Petri Nets and Model Checking

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- concurrency
- synchronization
- modeling distributed systems

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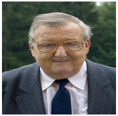
- Invented by C.A.Petri 

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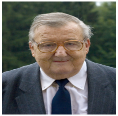
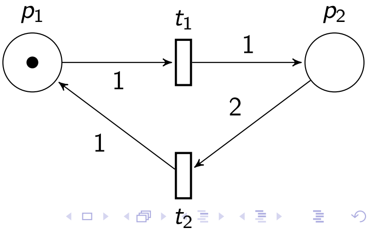
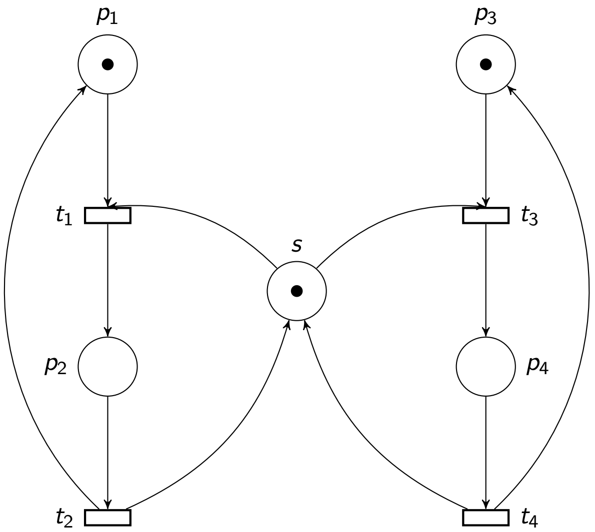
- Invented by C.A.Petri 

They are consisting of:

- places
- transitions
- arcs
- tokens
- initial marking



Petri Nets - Mutual Exclusion



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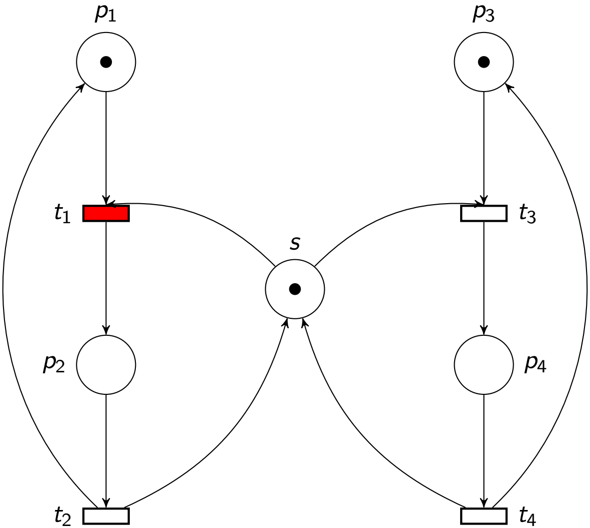

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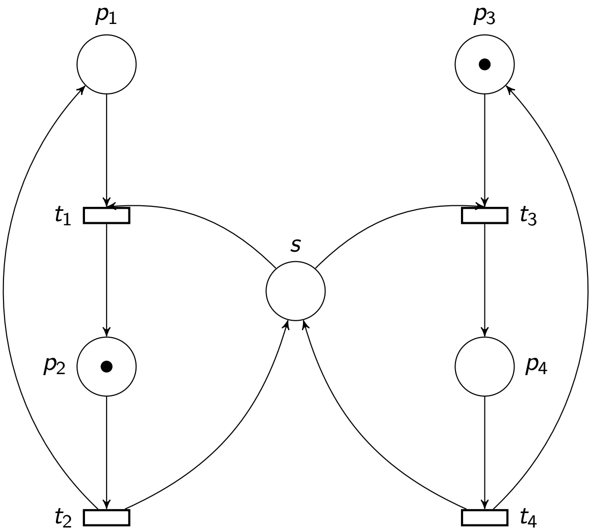

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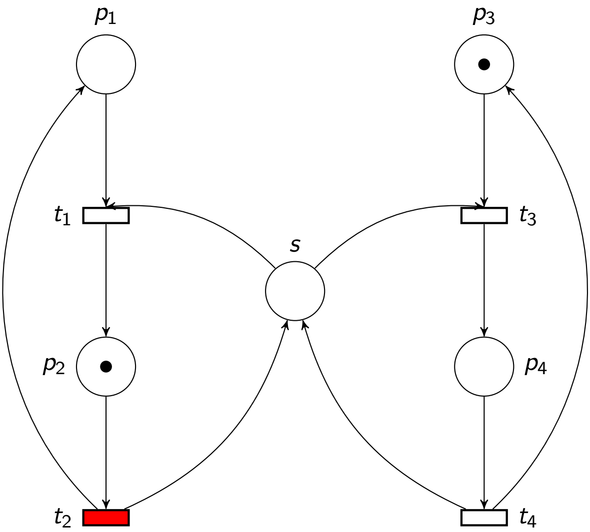

