    - yes, and with next use inside is block (and indicate instruction where)
    - yes, but with no use inside this block
    - not live
  - even more info: not just that but indicate, where's the next use



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Section

Global analysis

Chapter 10 “Code generation”
Course “Compiler Construction”
Martin Steffen
Spring 2022

From “local” to “global” data flow analysis



- data stored in variables, and “flows from definitions to uses”
- liveness analysis
  - one *prototypical* (and important) data flow analysis
  - so far: *intra-block* = straight-line code
- related to
  - *def-use* analysis: given a “definition” of a variable at some place, where it is (potentially) used
  - *use-def*: (the inverse question, “reaching definitions”
- other similar questions:
  - has a value of an expression been calculated before (“available expressions”)
  - will an expression be used in all possible branches (“very busy expressions”)

Global data flow analysis

- block-local
  - block-local analysis (here liveness): *exact* information possible
  - block-local liveness: *1 backward scan*
  - important use of liveness: *register allocation*, temporaries typically don't survive blocks anyway
- global: working on complete CFG

2 complications

- branching: *non-determinism*, unclear which branch is taken
- loops in the program (loops/cycles in the graph): simple *one pass* through the graph does not cut it any longer

- *exact* answers no longer possible (undecidable)

$\Rightarrow$ work with safe approximations

- this is: general characteristic of DFA



Generalizing block-local liveness analysis

- *assumptions* for block-local analysis
  - all program variables (assumed) *live* at the end of each basic block
  - all temps are assumed *dead* there.
- now: we do better, info across blocks

at the end of each block:

which variables may be used in subsequent block(s).

- now: re-use of temporaries (and thus corresponding registers) across blocks possible
- remember local liveness algo: determined liveness status per var/temp *at the end* of each "line/instruction"



Connecting blocks in the CFG: *inLive* and *outLive*

- CFG:
  - pretty conventional graph (nodes and edges, often designated start and end node)
  - *nodes* = basic blocks = contain straight-line code (here 3AIC)
  - being conventional graphs:
    - conventional representations possible
    - E.g. nodes with lists/sets/collections of immediate *successor nodes* plus immediate *predecessor nodes*
- remember: local liveness status
  - can be different *before* and *after* one single instruction
  - liveness status *before* expressed as dependent on status *after*

  $\Rightarrow$ backward scan
- Now per block: *inLive* and *outLive*



inLive and *outLive*

- tracing / approximating set of live variables[4] at the *beginning* and *end* per basic block
- *inLive* of a block: depends on
  - *outLive* of that block and
  - the SLC inside that block
- *outLive* of a block: depends on *inLive* of the *successor* blocks

Approximation: To err on the safe side

Judging a variable (statically) live: always *safe*. Judging wrongly a variable *dead* (which actually will be used): unsafe

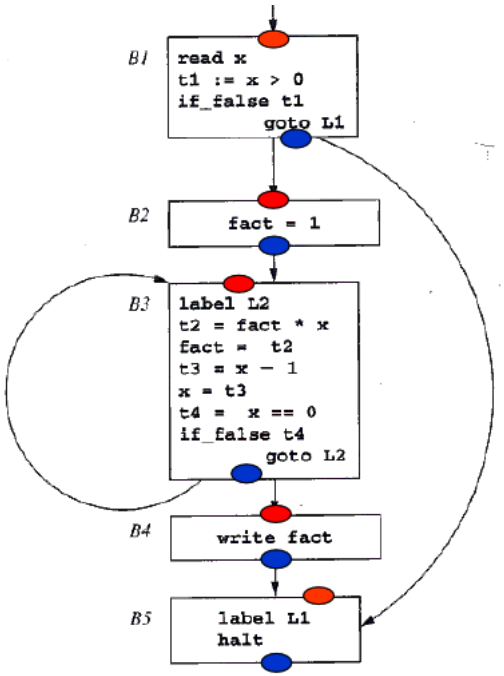
- goal: smallest (but safe) possible sets for *outLive* (and *inLive*)

[4] To stress "approximation": *inLive* and *outLive* contain sets of *statically* live variables. If those are dynamically live or not is undecidable.



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Example: Faculty CFG



- $inLive$ and $outLive$
- picture shows arrows as *successor nodes*
- needed *predecessor nodes* (reverse arrows)

node/block	predecessors
B_1	$\emptyset$
B_2	$\{B_1\}$
B_3	$\{B_2, B_3\}$
B_4	$\{B_3\}$
B_5	$\{B_1, B_4\}$

Block local info for global liveness/data flow analysis

- 1 CFG per procedure/function/method
- as for SLC: algo works backwards
- for each block: underlying block-local liveness analysis

3-valued block local status per variable

result of block-local live variable analysis

1. *locally live* on entry: variable used (before overwritten or not)
2. *locally dead* on entry: variable overwritten (before used or not)
3. status not locally determined: variable neither assigned to nor read locally

- for efficiency: *precompute* this info, before starting the global iteration $\Rightarrow$ avoid *recomputation* for blocks in loops



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Global DFA as iterative "completion algorithm"

- different names for the general approach
  - *closure* algorithm, *saturation* algo
  - *fixpoint* iteration
- basically: a big loop with
  - iterating a step approaching an intended solution by making current approximation of the solution *larger*
  - until the solution stabilizes
- similar (for example): calculation of first- and follow-sets
- often: realized as *worklist algo*
  - named after central data-structure containing the "work-still-to-be-done"
  - here possible: worklist containing nodes untreated wrt. liveness analysis (or DFA in general)

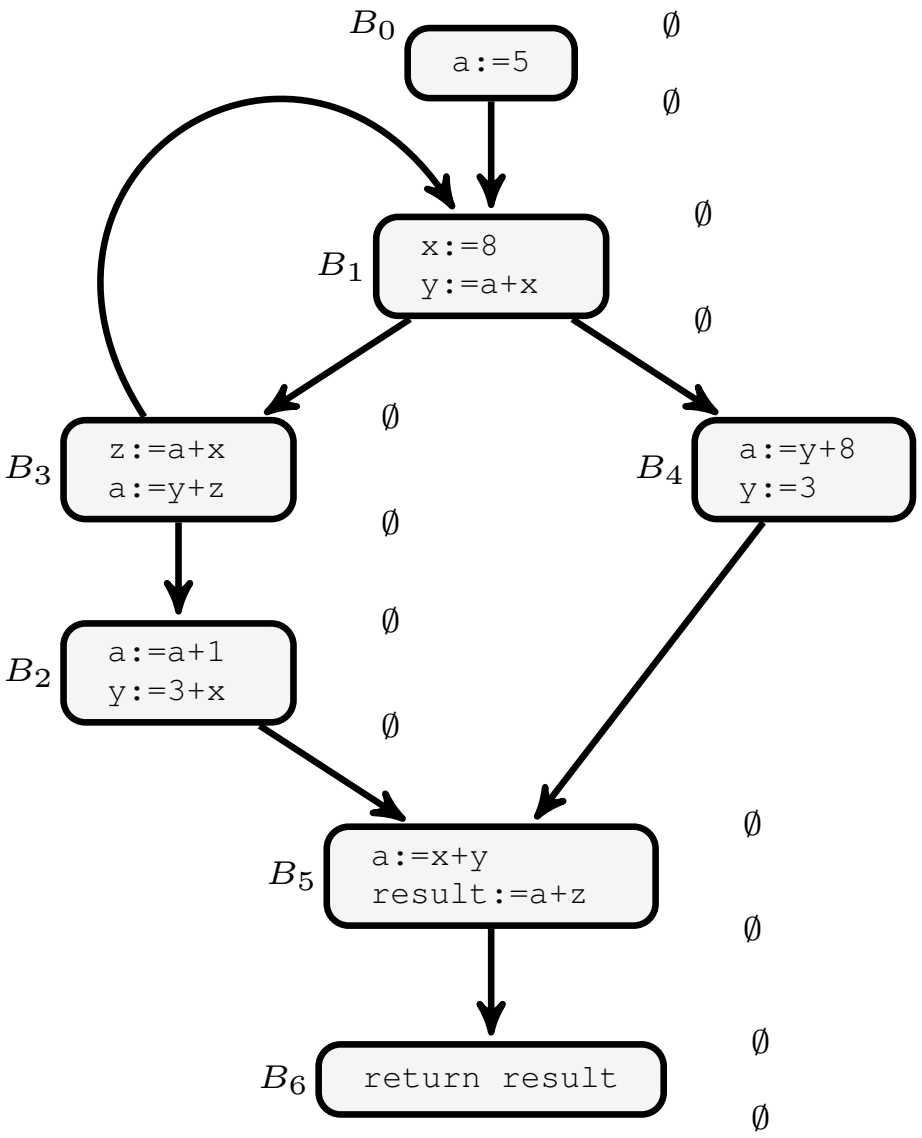


Example

```
1      a := 5
2  L1: x := 8
3      y := a + x
4      if_true x=0 goto L4
5      z := a + x          // B3
6      a := y + z
7      if_false a=0  goto L1
8      a := a + 1          // B2
9      y := 3 + x
10 L5  a := x + y
11     result := a + z
12     return result       // B6
13 L4: a := y + 8
14     y := 3
15     goto L5
```



CFG: initialization



- $inLive$ and $outLive$: *initialized* to $\emptyset$ everywere
- note: start with (most) *unsafe* estimation
- extra (return) node
- but: analysis here *local per procedure*, only

Iterative algo

General schema

Initialization start with the "minimal" estimation ($\emptyset$ everywhere)

Loop pick one node & update (= enlarge) liveness estimation in connection with that node

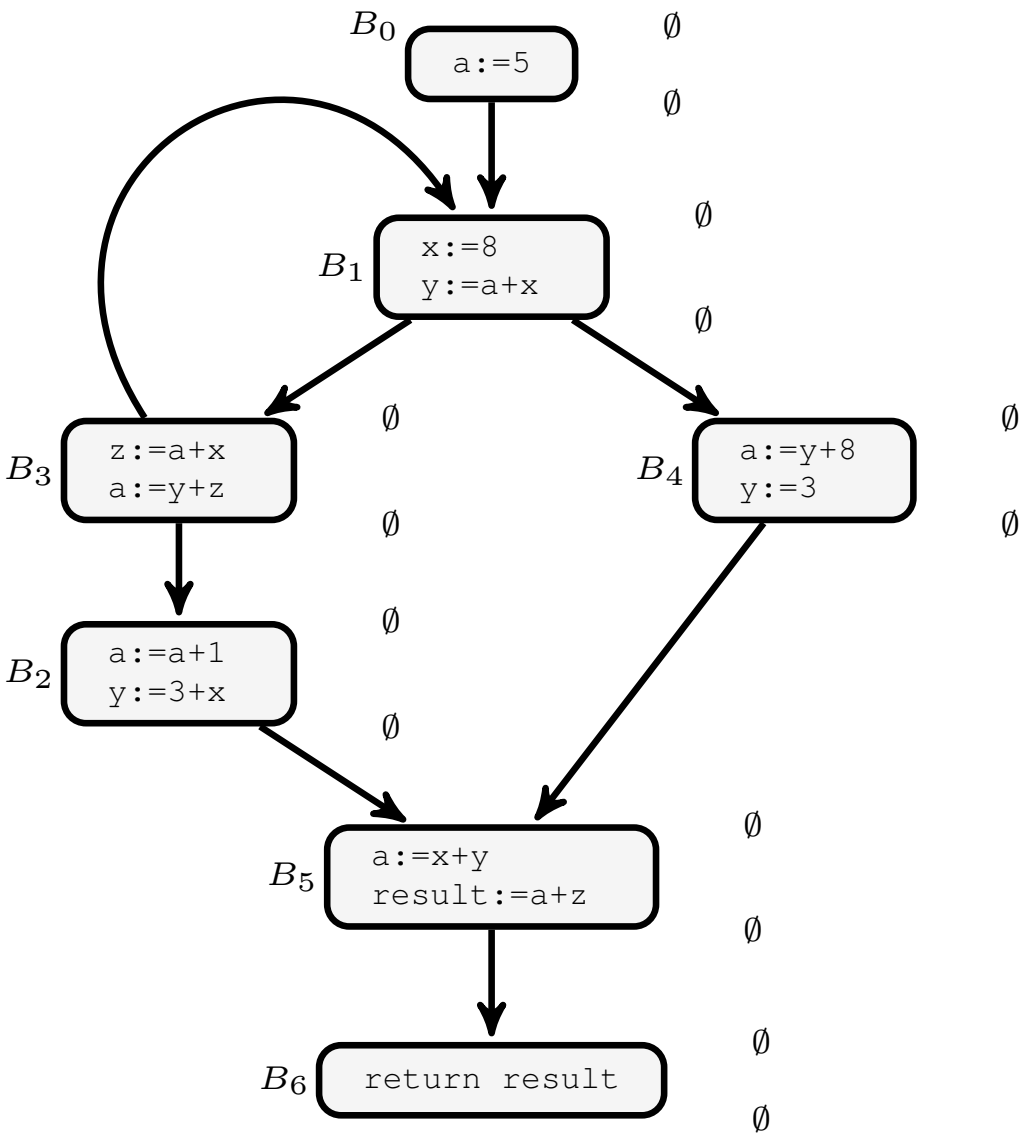
Until finish upon stabilization (= no further enlargement)

- order of treatment of nodes: in princple arbitrary[5]
- in tendency: following edges backwards
- comparison: for linear graphs (like inside a block):
  - no repeat-until-stabilize loop needed
  - 1 simple backward scan enough

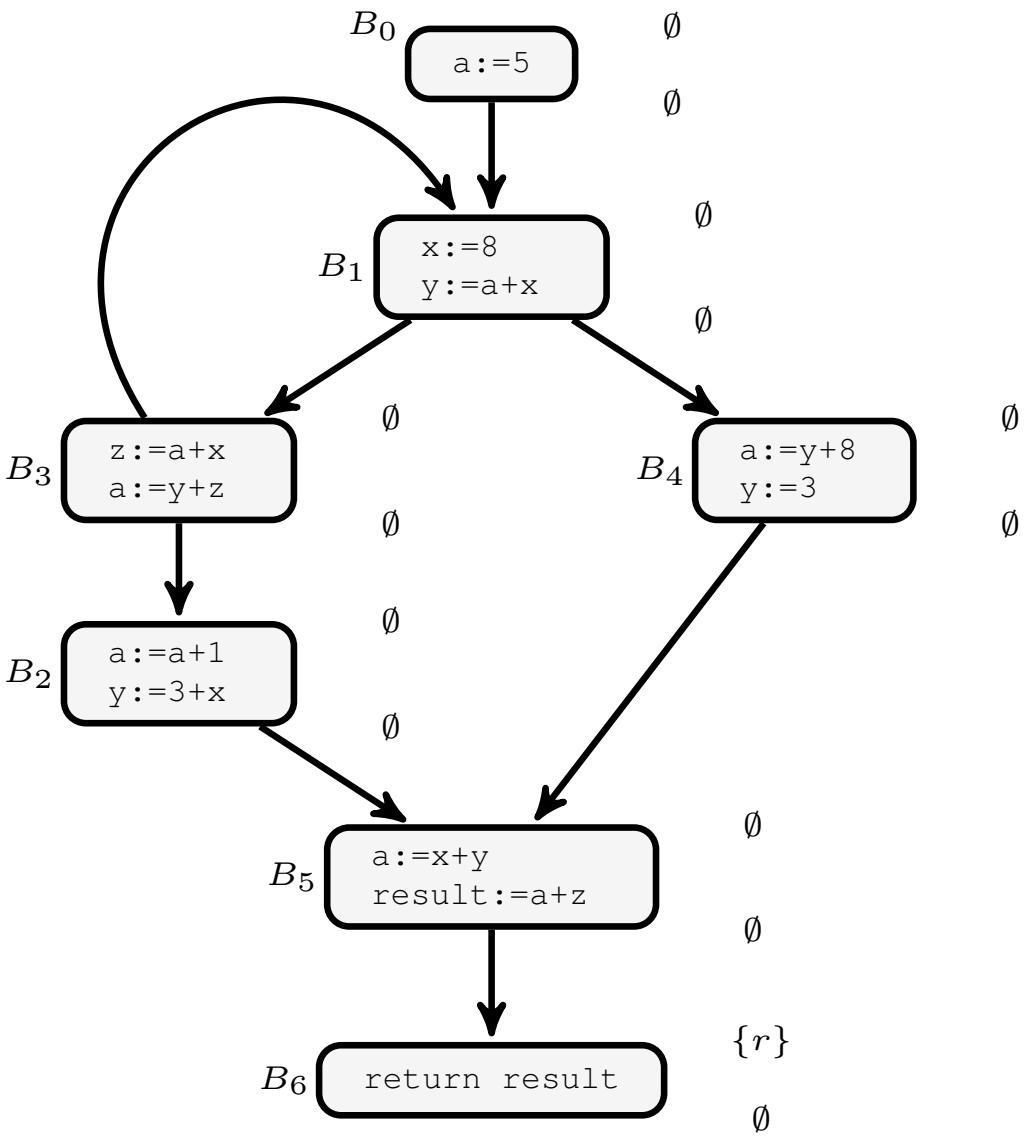
[5] There may be more efficient and less efficient orders of treatment.



