INF5300 – spring 2015 | Exercises | Graph-based SSL and image segmentation

1) Do the intermediate steps in deriving E($\mathbf{y}$) = $\mathbf{y}^T\mathbf{L}\mathbf{y}$ presented on slide 14 in the lecture foils. (Convince a fellow student that the statement is true.)

2) Do all the intermediate steps when finding the $\mathbf{y}$ that minimize E($\mathbf{y}$) s.t. $\mathbf{y}_l$ fixed (slides 16,17) . That is, go from E($\mathbf{y}$)=$\mathbf{y}^T\mathbf{L}\mathbf{y}$ and the division of $\mathbf{y}$ into $\mathbf{y}_l$ and $\mathbf{y}_u$ on page 16 and prove that the solution given on page 17 is correct. Also, convince yourself that the two "Note that"s on pages 16 and 17 are true.

3) On the eigendecomposition of the Laplacian, cf. slides 23 and 24:

a) In Matlab, create the **D**, **W** and **L** (Laplacian) matrices corresponding to the depicted graph. (Assume unit edge-weights.)

b) Do an eigendecomposition of the **L** matrix and verify that the eigenvectors of the two zero-valued eigenvalues looks similar to the ones in the figure.

c) Explain why these eigenvectors have zero eigenvalues based on the result on slide 23 and an "intuitive" understanding of the non-smoothness measure E($\mathbf{y}$).

d) Remove one of the non-zero edges and study the eigenvalues and corresponding eigenvectors. Remove one more edge and repeat. Based on this (and perhaps some more experiments); what is the connection between the number of zero eigenvalues and the number of connected components (separate subgraphs)?

e) Put a "weak" edge (having a small weight) in place of the missing link in the original graph. How many zero-valued eigenvalues do we have? What do the eigenvectors corresponding to the two smallest eigenvalues look like? How could this be used in a (fully) unsupervised segmentation?

4) Find the $\mathbf{y}$ minimizing C($\mathbf{y}$) on slide 25 by taking the derivative and equating it to zero. That is, do the steps getting to the solution found on the same slide.

5) Download the file «INF5300_LPBasedImageSegmentation.zip» linked to on the course's web page ("Undervisningsplan"). The zip file contains the Matlab scripts and image files behind the example of user-guided image segmentation shown during the lecture.

a) Run the script, and familiarize yourself with it.

b) Alter the code to implement the «class mass normalization» scheme; either by converting the solution vector of (soft) labels into a "soft" indicator matrix, or formulate an equivalent scheme by altering the threshold (which is, in the code, now simply set to 0).

c) Change the script by replacing the solution based on matrix-inversion by an iterative solver for the quadratic criterion. That is, implement "Algorithm 11.2".

d) Try changing the initial labels (the red and green pixels in the label-init image) so that we segment some other structure in the image.

e) Study the effect of changing some of the other parameters, like the size of the texture patches, sigma value, image scale factor, standardizing the texture features, etc.