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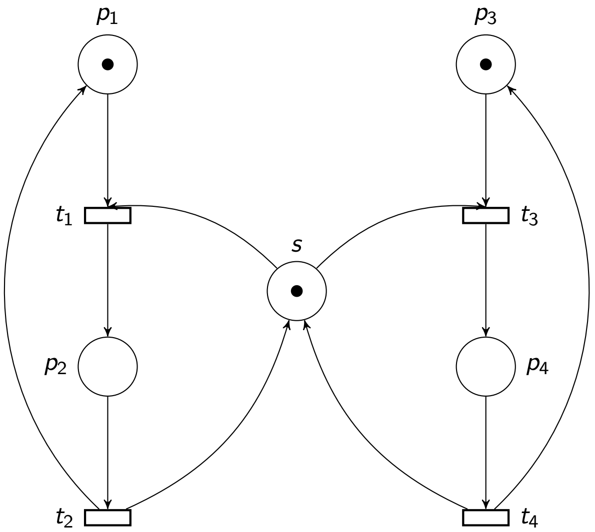

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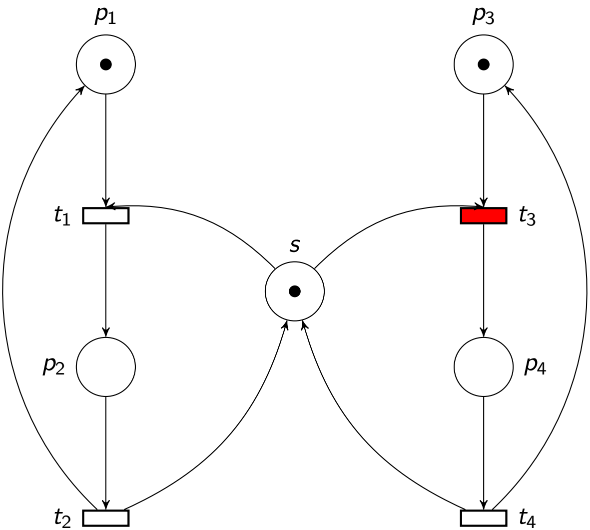

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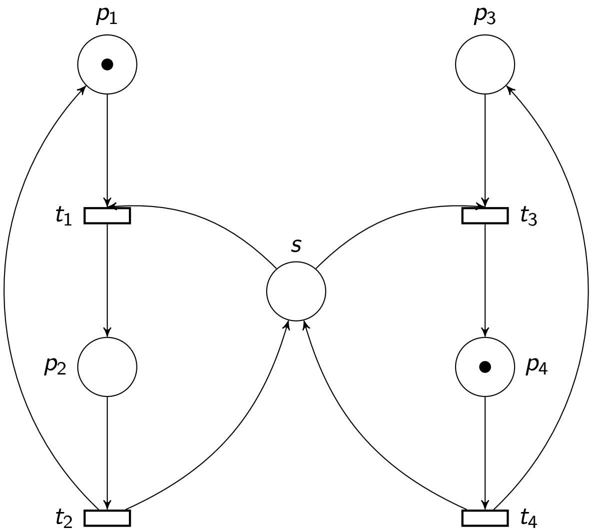

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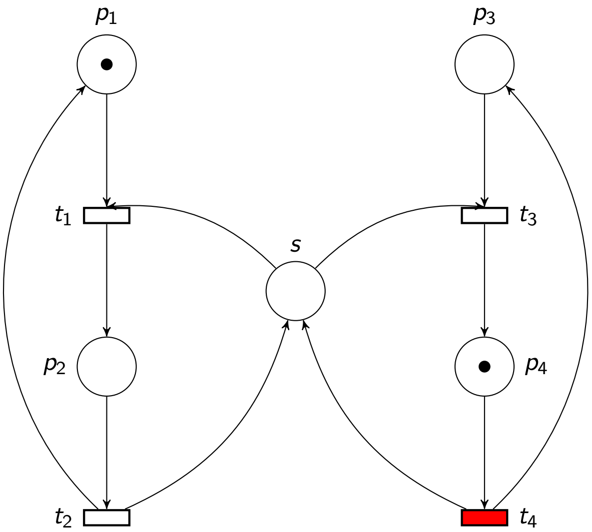

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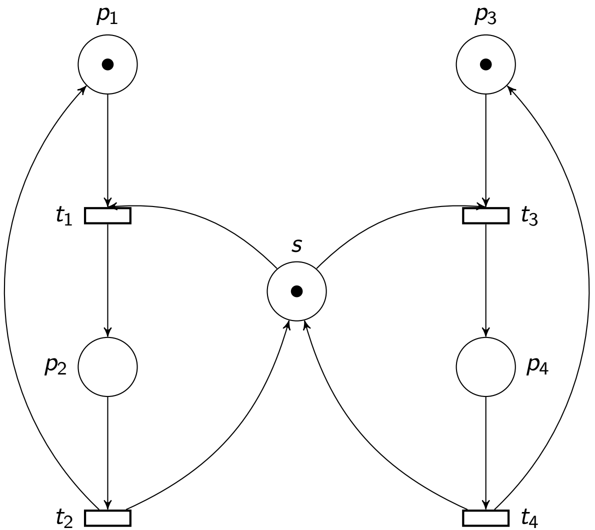
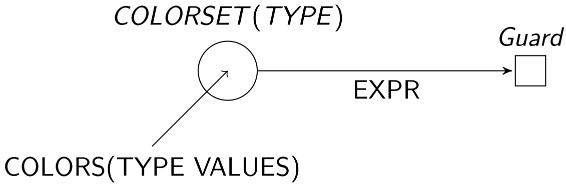