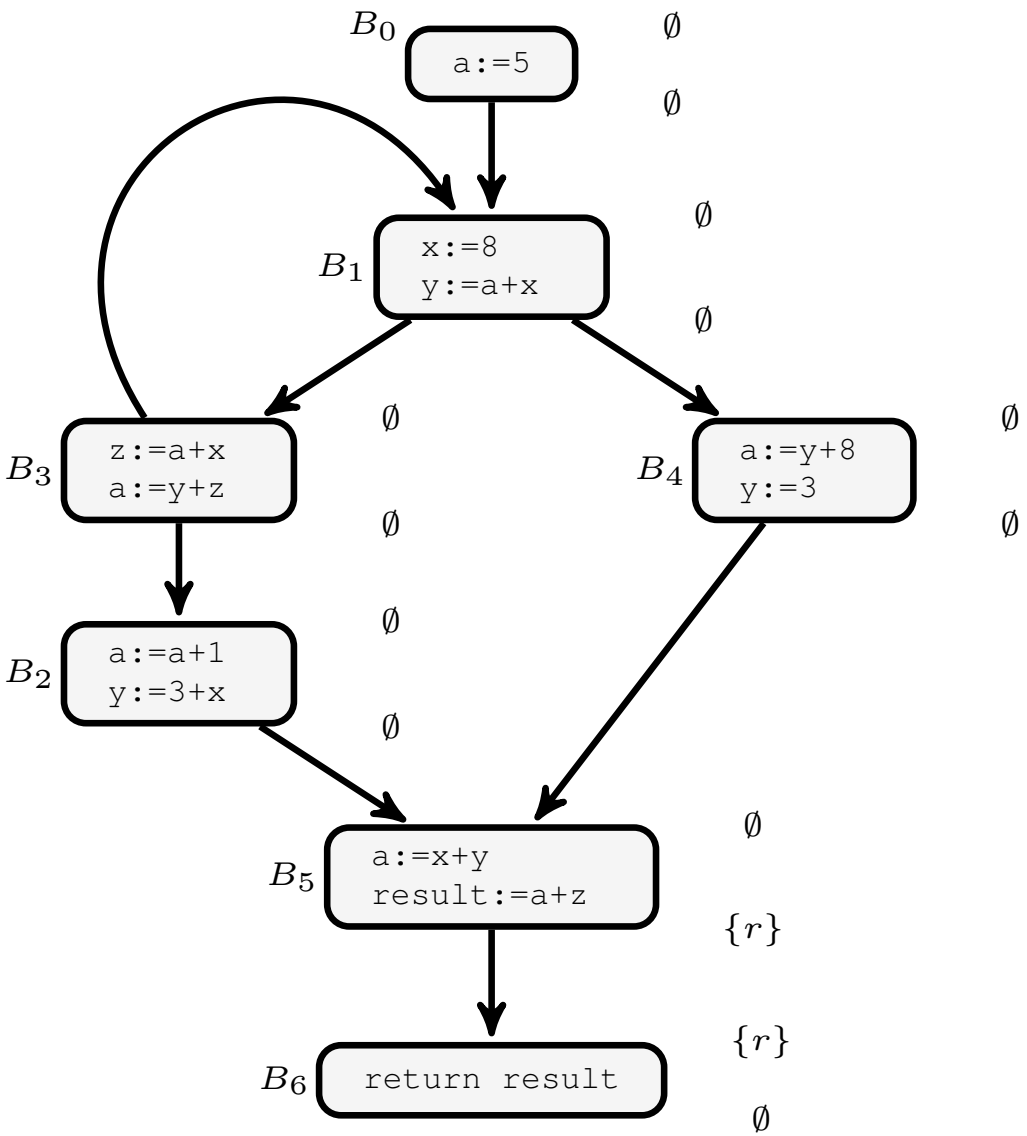
Liveness: run



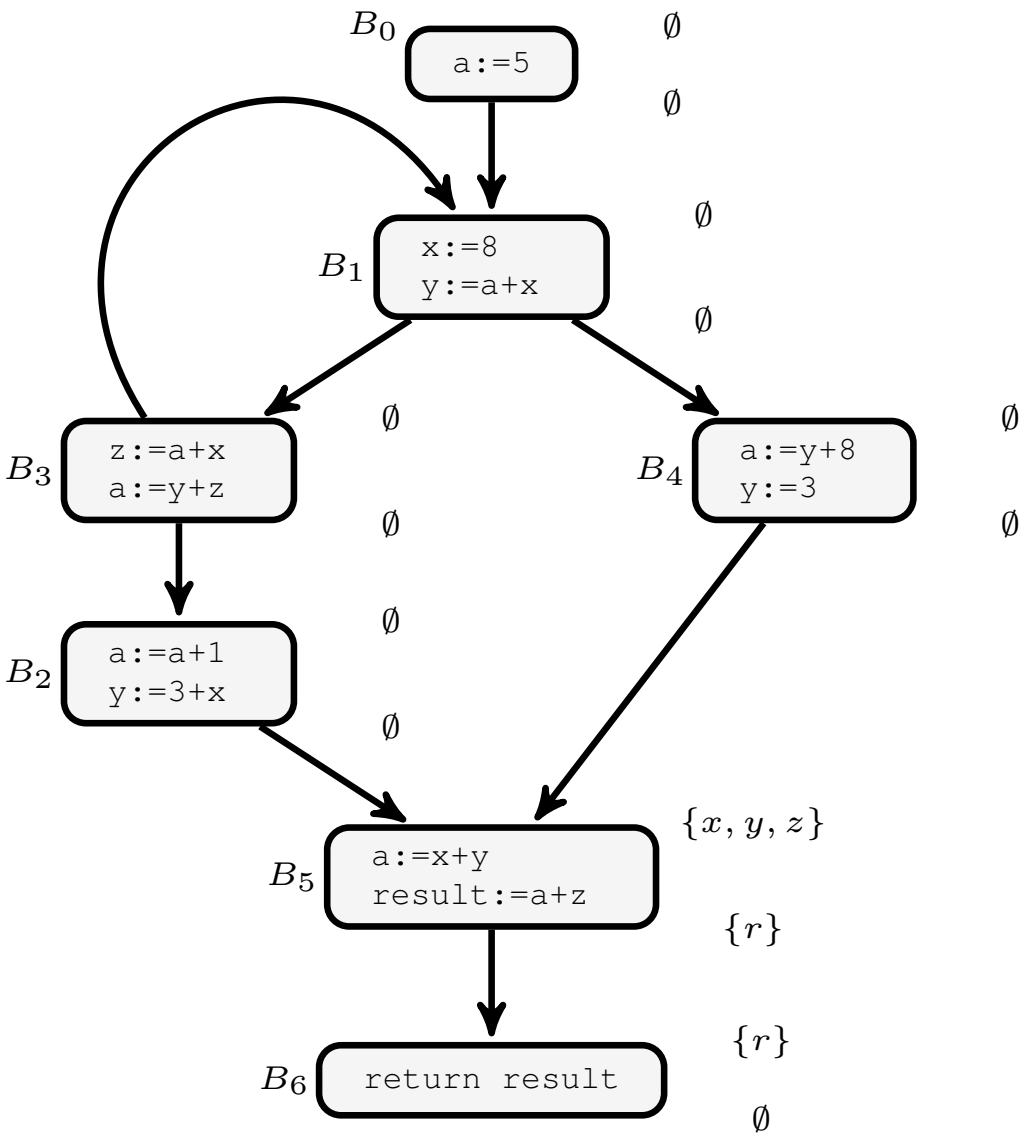
Liveness: run



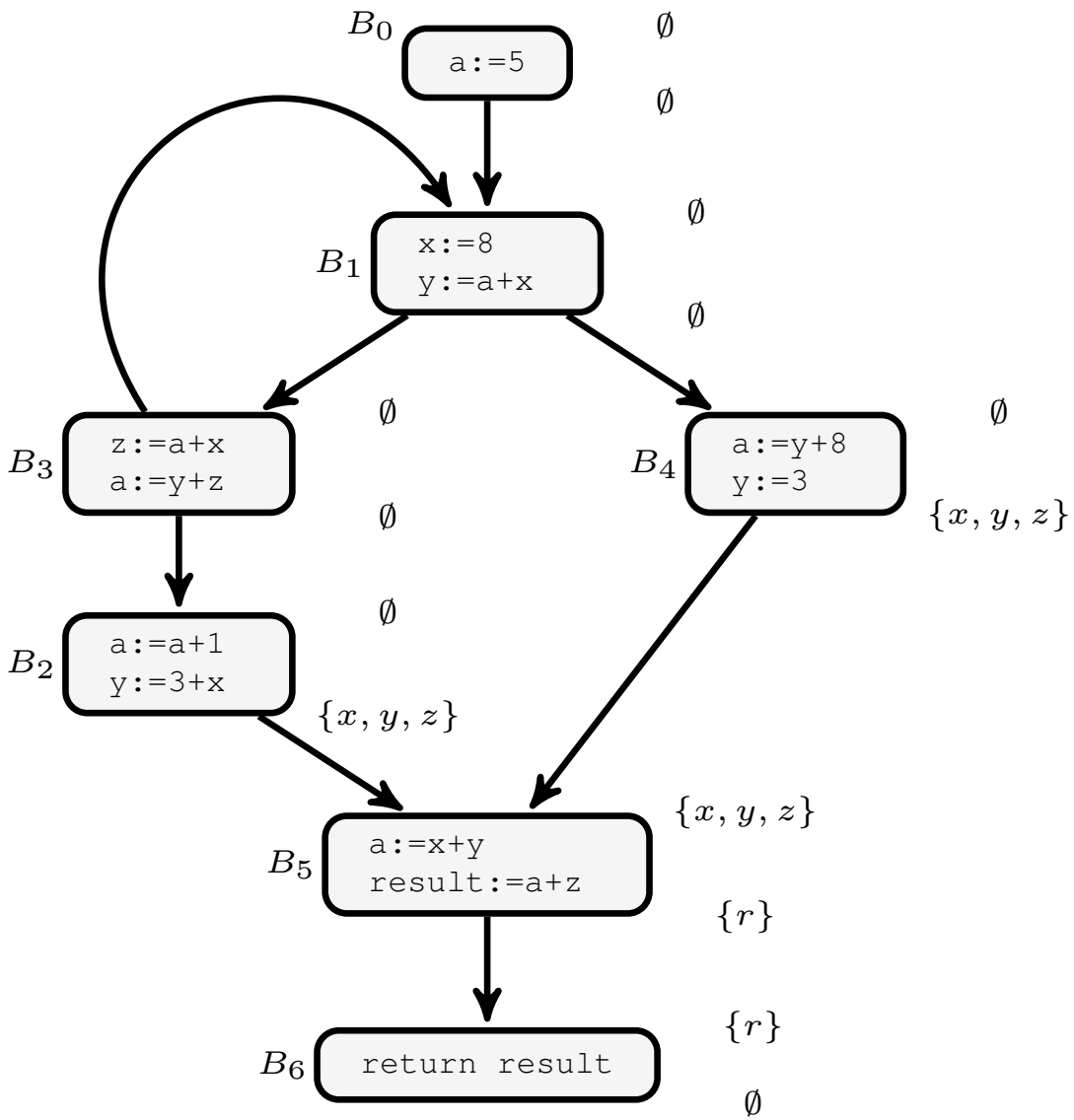
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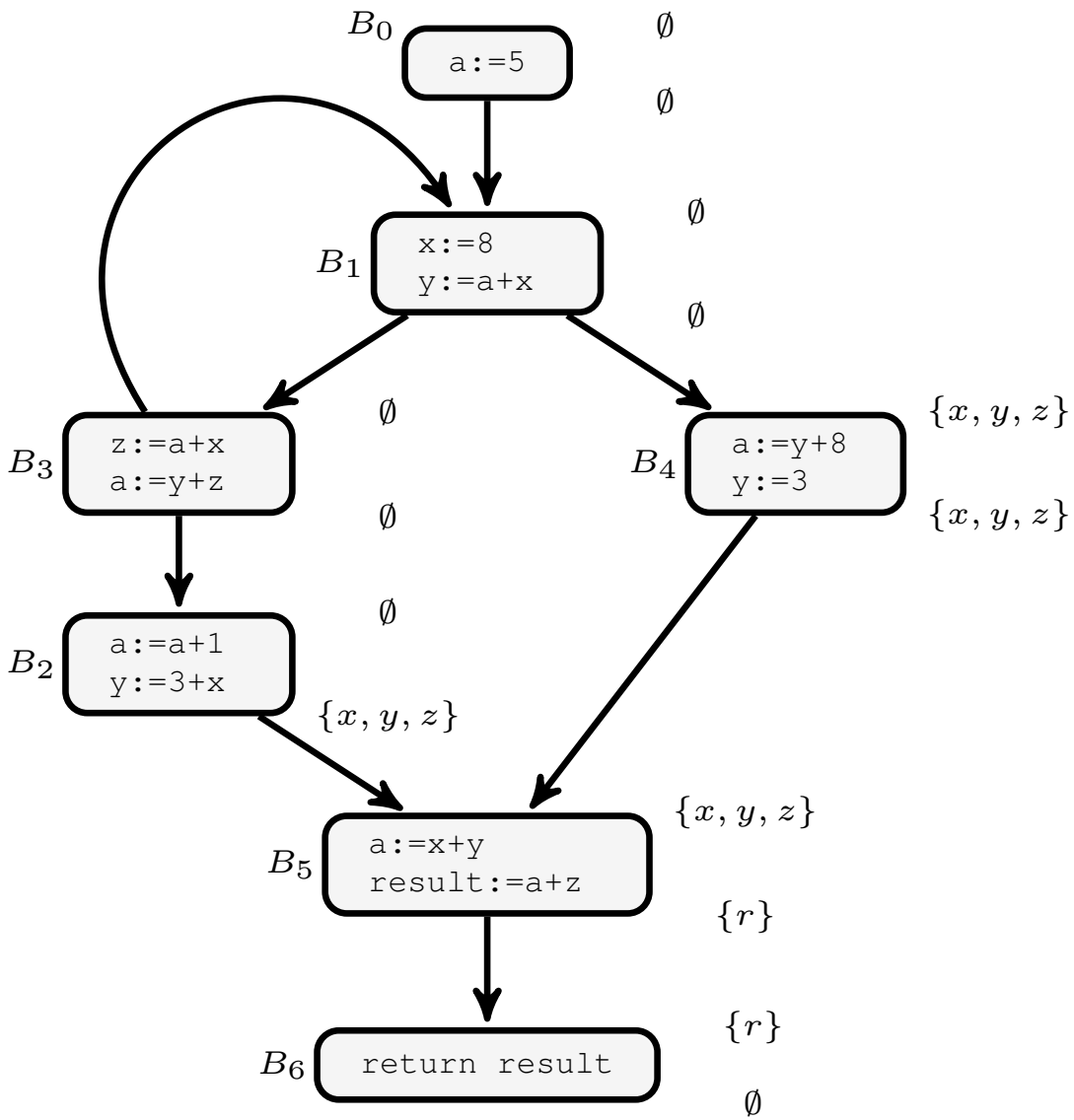
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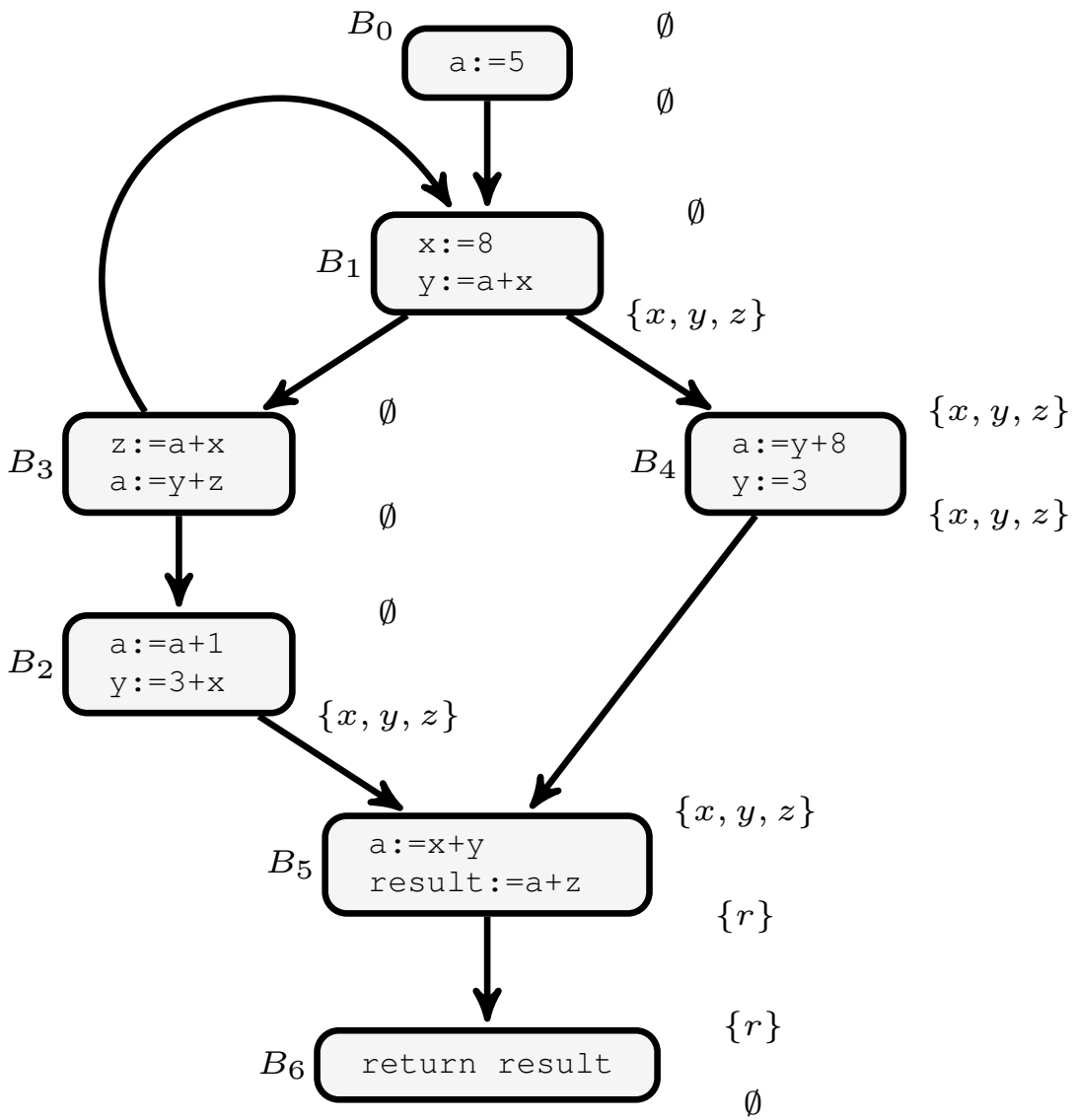
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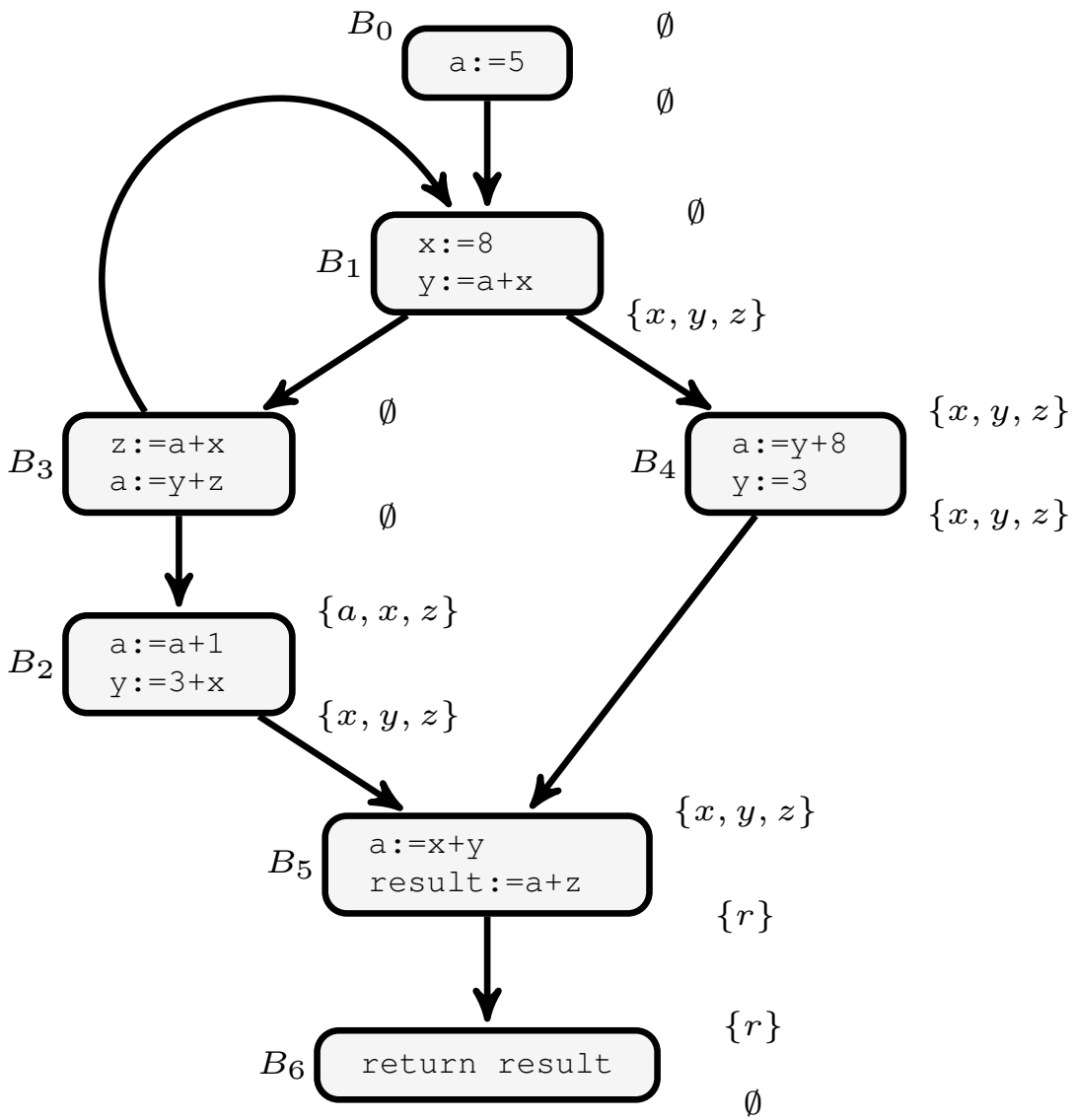
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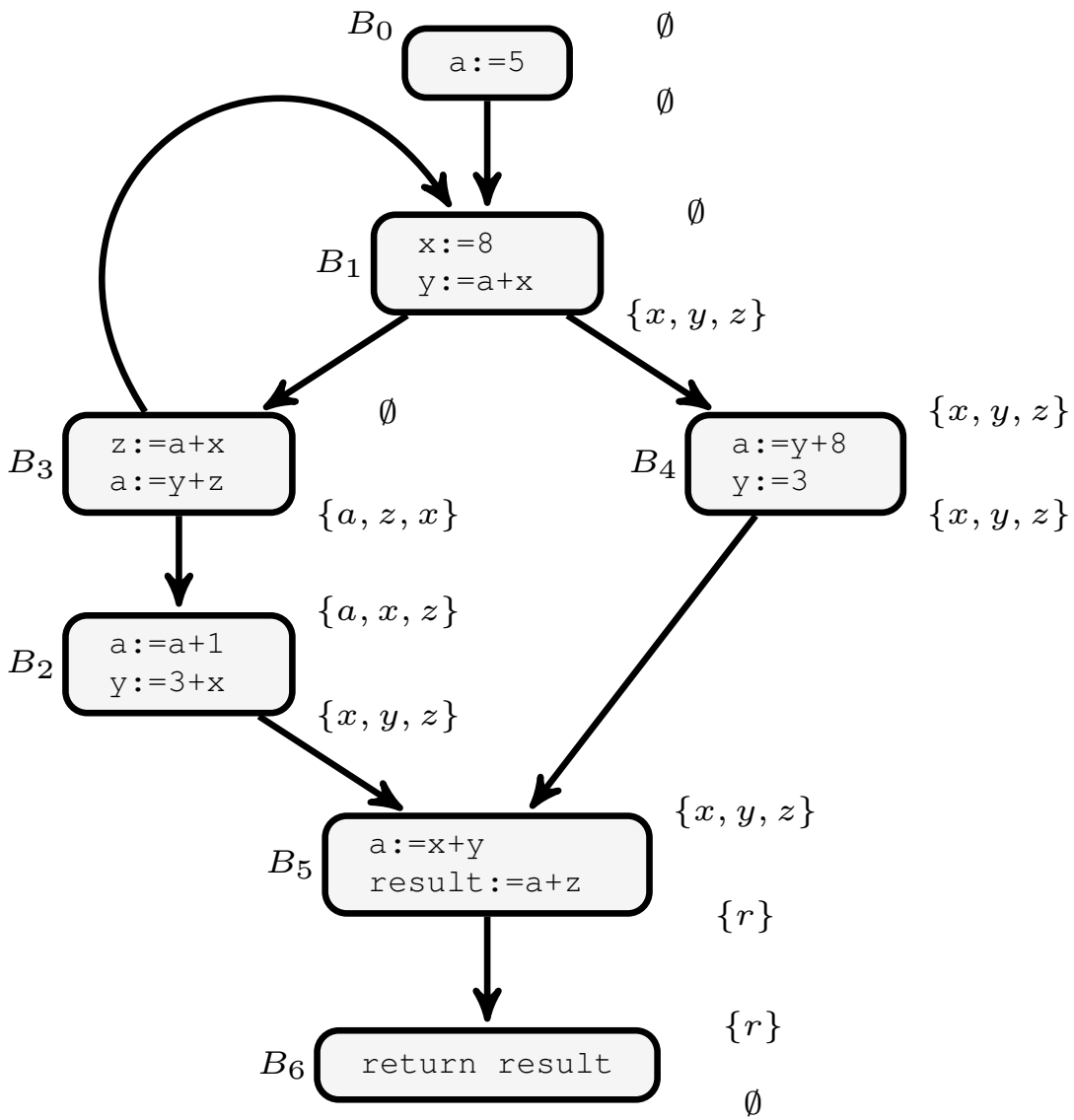
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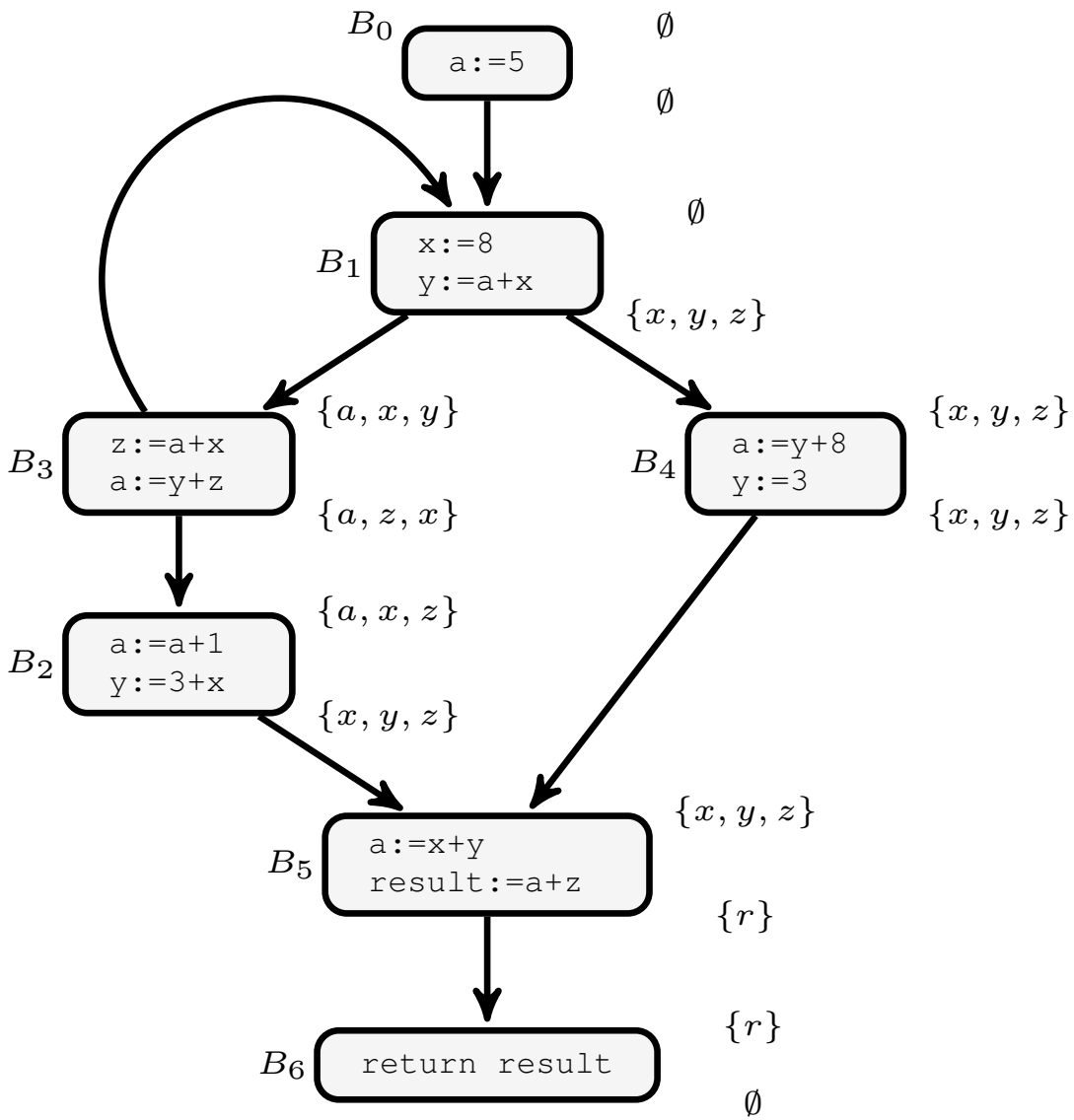
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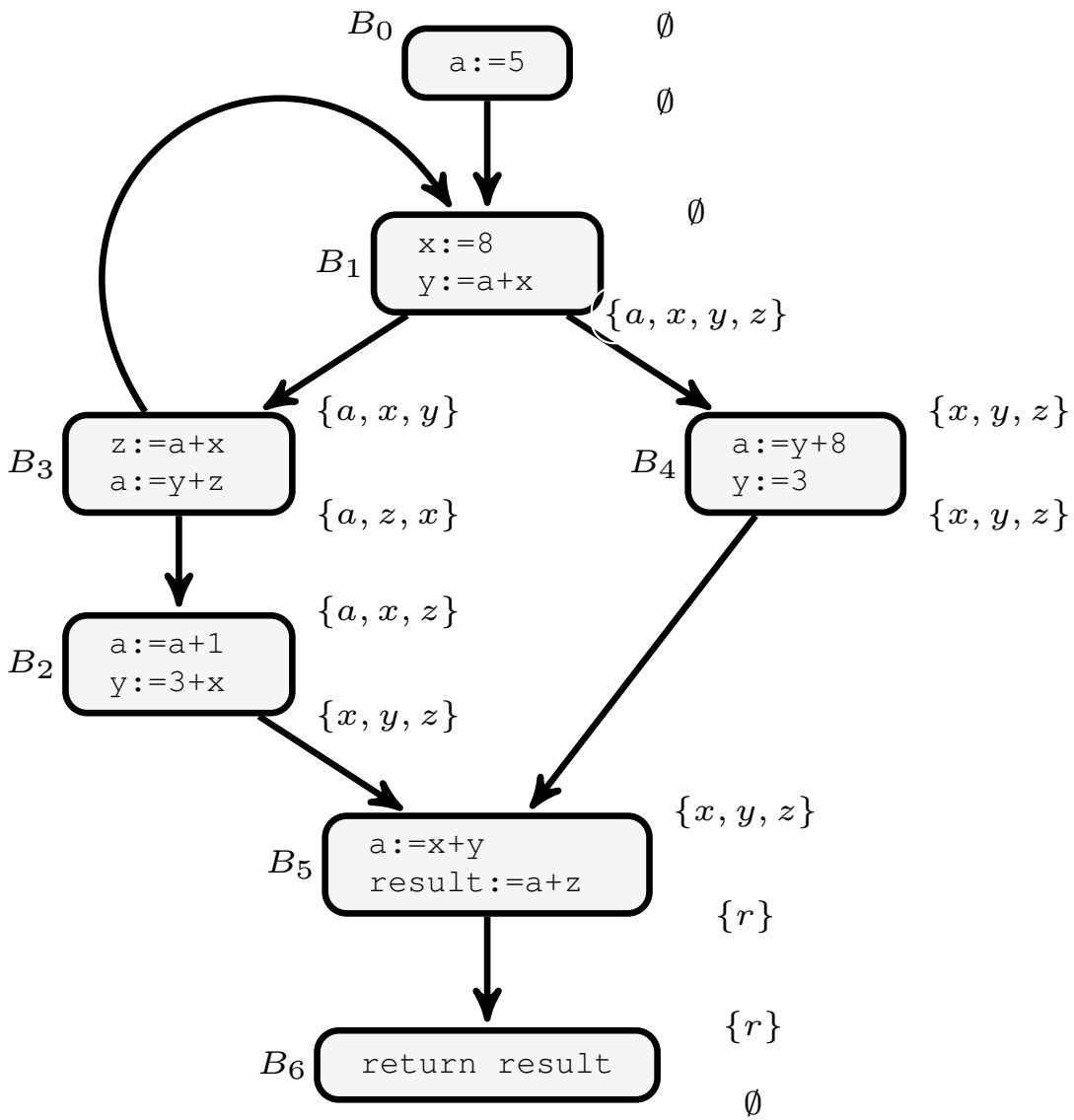
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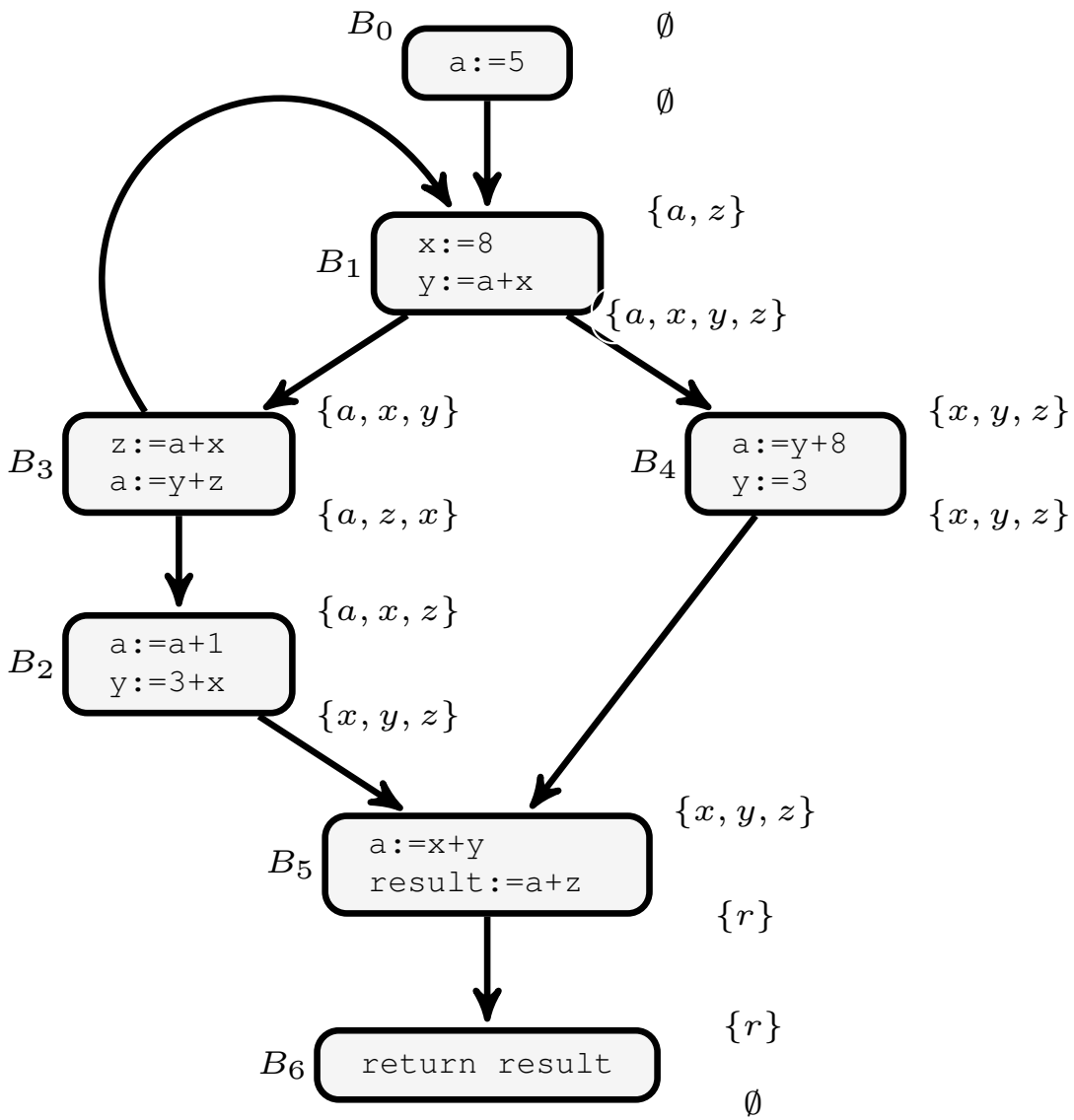
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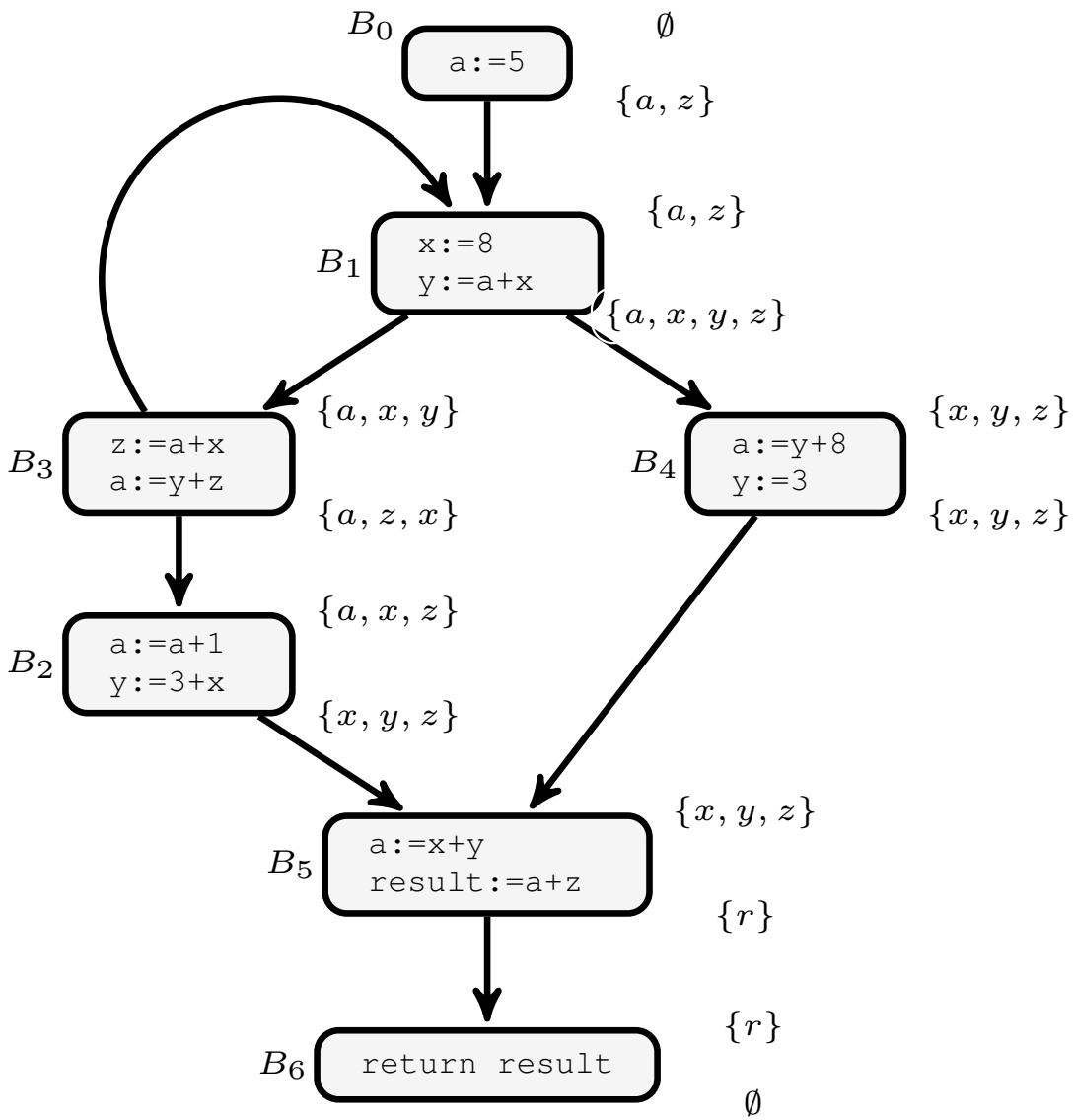
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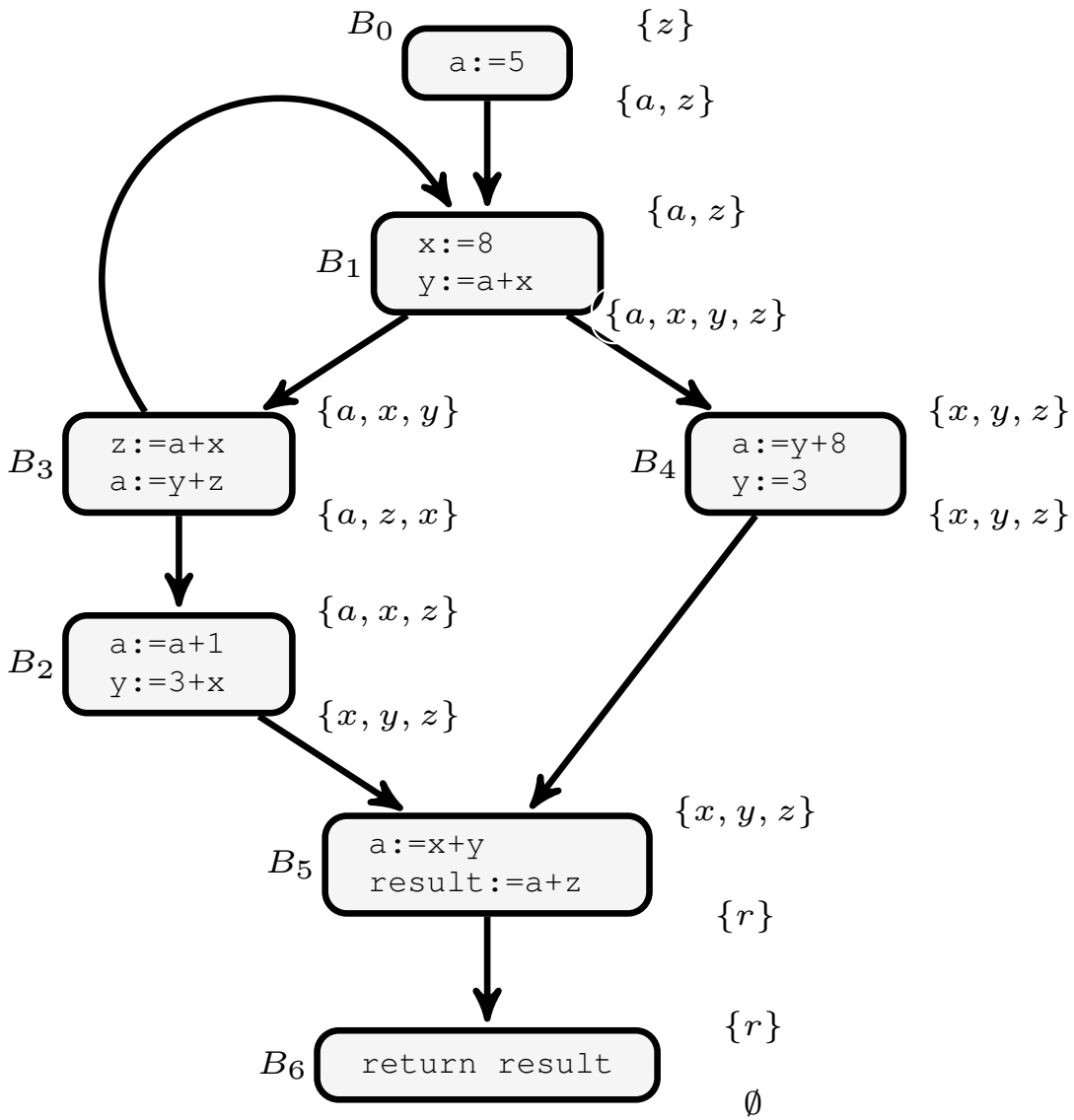
Liveness: run