Colored Petri nets

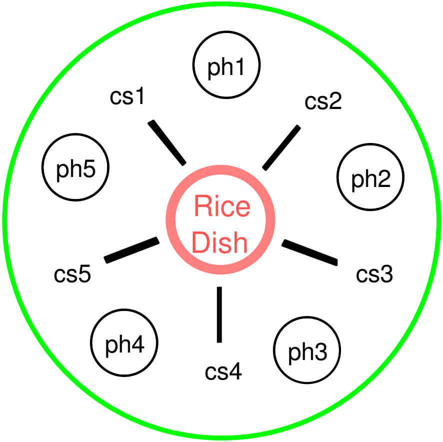
High-level Petri nets

The extension of Petri nets (called *place/transition nets*) with abstract data types.

Colored Petri nets

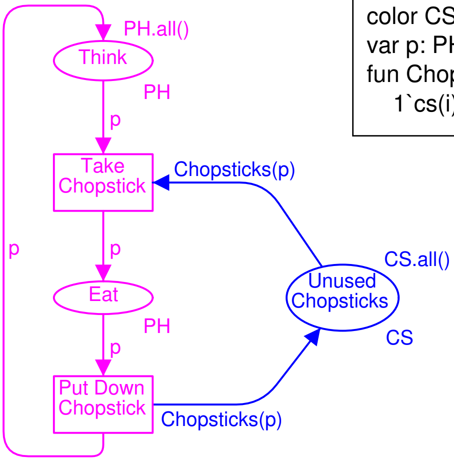


Example: Dining Philosophers



Example: Dining Philosophers

```
val n = 5;
color PH = index ph with 1..n;
color CS = index cs with 1..n;
var p: PH;
fun Chopsticks(ph(i)) =
   1`cs(i)++1`cs(if i=n then 1 else i+1);
```



State Space

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A directed graph having a node for each reachable marking and an arc for each occurring binding element.

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The *strongly-connected-component graph (SCC graph)* is the graph derived from the state space where each node is a SCC of the state space.

SCC graph

- is an acyclic graph
- fewer nodes than the ss mean that there exist infinite occurrence sequences
- more efficient since often much smaller than the ss

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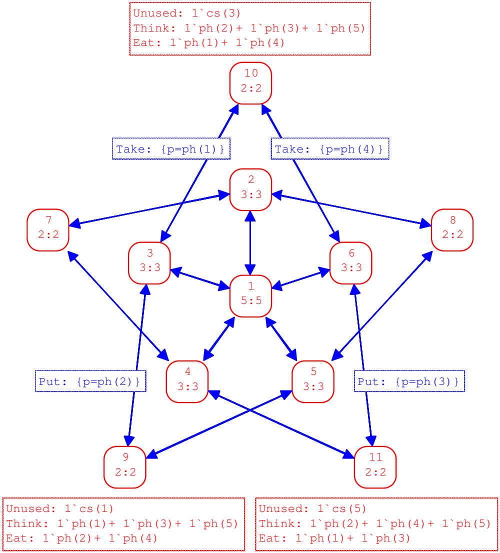

Behavioral Properties

Boundedness properties

How many and which tokes a place may hold when all reachable markings are considered.

Home Properties

A *home marking* is a marking that can be reached from any reachable marking

- *All the markings in a (single) terminal SCC are home markings*

Behavioral Properties

Liveness Properties

A *dead marking* is a marking in which no binding elements are enabled.
Similarly *dead transition*.
A transition is *live* if, starting from any reachable marking, we can always find an occurrence sequence containing it.

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Fairness Properties

How often transitions occur in infinite occurrence sequences.
A transition is *impartial* if it occurs infinitely often in all infinite occurrence sequences.

- *Removal of this transition implies no infinite occurrence sequences!*

Example: Dining Philosophers

|PH|	Nodes	Arcs
2	3	4
3	4	6
4	7	16
5	11	30
6	18	60
7	29	112
8	47	208
9	76	378
10	123	680
15	1,364	11,310

State Space Reduction Methods

- Sweep-Line method

A *progress measure* is a function that maps each marking into a *progress value*.
For a given marking, the progress value of any successor marking must be greater or equal to its progress value.

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Equivalence classes used for symmetric markings and symmetric binding elements.

  - the ss can be significantly reduced
  - can check all behavioral properties that are invariant under symmetry
  - computing canonical representations of markings and binding elements is computationally expensive

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- Equivalence method

A generalization of the symmetry method. Here, no requirement that the equivalence relations should be induced by symmetries.

Thank you!