Liveness: run



Liveness: run





Liveness example: remarks

- the shown traversal strategy is (cleverly) backwards
- example resp. example run simplistic:
- the *loop* (and the choice of "evaluation" order):

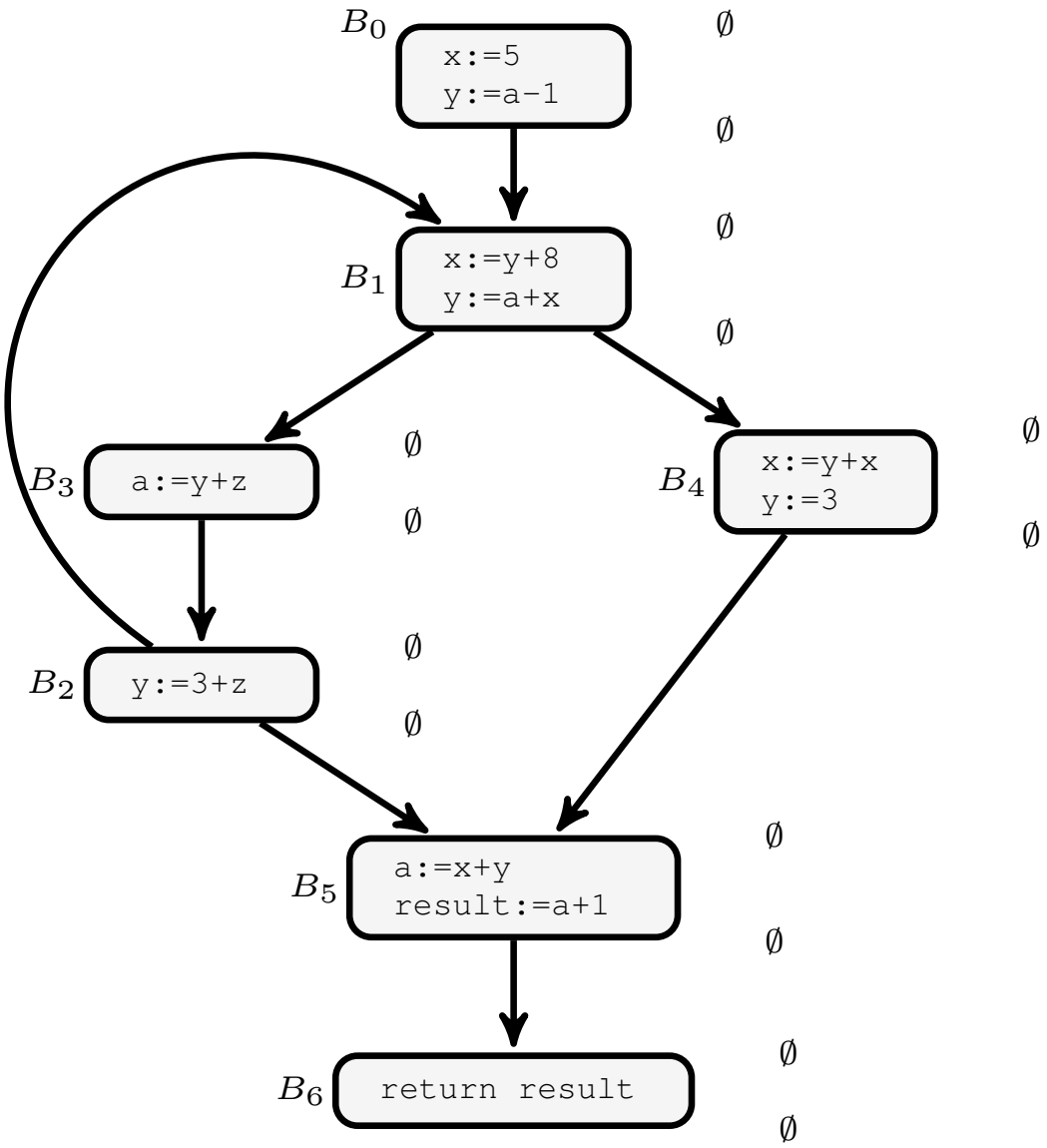
"harmless loop"

after having updated the $outLive$ info for B_1 following the edge from B_3 to B_1 *backwards* (propagating flow from B_1 back to B_3) does not increase the current solution for B_3

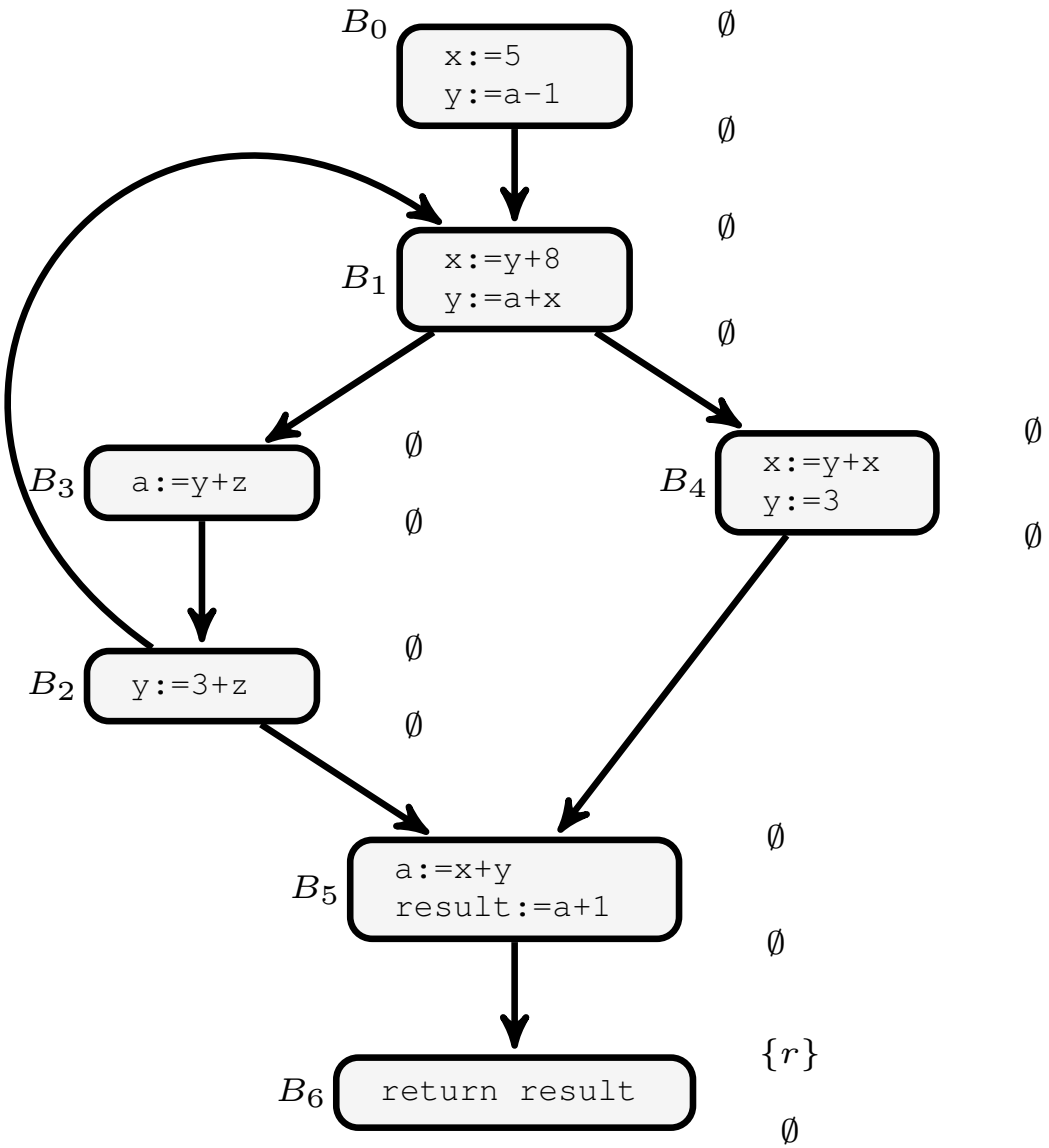
- no need (in this particular order) for continuing the iterative search for stabilization
- in other examples: loop iteration cannot be avoided
- note also: end result (after stabilization) independent from evaluation order! (only some strategies may stabilize faster...)



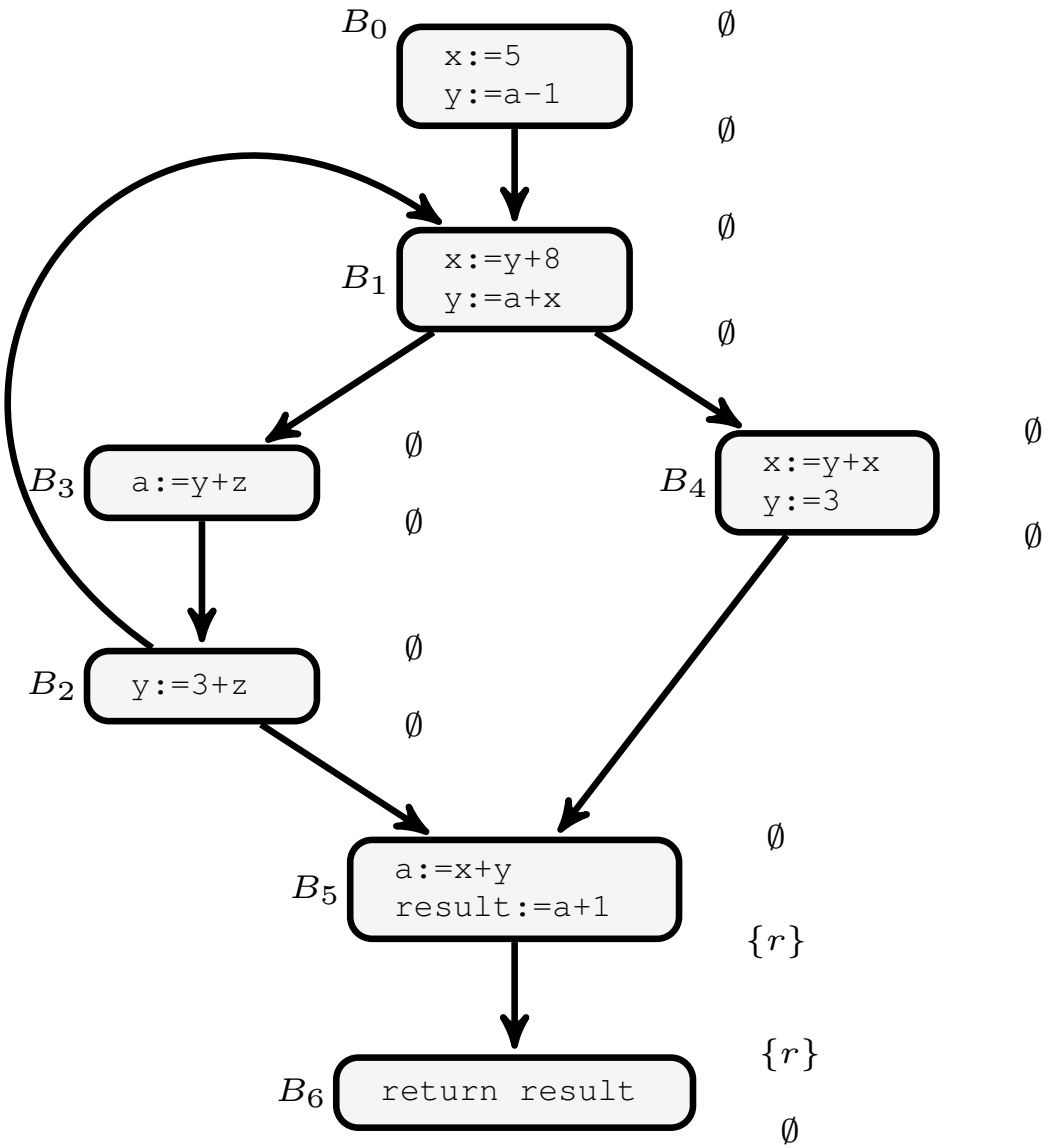
Another, more interesting, example



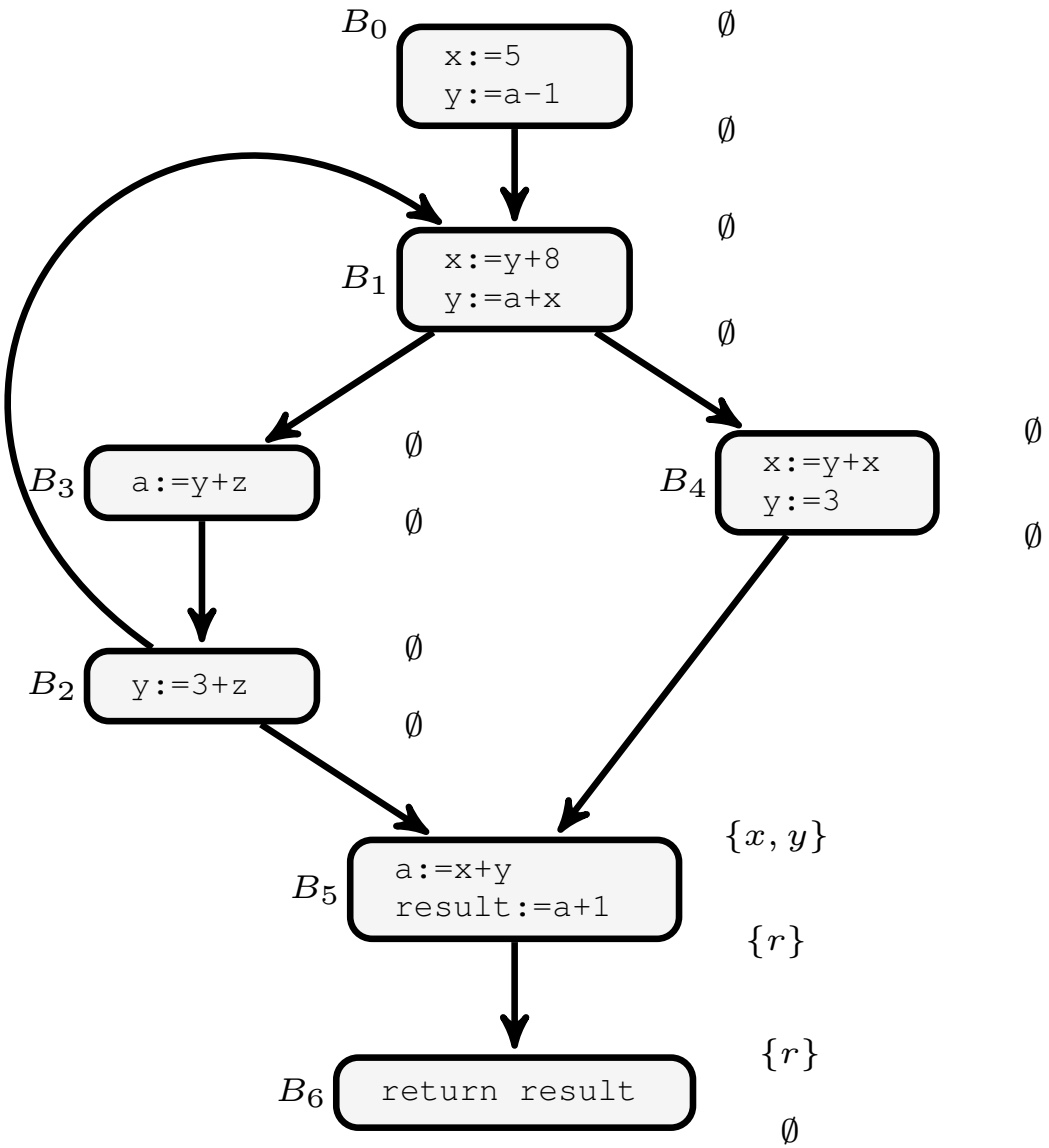
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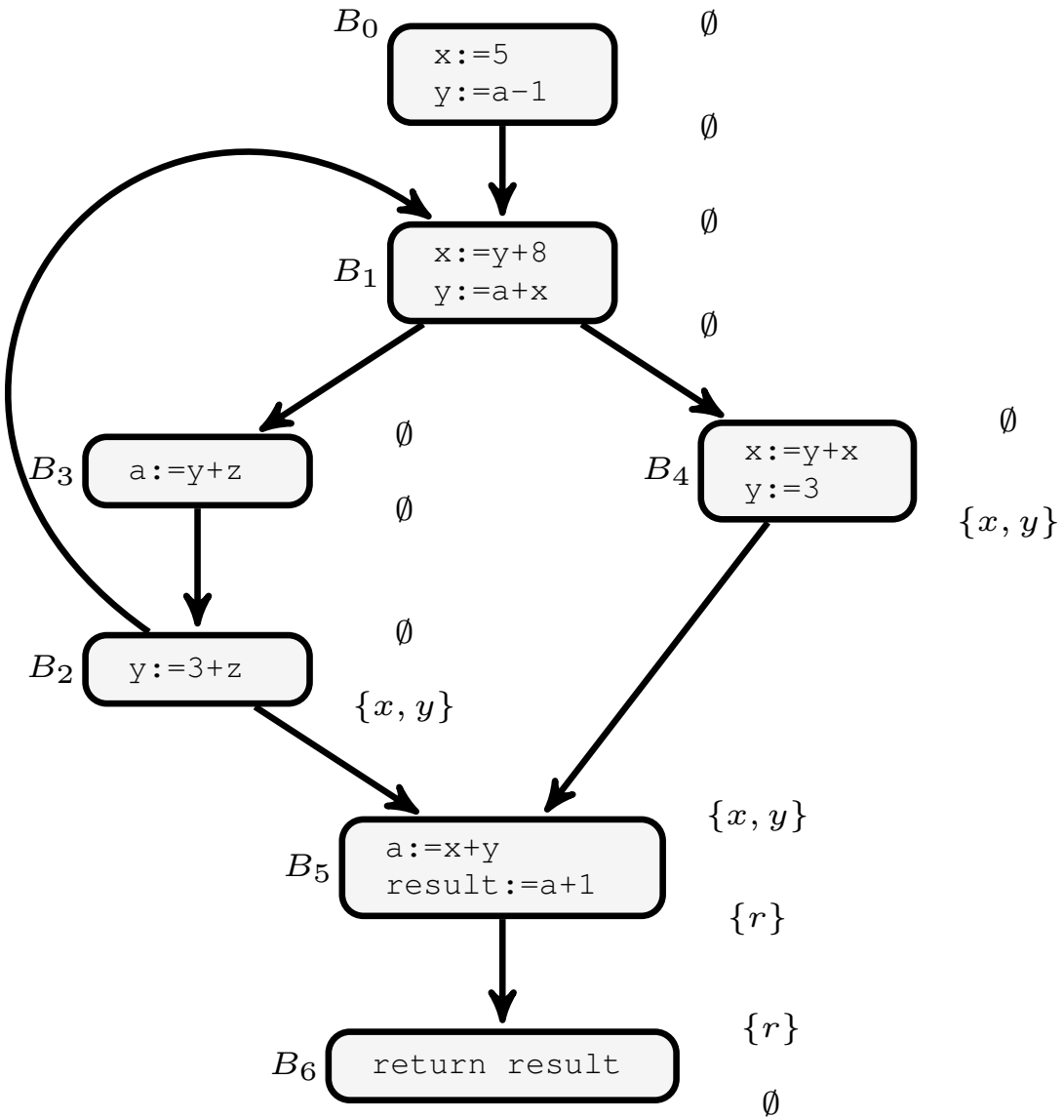
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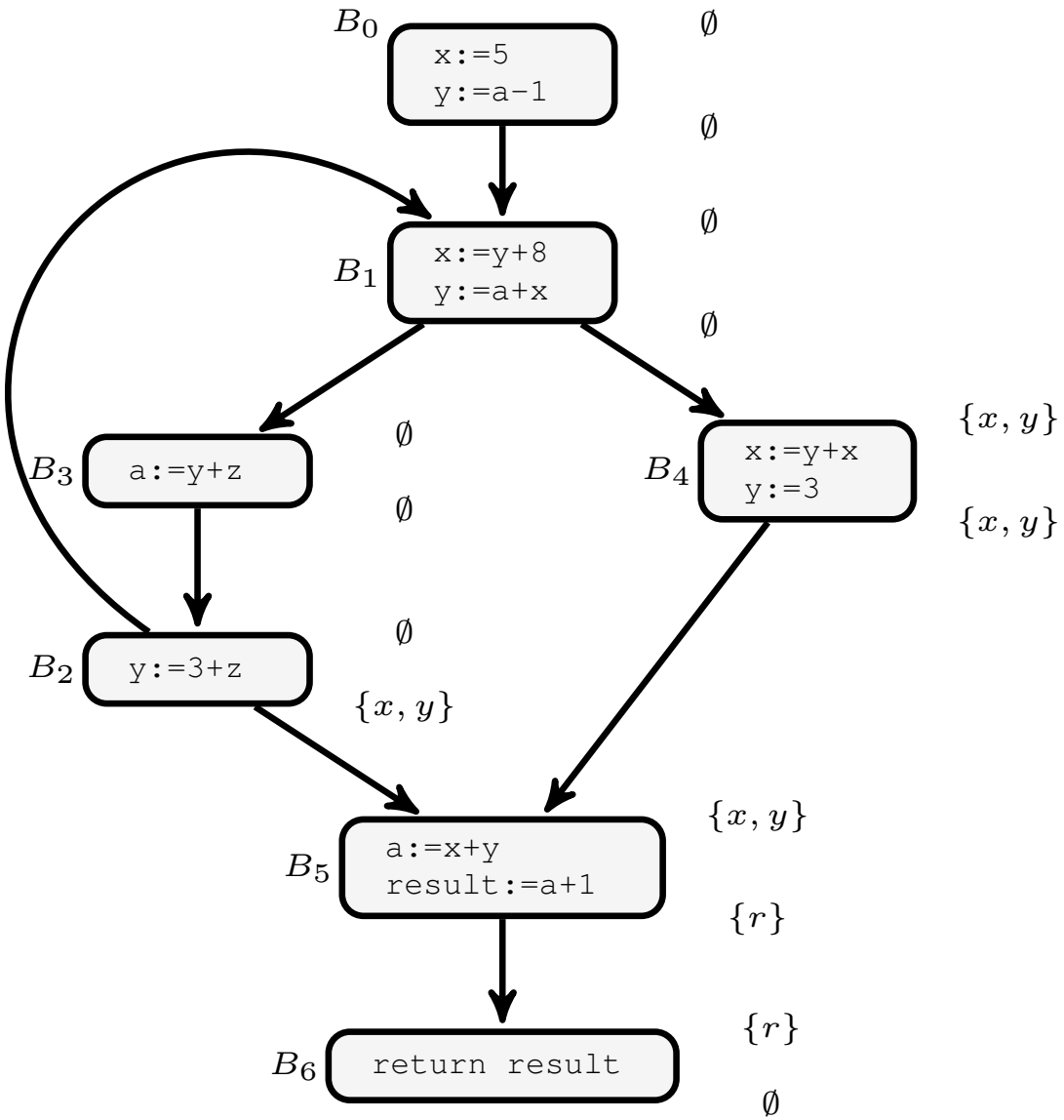


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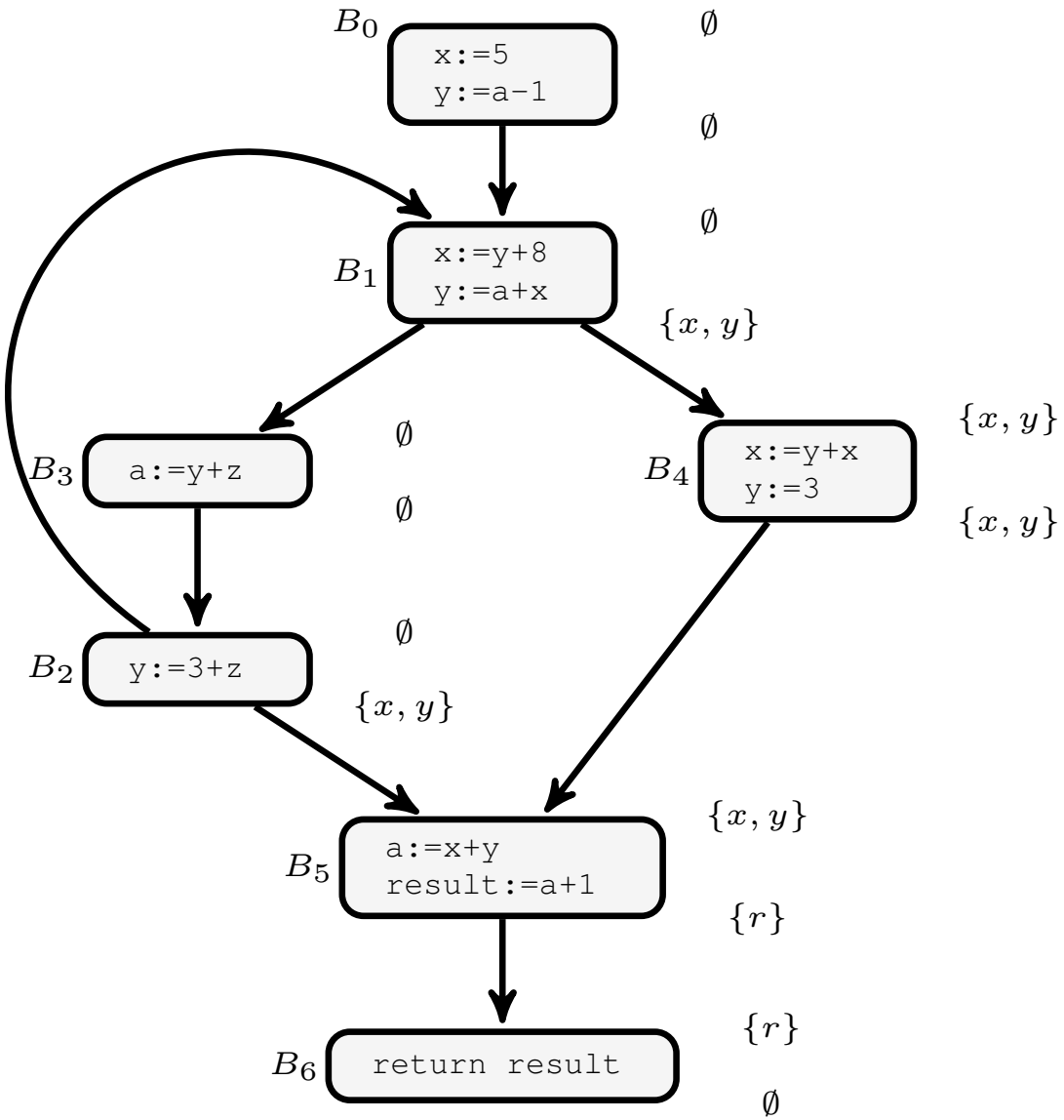


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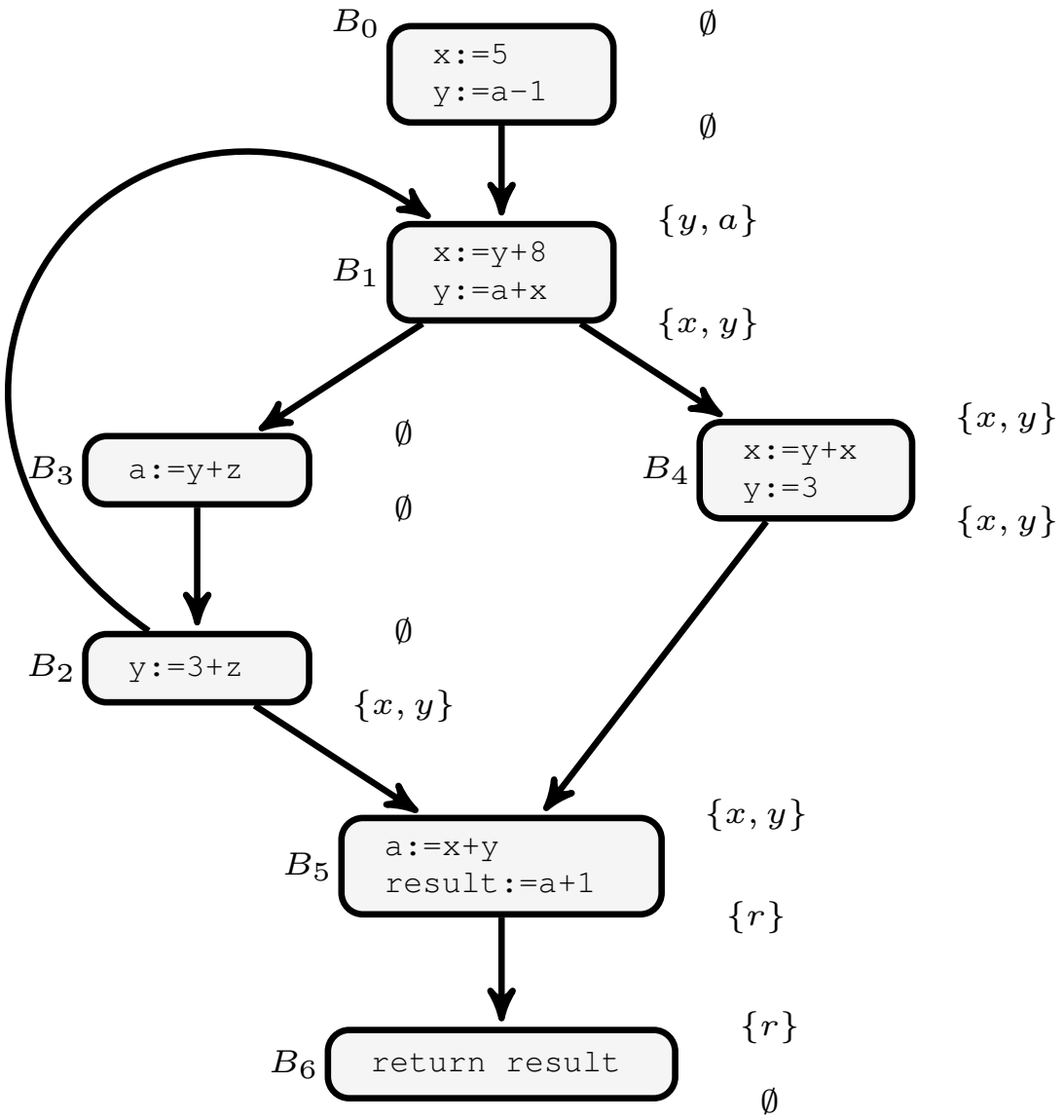




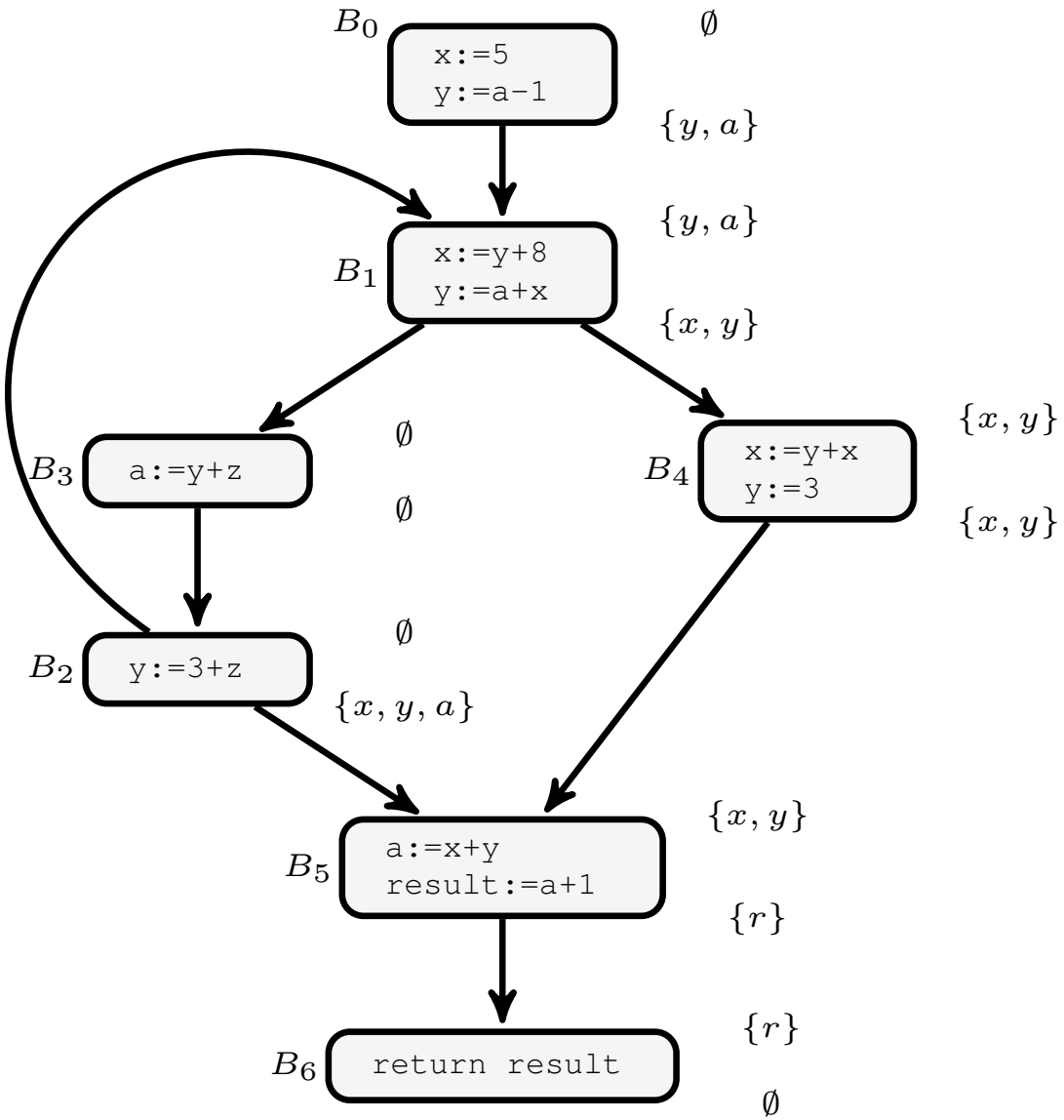
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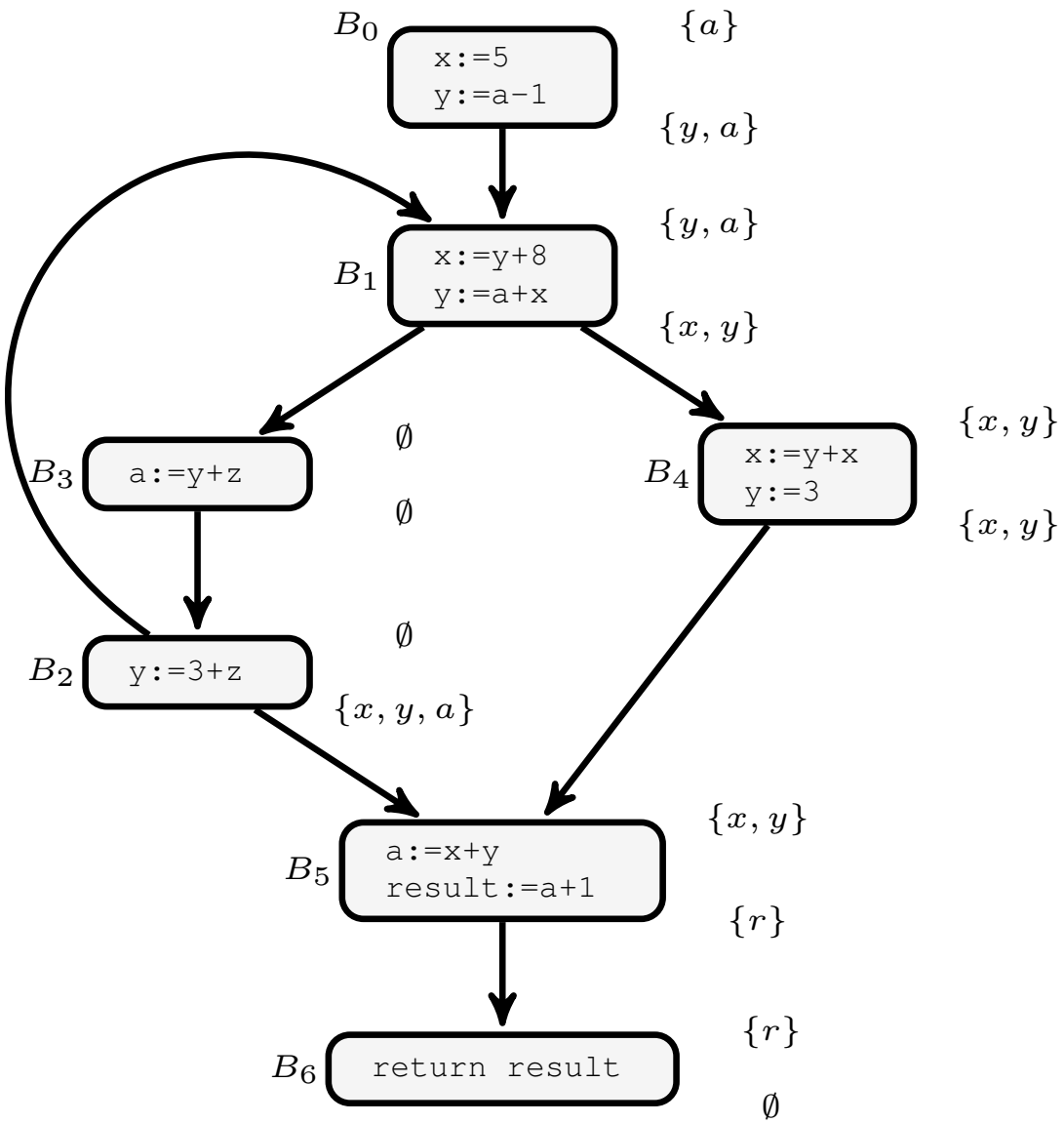
Another, more interesting, example



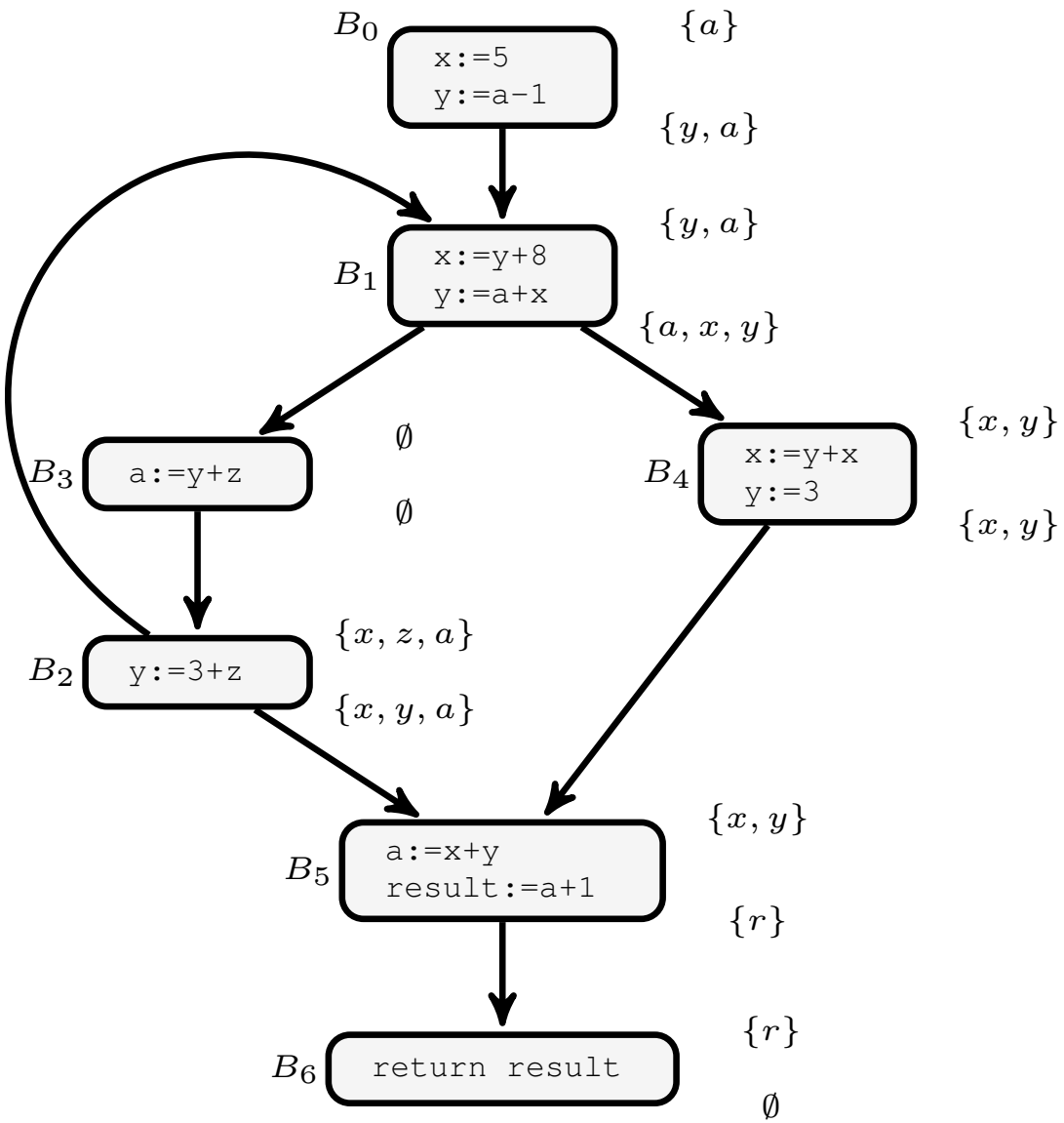
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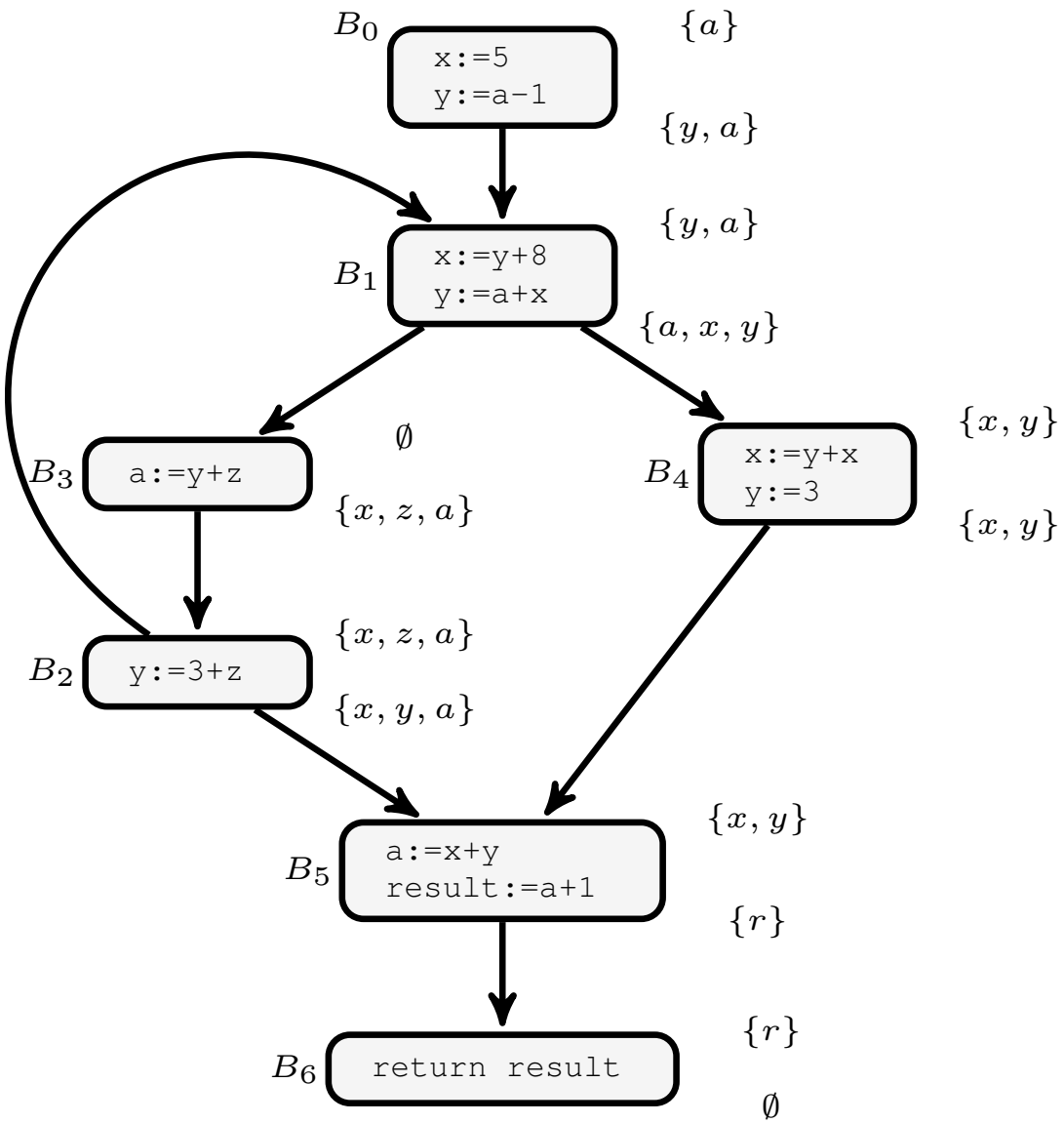
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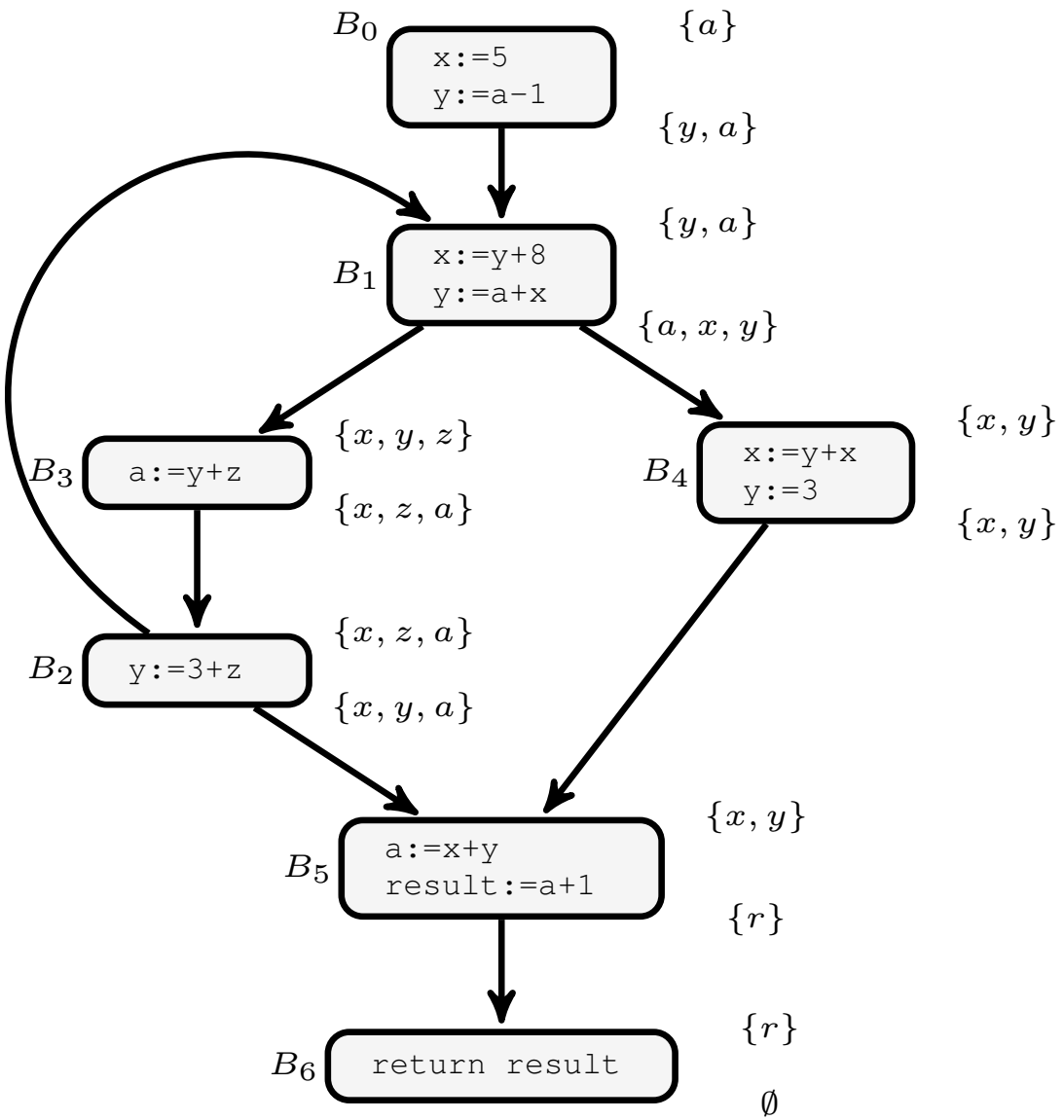
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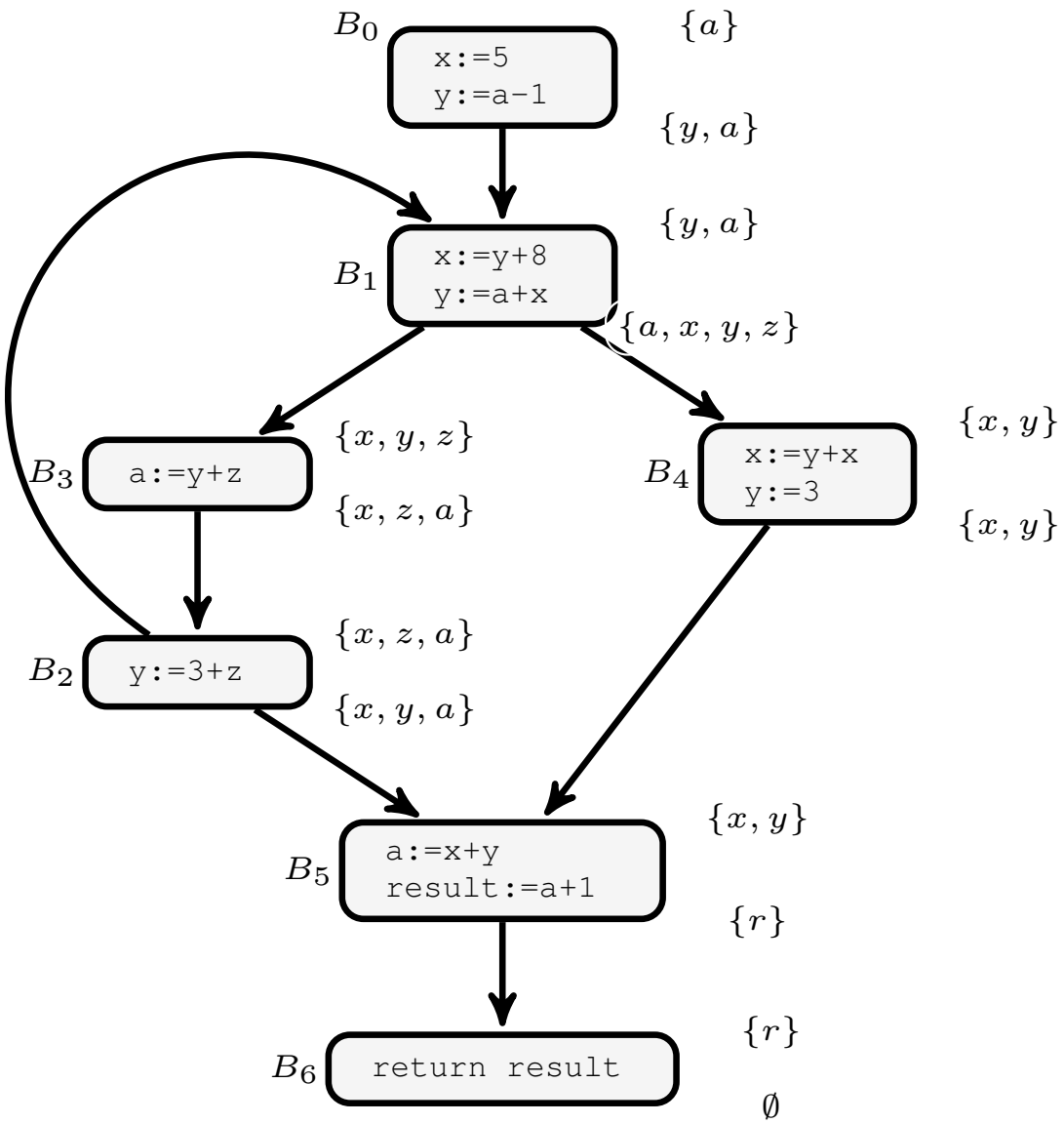
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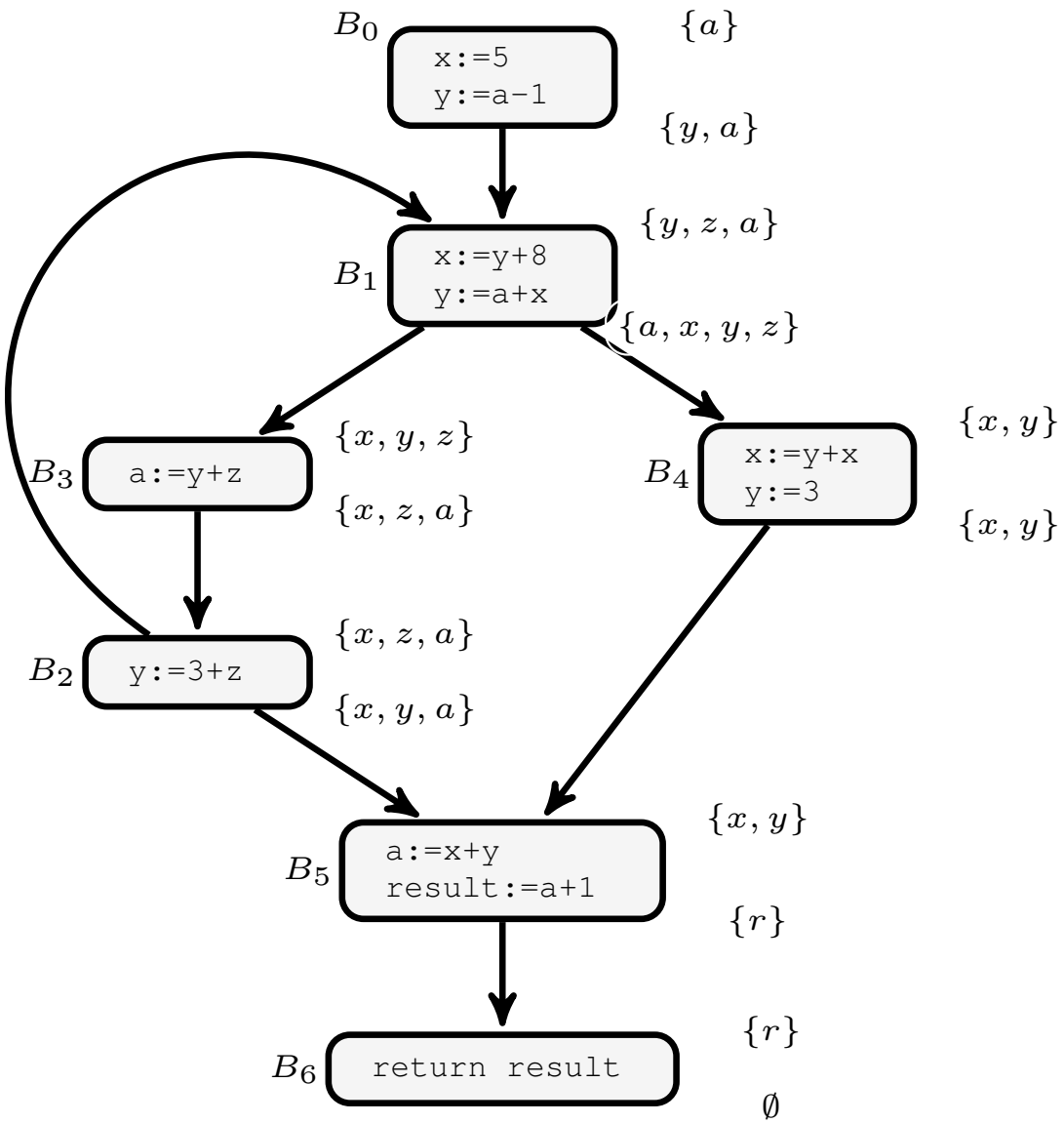
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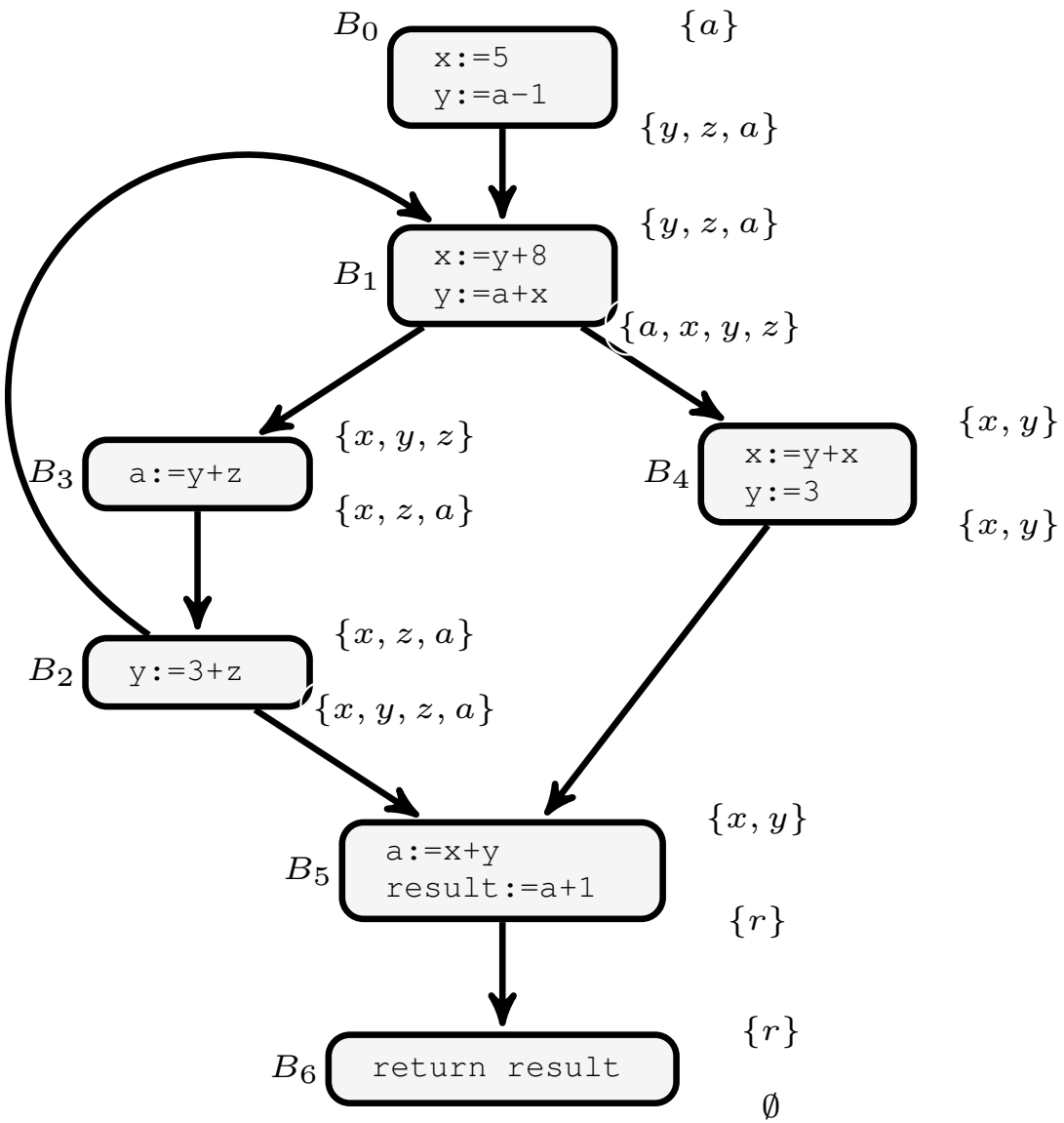
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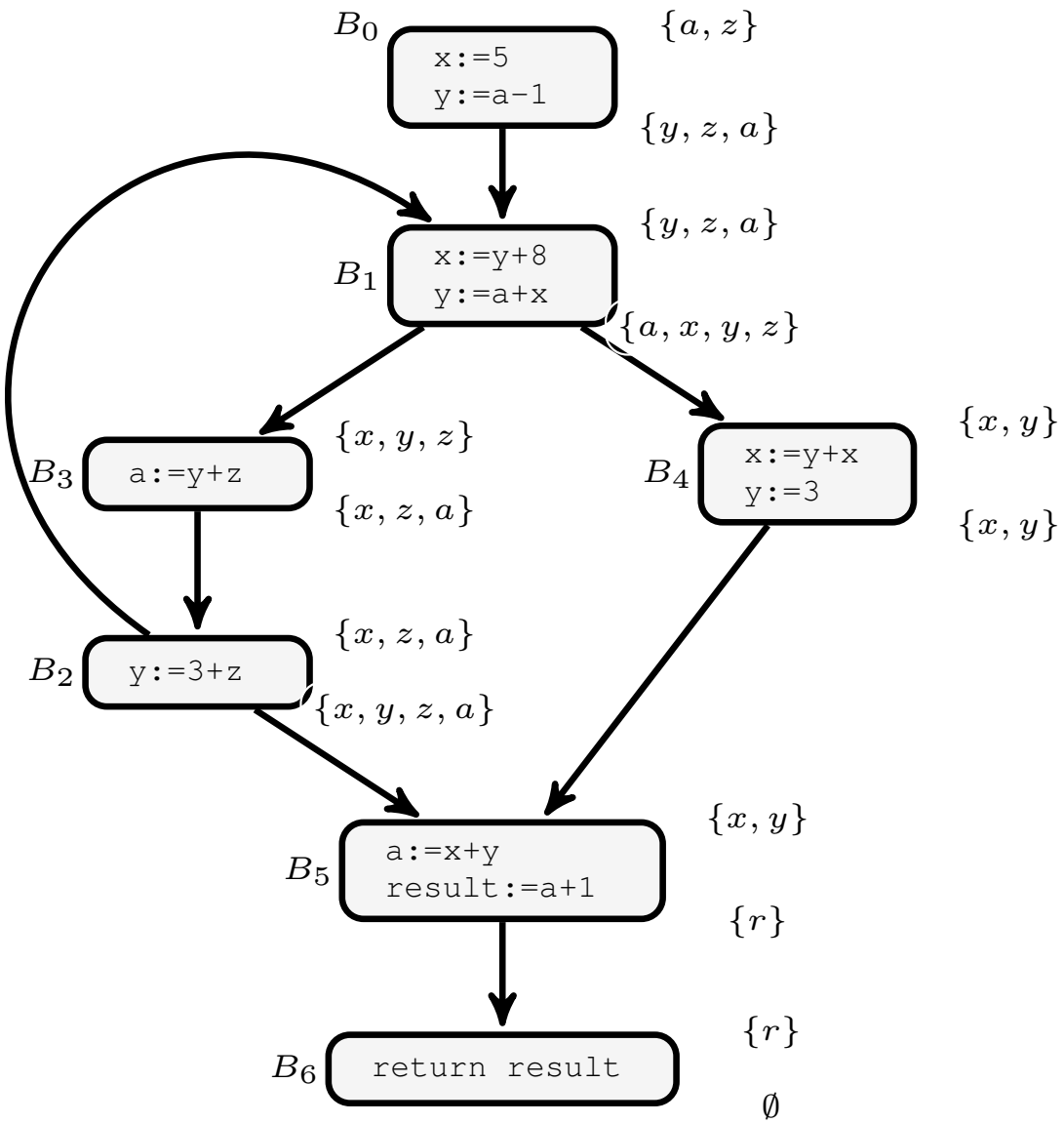
Another, more interesting, example



Another, more interesting, example



Another, more interesting, example



Example remarks

- loop: this time leads to updating estimation more than once
- evaluation order not chosen ideally



Precomputing the block-local "liveness effects"

- *precomputation* of the relevant info: efficiency
- traditionally: represented as *kill* and *generate* information
- here (for liveness)
  1. kill: variable instances, which are overwritten
  2. generate: variables used in the block (before overwritten)
  3. rests: all other variables won't change their status

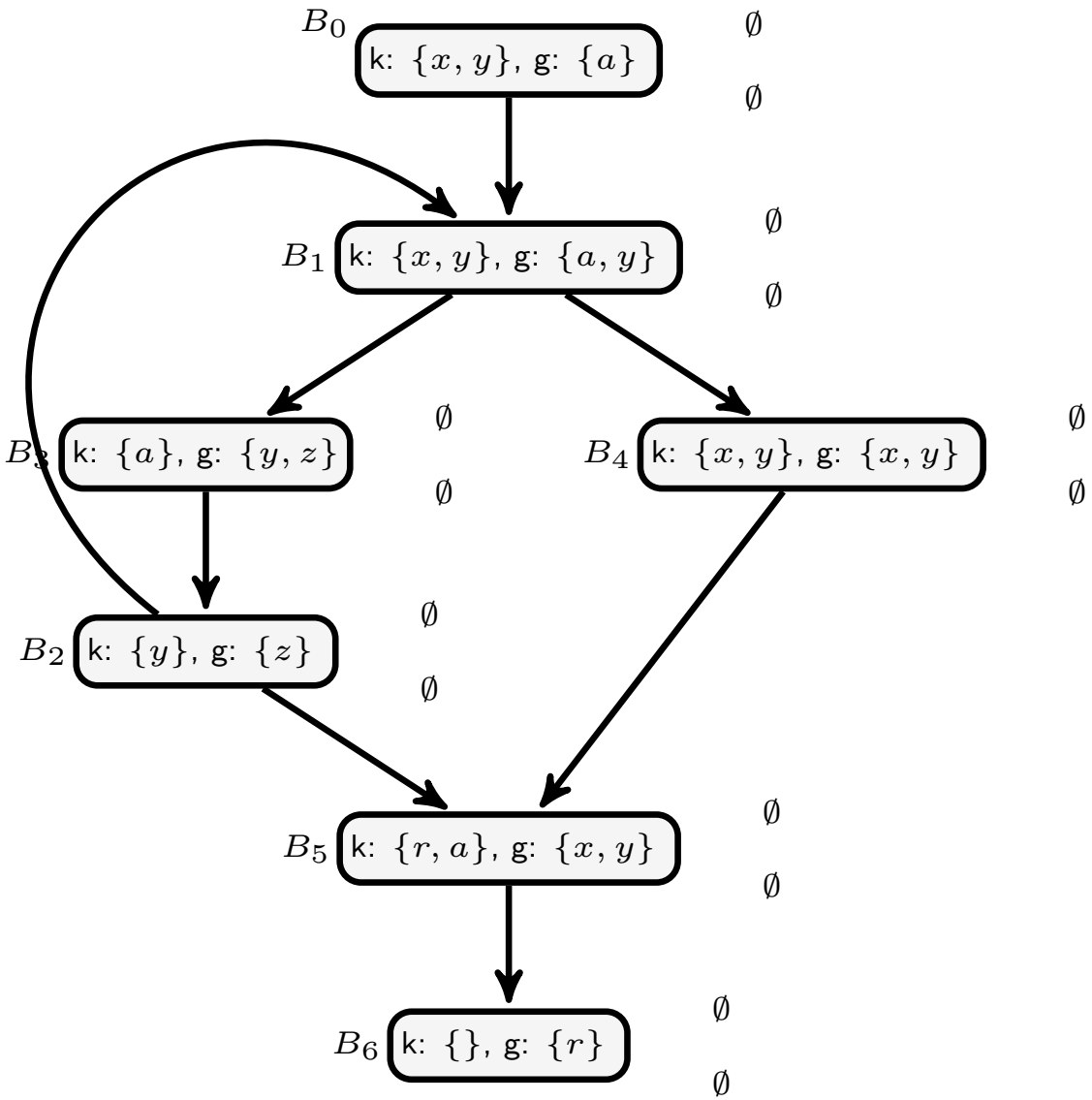
Constraint per basic block (transfer function)

$$inLive = outLive \backslash kill(B) \cup generate(B)$$

- note:
  - order of kill and generate in above's equation
  - a variable killed in a block may be "revived" in a block
- simplest (one line) example: `x := x +1`

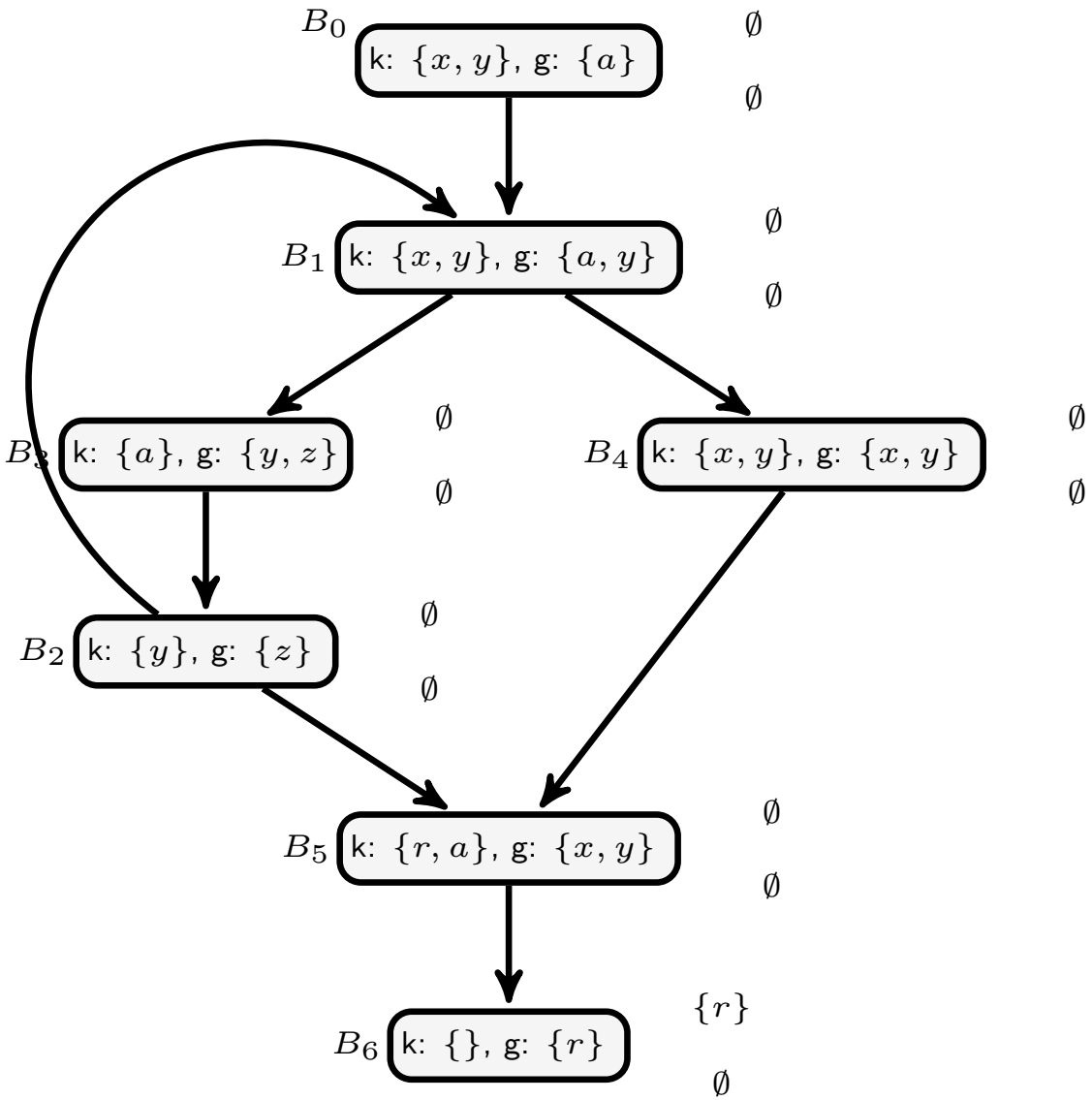


Example once again: kill and gen



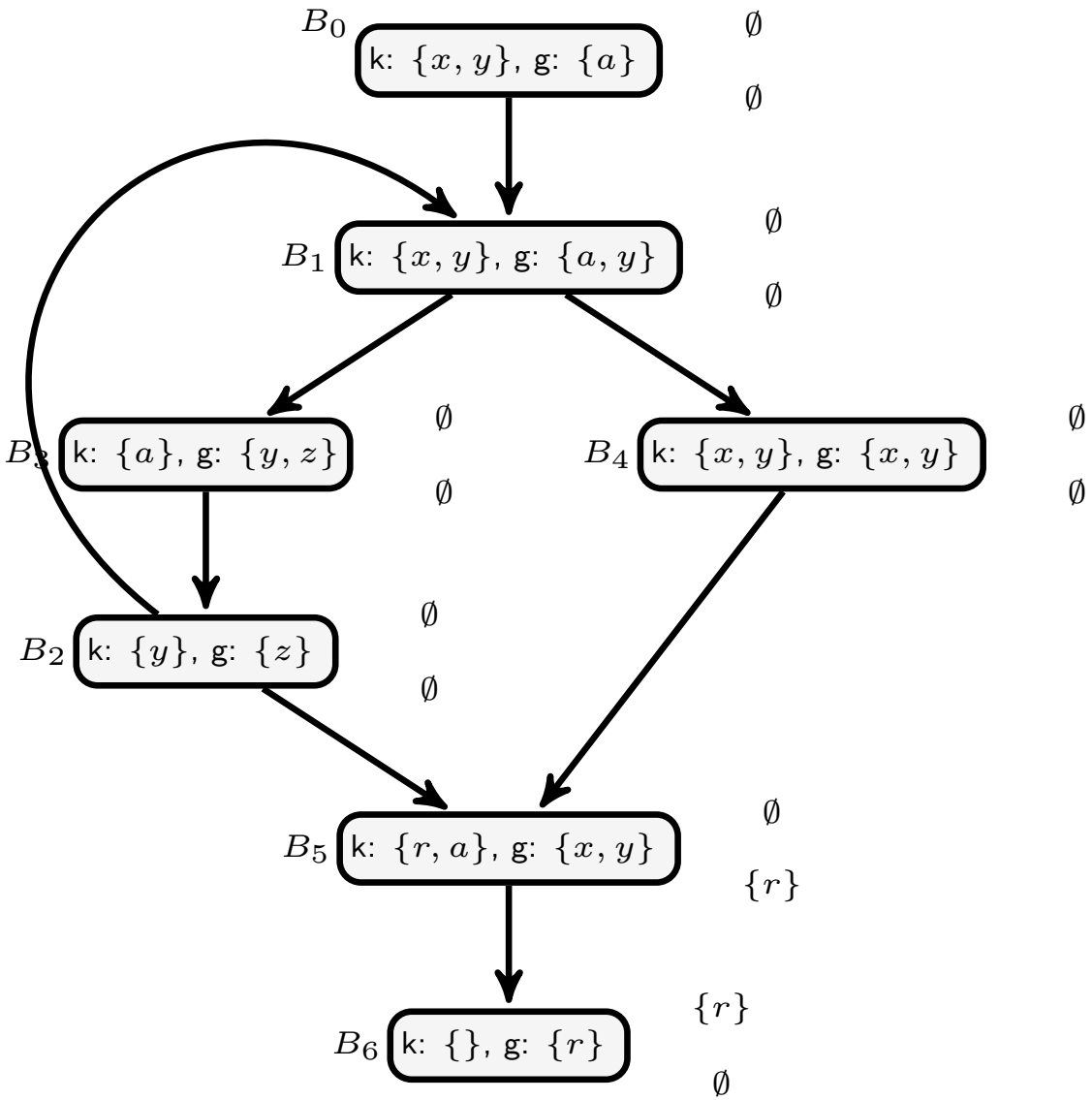


Example once again: kill and gen



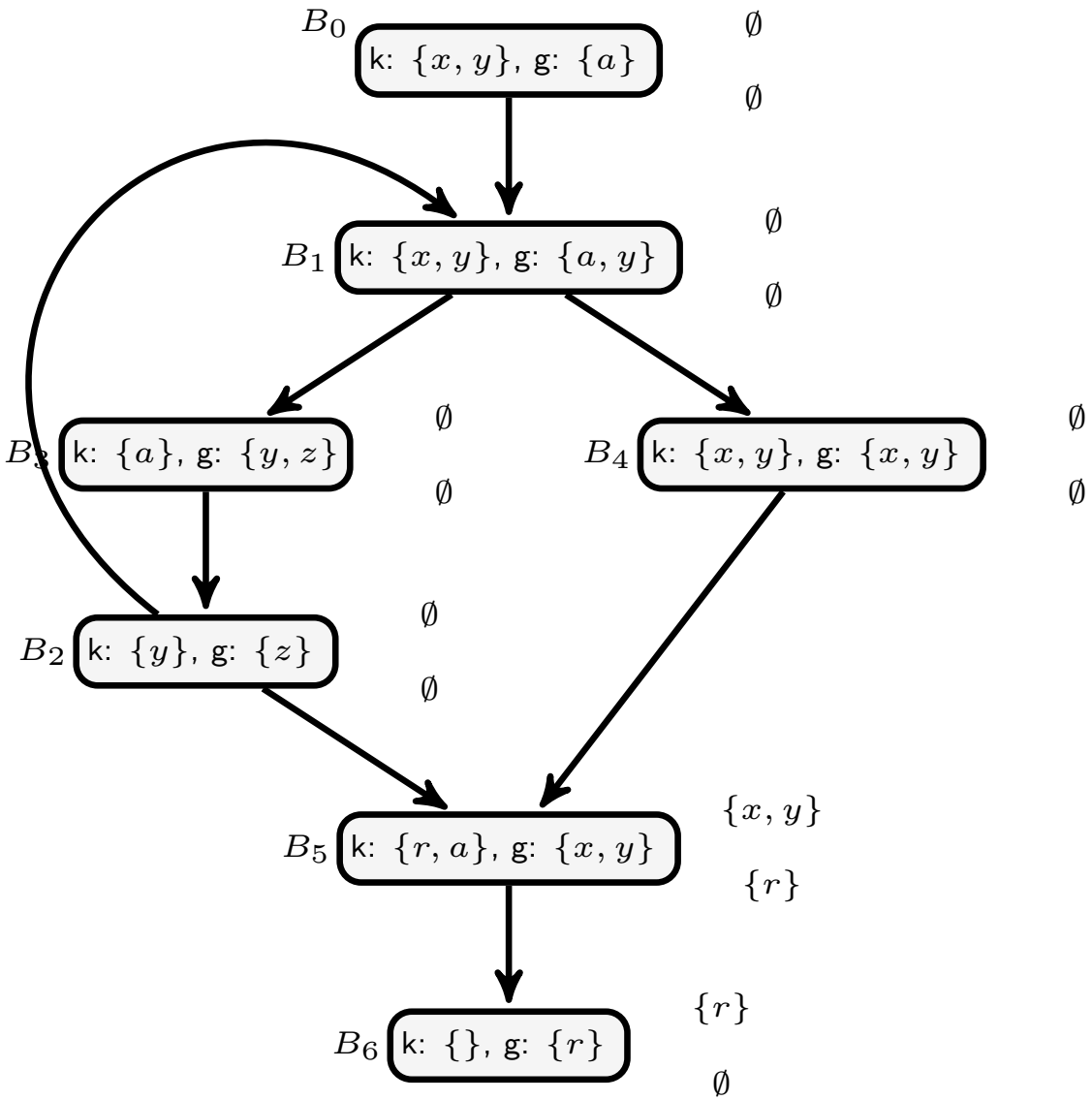


Example once again: kill and gen

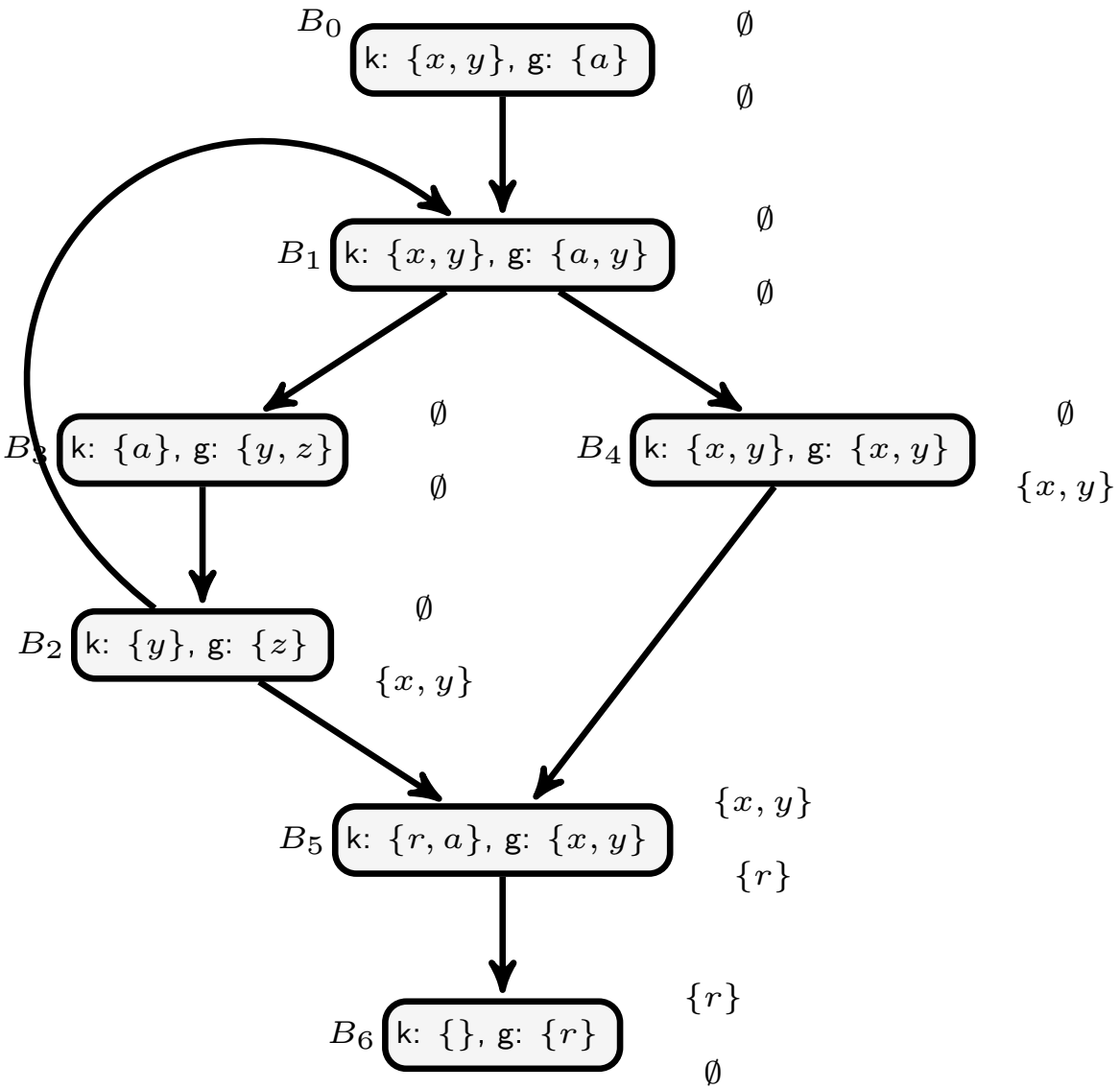




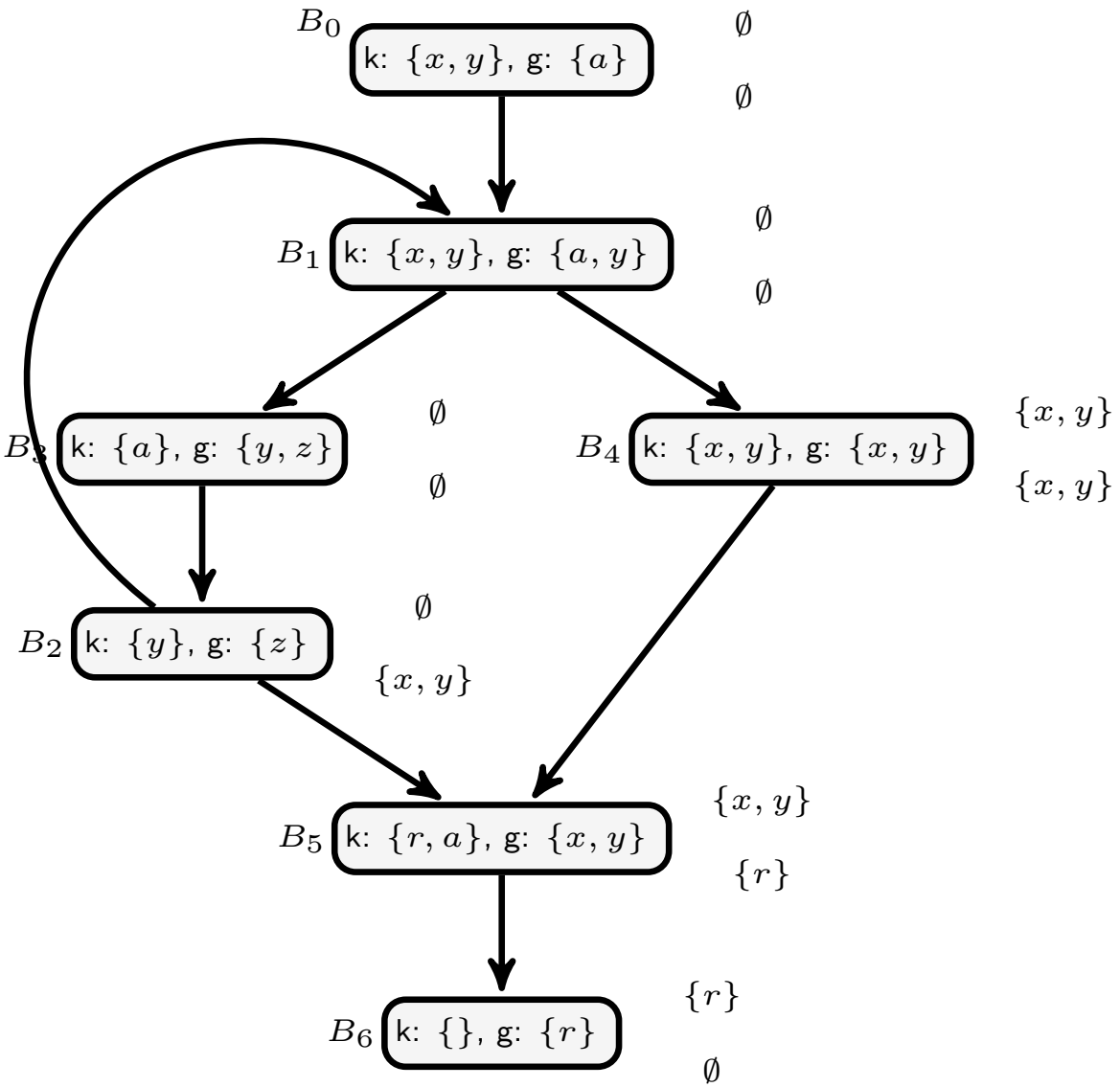
Example once again: kill and gen



Example once again: kill and gen

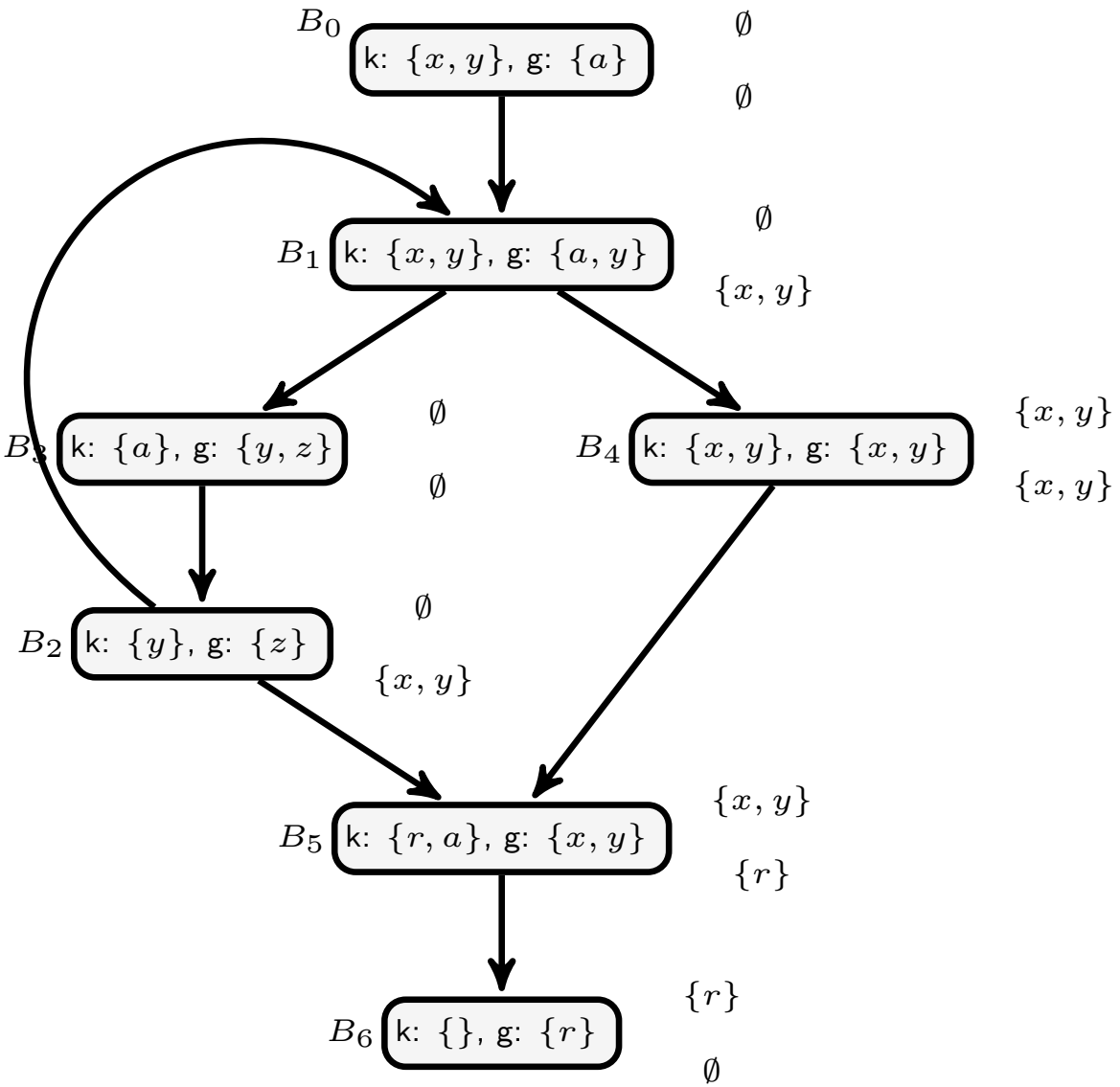


Example once again: kill and gen



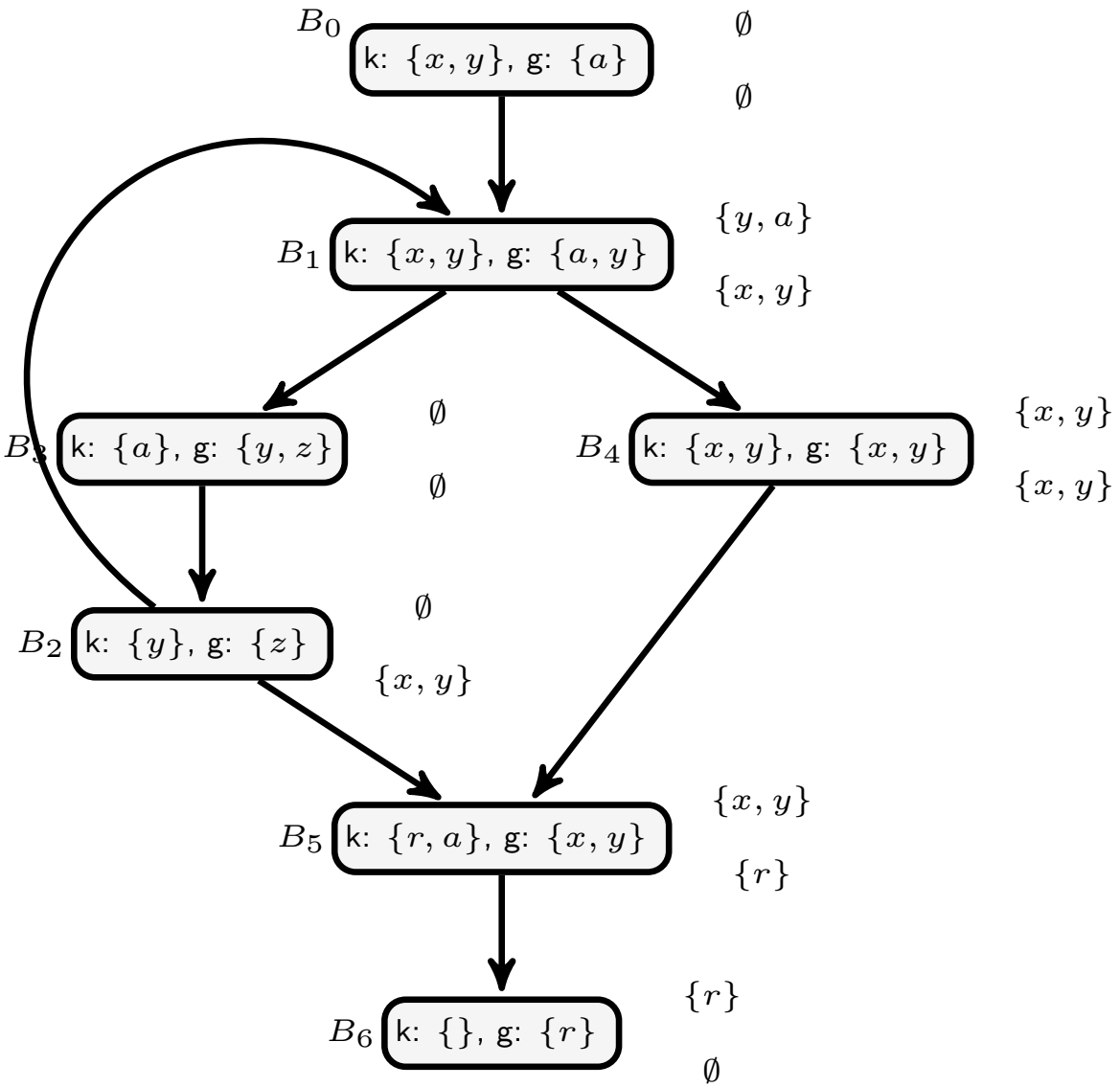


Example once again: kill and gen



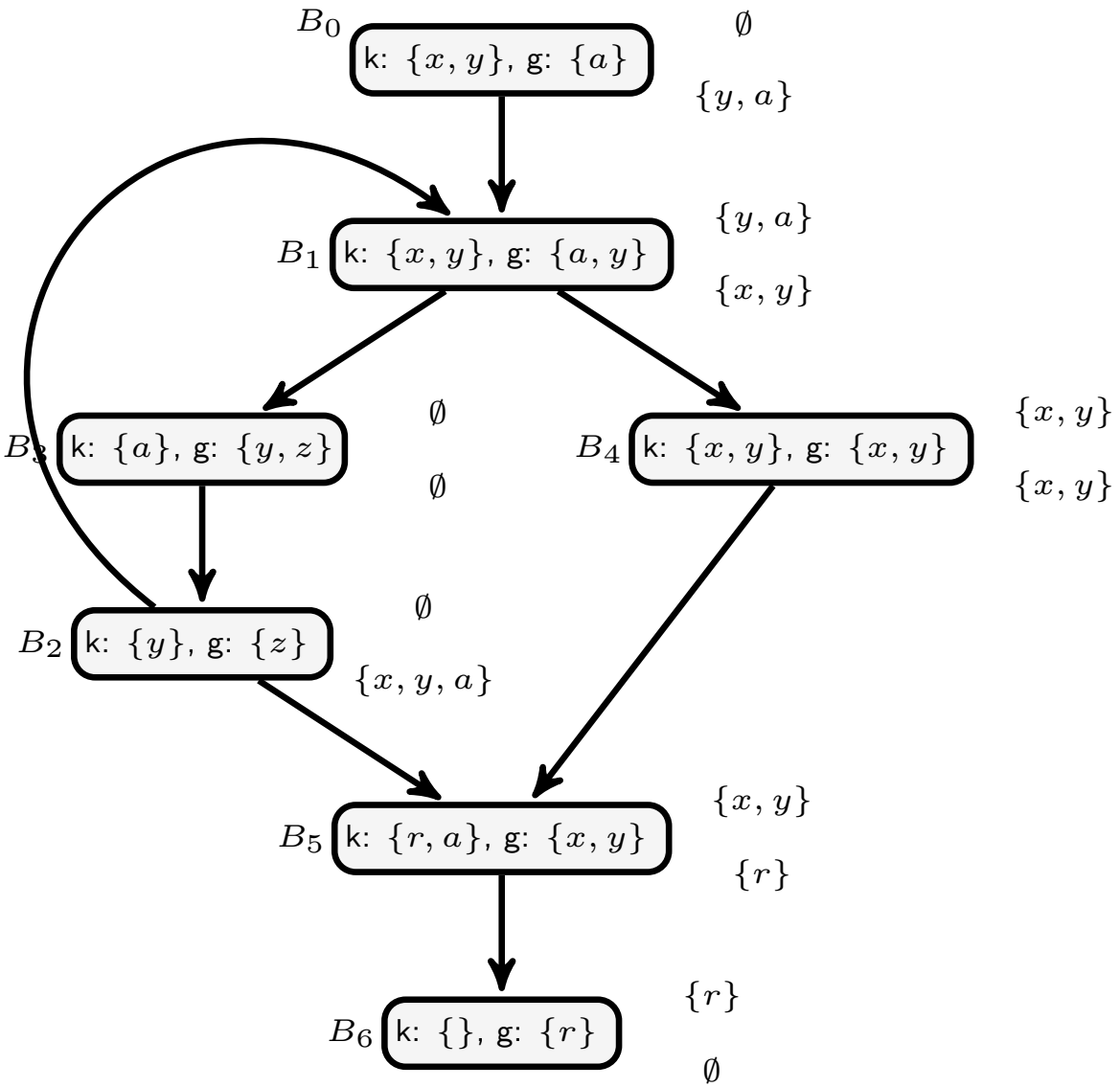


Example once again: kill and gen

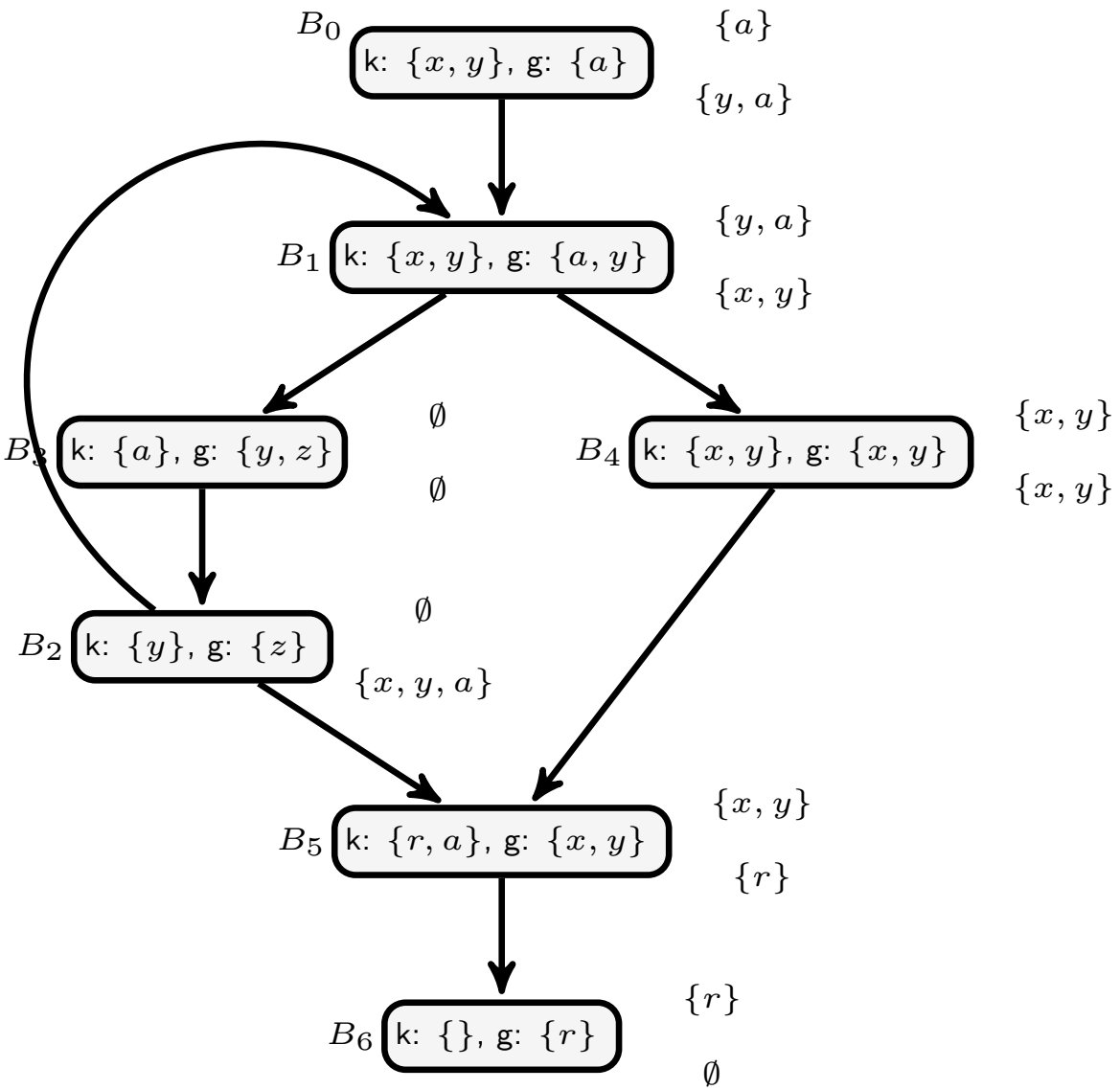




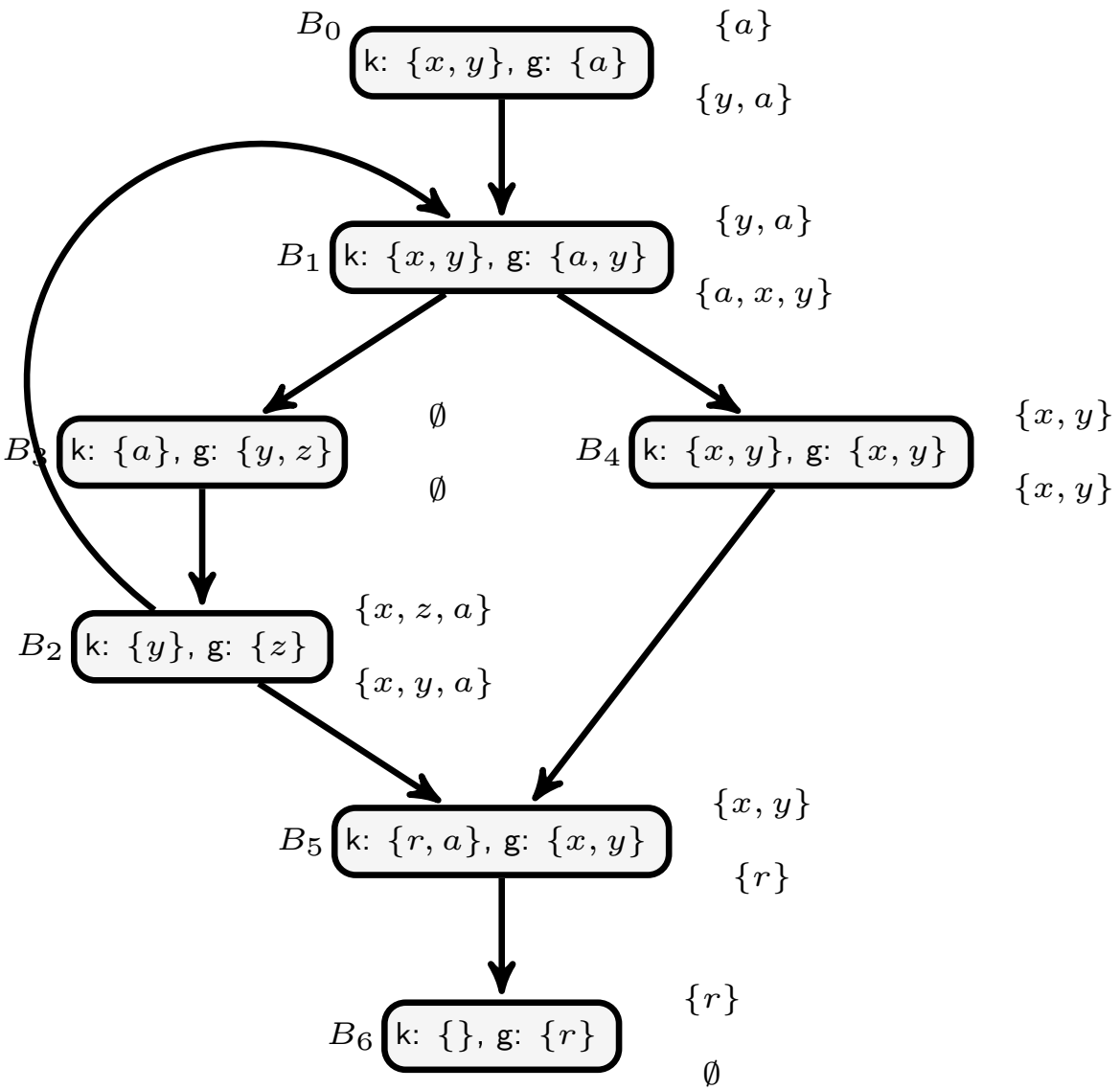
Example once again: kill and gen



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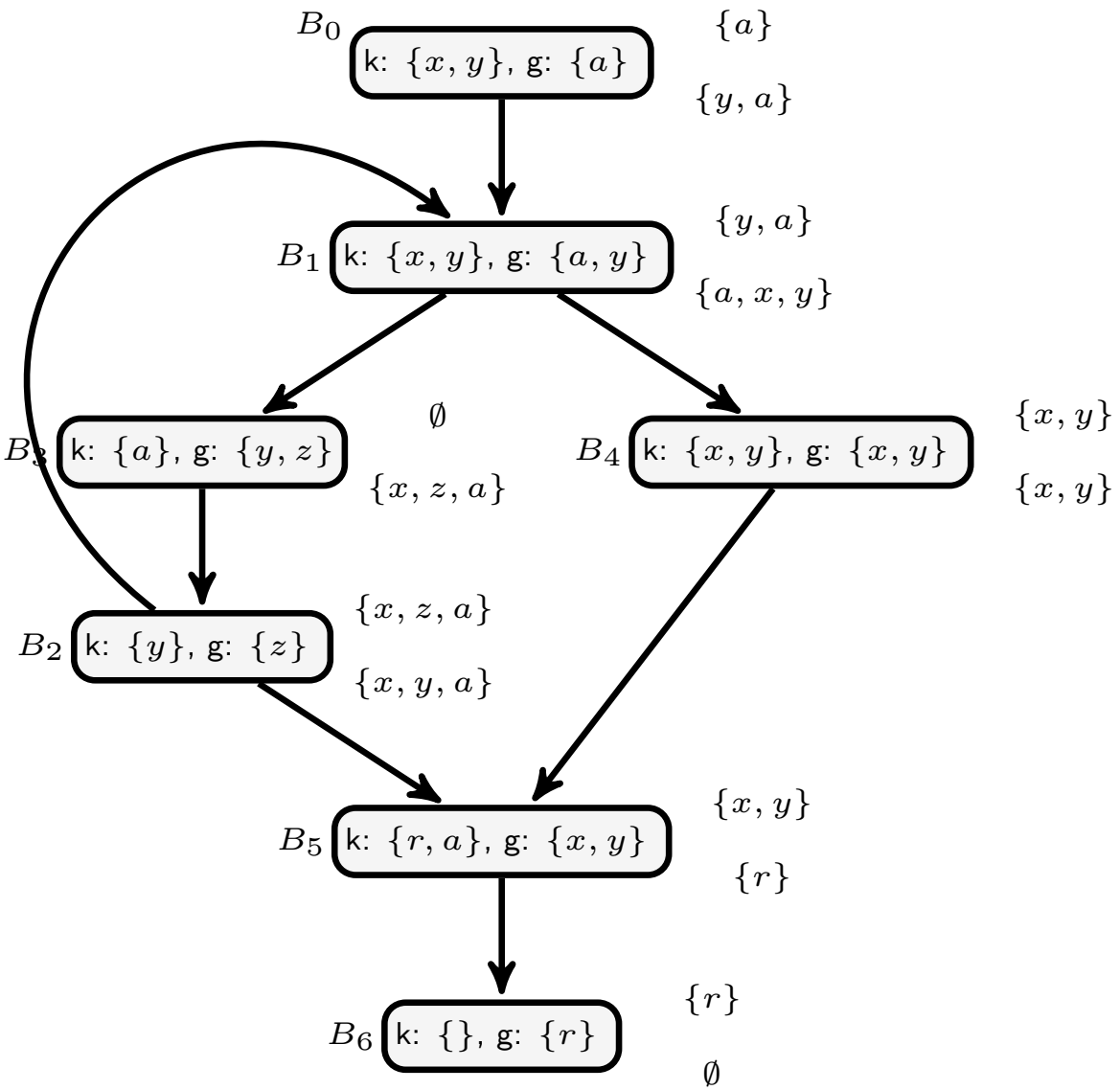


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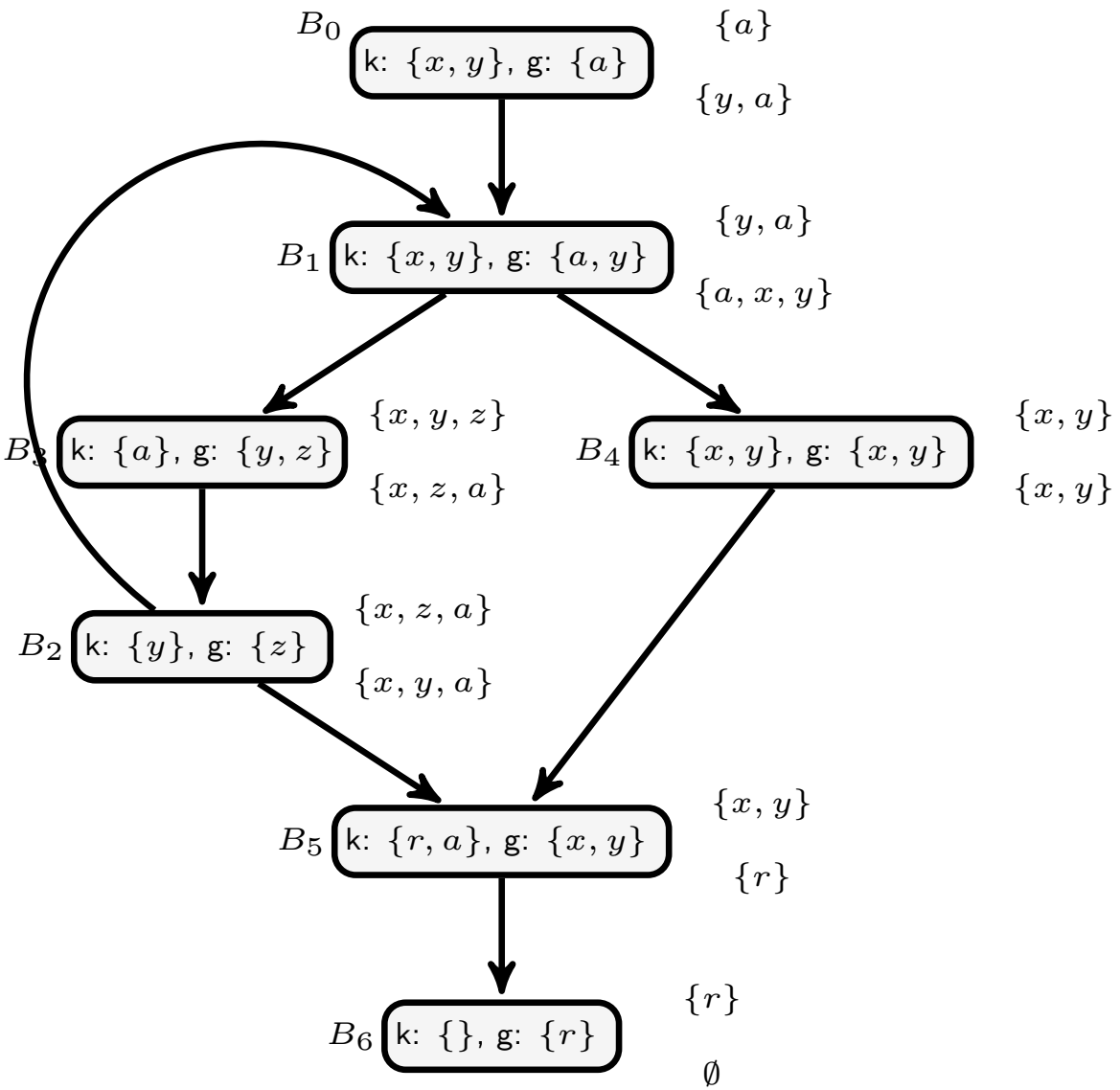


Example once again: kill and gen

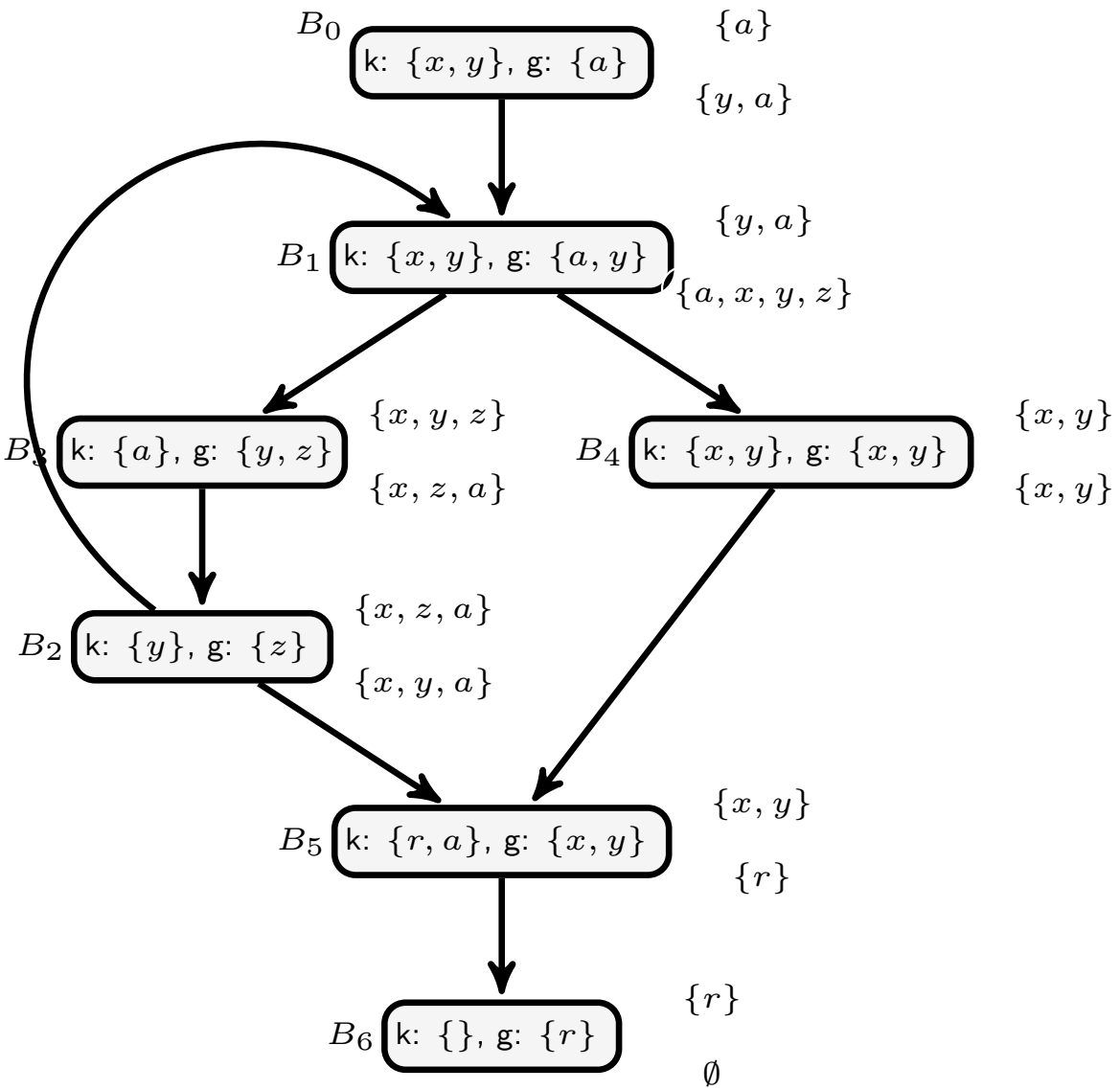




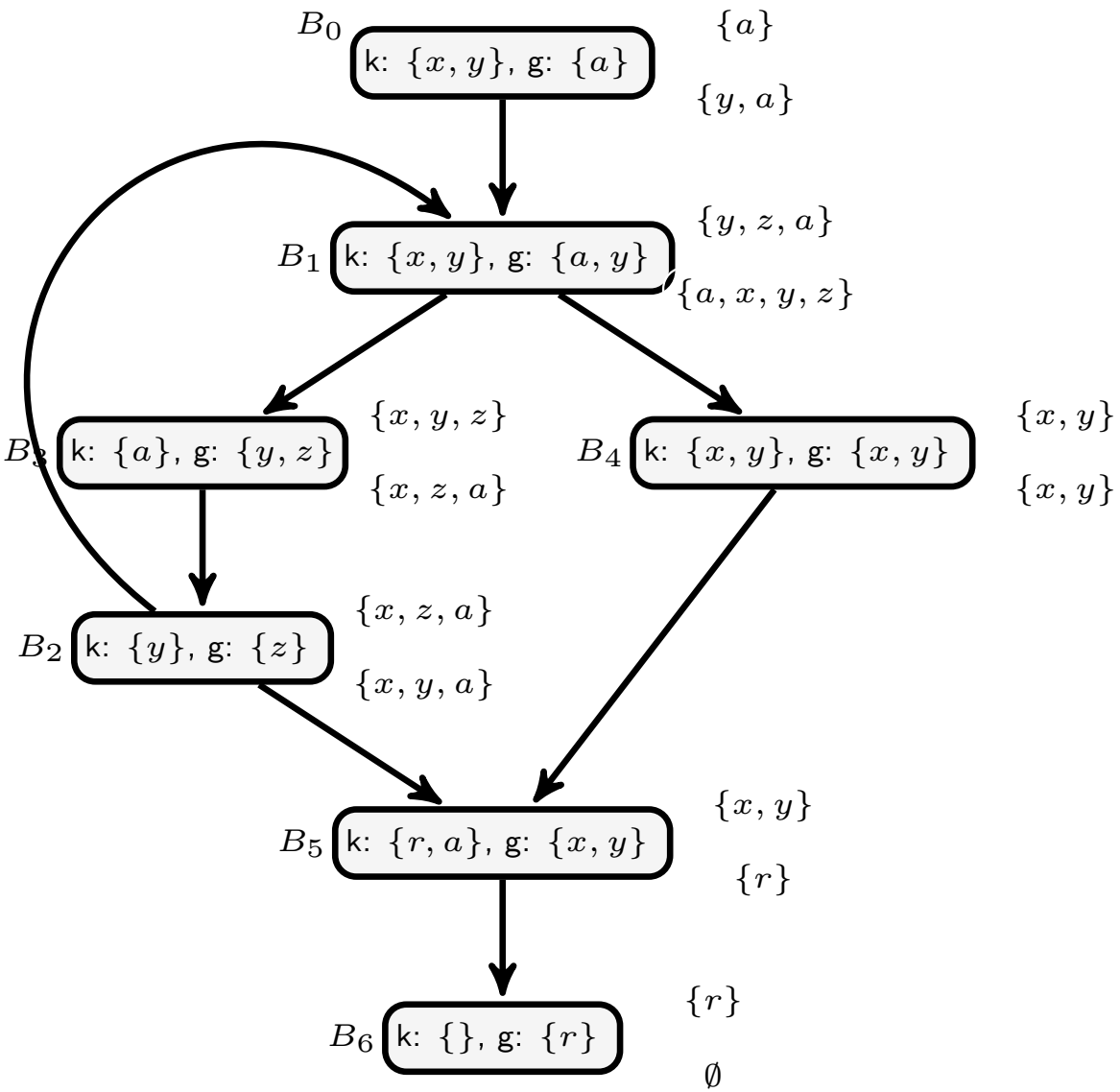
Example once again: kill and gen



Example once again: kill and gen

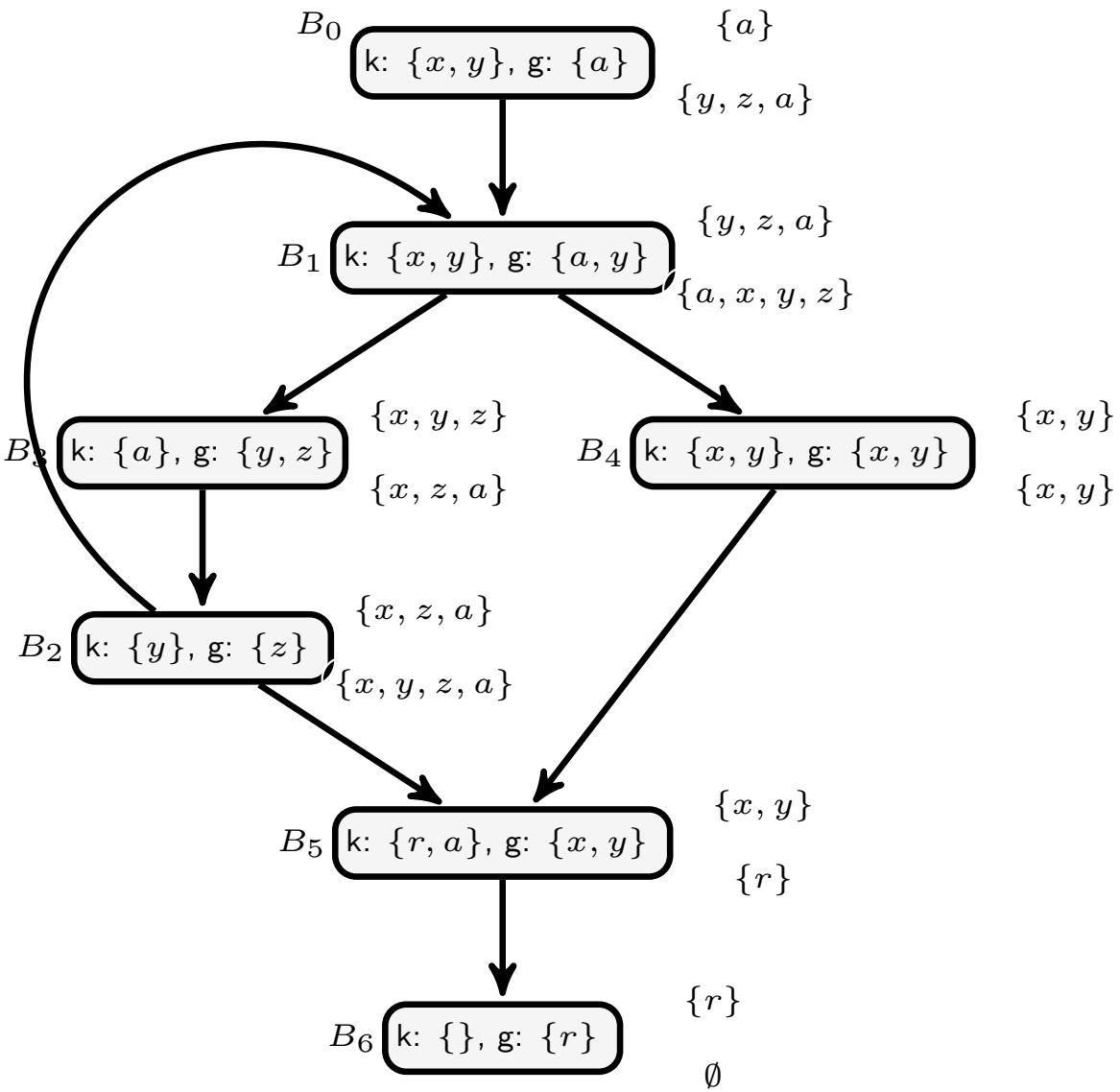


Example once again: kill and gen

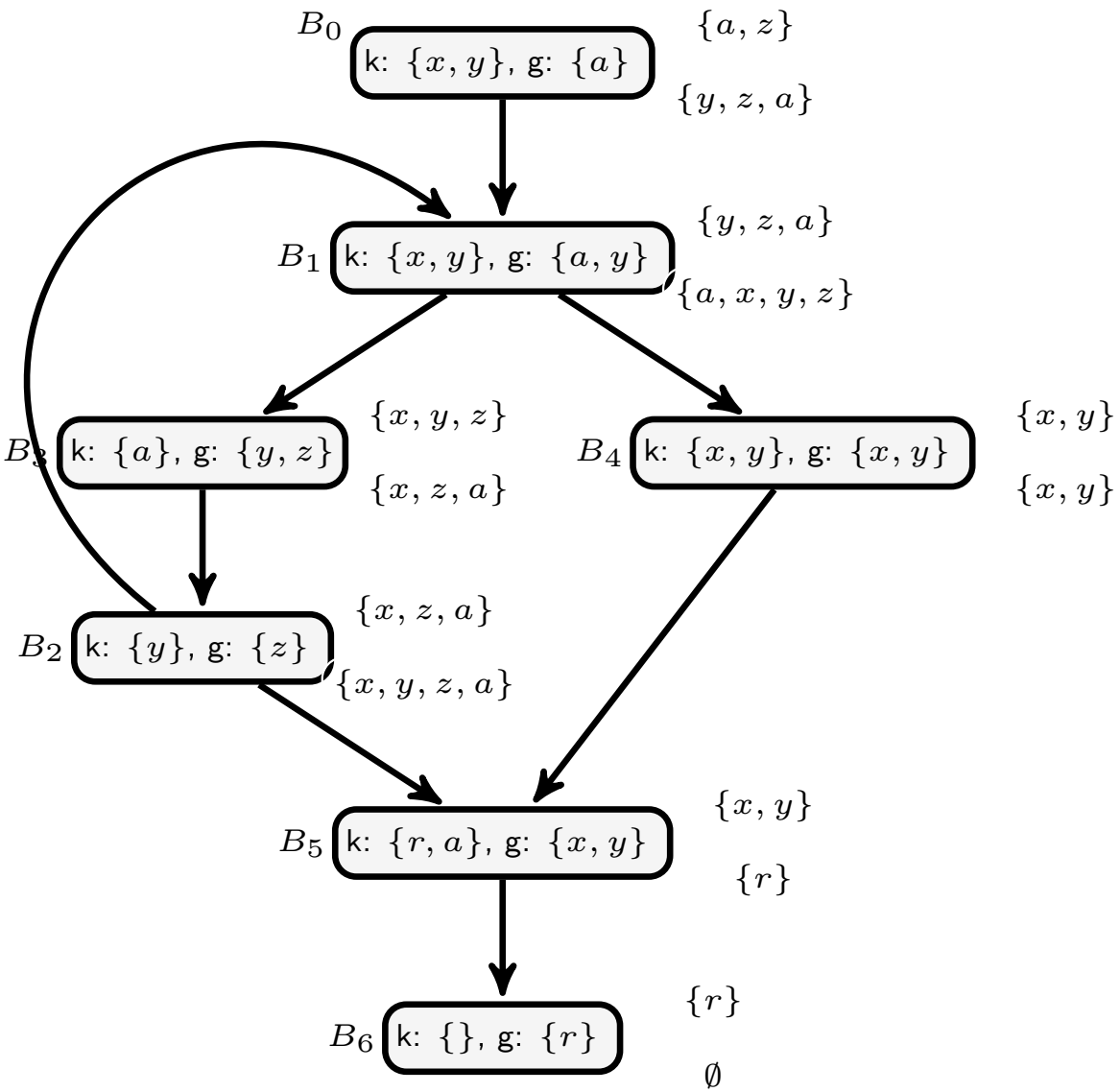




Example once again: kill and gen



Example once again: kill and gen





Section

Code generation algo

Chapter 10 “Code generation”
Course “Compiler Construction”
Martin Steffen
Spring 2022

Simple code generation algo

- simple algo: *intra-block* code generation
- core problem: register use
- register allocation & assignment
- hold calculated values in registers longest possible
- intra-block only $\Rightarrow$ at exit:
  - all *variables* stored back to main memory
  - all temps assumed "lost"
- remember: assumptions in the intra-block liveness analysis



Limitations of the code generation

- local intra block:
  - no analysis across blocks
  - no procedure calls, etc.
- no complex data structures
  - arrays
  - pointers
  - ...

some limitations on how the algo itself works for one block

- for read-only variables: never put in registers, even if variable is *repeatedly* read
  - algo works only with the temps/variables given and does not come up with new ones
  - for instance: DAGs could help
- no *semantics* considered
  - like *commutativity*: $a + b$ equals $b + a$



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Purpose and "signature" of the *getreg* function

- one *core* of the code generation algo
- simple code-generation here $\Rightarrow$ simple *getreg*

getreg function

available: *liveness/next-use* info

Input: TAIC-instruction $x := y \ \mathbf{op} \ z$

Output: return *location* where x is to be stored

- location: register (if possible) or memory location



Coge generation invariant



it should go without saying . . . :

Basic safety invariant

At each point, "live" variables (with or without next use in the current block) must exist in at least one location

- another invariant: the location returned by getreg: the one where the result of a 3AIC assignment ends up

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Register and address descriptors

- code generation/$getreg$: keep track of
  1. register contents
  2. addresses for names

Register descriptor

- tracking current "content" of reg's (if any)
- consulted when new reg needed
- as said: at block entry, assume all regs unused

Address descriptor

- tracking location(s) where current value of name can be found
- possible locations: register, stack location, main memory
- >1 location possible (but not due to overapproximation, exact tracking)



Code generation algo for $x := y$ op z

1. determine location (preferably register) for result

```
l = getreg( ``x := y op z'')
```

2. make sure, that the value of y is in l :
   - consult address descriptor for $y \Rightarrow$ current locations l_y for y
   - choose the best location l_y from those (preferably register)
   - if value of y *not* in l, generate

```
MOV l_y , l
```

3. generate

```
OP l_z , l // l_z: a current location of z (prefer reg's)
```

   - update address descriptor $[x \mapsto_{\cup} l]$
   - if l is a reg: update reg descriptor $l \mapsto x$

4. exploit liveness/next use info: update register descriptors

Skeleton code generation algo for $x := y \text{ op } z$

```
1 l = getreg(``x:= y op z'') // target location for x
2 if l ∉ T_a(y) then let l_y ∈ T_a(y)) in emit ("MOV l_y, l");
3 let l_z ∈ T_a(z) in emit ("OP l_z,l");
```

- "skeleton"
  - *non-deterministic*: we ignored how to choose l_z and l_y
  - we ignore *book-keeping* in the *name* and *address* descriptor tables ($\Rightarrow$ step 4 also missing)
  - details of $getreg$ hidden.



```
1 l = getreg(``x:= y op z'') // generate target location for x
2 if l ∉ T_a(y)
3 then let l_y ∈ T_a(y)) // pick a location for y
4       in  emit (MOV l_y , l)
5 else skip;
6 let l_z ∈ T_a(z) in emit ("OP l_z , l");
7 T_a := T_a[x ↦∪ l];
8 if   l is a register
9 then T_r := T_r[l ↦ x]
```



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Exploit liveness/next use info: recycling registers

- register descriptors: don't update themselves during code generation
- once set (e.g. as $R_0 \mapsto t$), the info stays, unless reset
- thus in step 4 for $z := x \ \mathbf{op} \ y$:



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Code generation algo for $x := y \text{ op } z$

```
1  l = getreg("i: x := y op z")   // i for instructions line number/label
2  if l ∉ T_a(y)
3  then let l_y = best (T_a(y))
4       in   emit ("MOV l_y, l")
5  else skip;
6  let l_z = best (T_a(z))
7  in emit ("OP l_z,l");
8  T_a := T_a\(_ ↦ l);
9  T_a := T_a[x ↦ l];
10 if   l is a register
11 then T_r := T_r[l ↦ x];
12
13 if   ¬T_live[i, y] and T_a(y) = r then T_r := T_r\(r ↦ y)
14 if   ¬T_live[i, z] and T_a(z) = r then T_r := T_r\(r ↦ z)
```

Updating and exploit liveness info by recycling reg's

if y and/or z are currently

- *not live* and are
- in *registers*,

$\Rightarrow$ "wipe" the info from the corresponding register descriptors

getreg algo: $x := y$ op z

- goal: return a location for x
- basically: check possibilities of register uses
- starting with the "cheapest" option

Do the following steps, in that order

1. **in place:** if x is in a register already (and if that's fine otherwise), then return the register
2. **new register:** if there's an unsused register: return that
3. **purge filled register:** choose more or less cleverly a filled register and save its content, if needed, and return that register
4. **use main memory:** if all else fails



getreg algo: $x := y$ op z in more details

1. if
   - y in register R
   - R holds *no alternative names*
   - y is *not live* and has no next use after the 3AIC instruction
   - $\Rightarrow$ return R
2. else: if there is an empty register R': return R'
3. else: if
   - x has a next use [or operator requires a register] $\Rightarrow$
     - find an occupied register R
     - store R into M if needed (MOV R, M))
     - don't forget to update M 's address descriptor, if needed
     - return R
4. else: x not used in the block *or* no suituable occupied register can be found
   - return x as location l

- choice of purged register: *heuristics*
- remember (for step 3): registers may contain value for > 1 variable $\Rightarrow$ *multiple* MOV's



Sample TAIC

```
d := (a-b) + (a-c) + (a-c)
```

```
1  t := a - b
2  u := a - c
3  v := t + u
4  d := v + u
```

line	a	b	c	d	t	u	v
[0]	$L(1)$	$L(1)$	$L(2)$	D	D	D	D
1	$L(2)$	$L(\perp)$	$L(2)$	D	$L(3)$	D	D
2	$L(\perp)$	$L(\perp)$	$L(\perp)$	D	$L(3)$	$L(3)$	D
3	$L(\perp)$	$L(\perp)$	$L(\perp)$	D	D	$L(4)$	$L(4)$
4	$L(\perp)$	$L(\perp)$	$L(\perp)$	$L(\perp)$	D	D	D



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	3AIC	2AC	reg. descr.		addr. descriptor						
			R_0	R_1	a	b	c	d	t	u	v
[0]			$\perp$	$\perp$	a	b	c	d	t	u	v
1	t := a - b										



Code sequence

	3AIC	2AC	reg. descr.		addr. descriptor						
			R_0	R_1	a	b	c	d	t	u	v
[0]			$\perp$	$\perp$	a	b	c	d	t	u	v
1	t := a − b	**MOV** a, R0	$[a]$		$[R_0]$						
		SUB b, R0	t		~~R_0~~				R_0		
2	u := a − c	**MOV** a, R1	·	$[a]$	$[R_0]$						
		SUB c, R1		u	~~R_0~~					R_1	
3	v := t + u	**ADD** R1, R0	v	·					~~R_0~~		R_0
4	d := v + u	**ADD** R1, R0	d					R_0			~~R_0~~
		MOV R0, d									
			R_i: unused		all var's in "home position"						

- address descr's: "home position" not explicitely needed.
- e.g. variable a to be found "at a " (if not stale), as indicated in line "0".
- in the table: only *changes* (from top to bottom) indicated
- after line 3:
  - t dead
  - t resides in R_0 (and nothing else in R_0)

  $\rightarrow$ reuse R_0
- Remark: info in [brackets]: "ephemeral"



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Bibliography

[1] Aho, A. V., Lam, M. S., Sethi, R., and Ullman, J. D. (2007). *Compilers: Principles, Techniques and Tools*. Pearson,Addison-Wesley, second edition.

[2] Aho, A. V., Sethi, R., and Ullman, J. D. (1986). *Compilers: Principles, Techniques, and Tools*. Addison-Wesley.