INF5410 Array signal processing.
Chapter 2.3 Non-linearity

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Compliance in closed chamber

- The gas law without heat transfer (adiabatic): $pV^\gamma = C$
  - $\gamma$ is the adiabatic exponent, $\gamma = 1.4$ for air
  - $p$ is pressure and $V$ is volume.
- A loudspeaker affects the volume, $V$
  - Our ears sense the resulting pressure, $p$.
- In loudspeakers nonlinearity affects the lower frequencies: small subwoofers
  - cone excursion increases with lower frequency
- The nonlinearity of the pressure – volume relationship $\Rightarrow$ nonlinear acoustics.
Harmonic generation

- Muir and Carstensen: Prediction of nonlinear acoustic effects at biomedical frequencies and intensities, Ultrasound in Medicine & Biology, 1980

State equation for gas

\[ \frac{p}{p_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma \]

- Taylor series for pressure variation:
  \[ p - p_0 = A \rho - \rho_0 + \frac{B}{2!} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \ldots \]
  - A = \rho_0^\gamma, B = \rho_0^\gamma(\gamma - 1); B/A = \gamma - 1
  - A nonlinear spring: replaces Hooke's law
  - Similar approach for fluids
Non-linear acoustics PDE

From Eqns. (5) and (6) we also obtain the "classical" equation in the single variable \( \phi \):

\[
\nabla^2 \phi - \frac{1}{c_0^2} \frac{\partial \phi}{\partial t} + \frac{D}{c_0^2} \nabla^2 \phi \frac{\partial \phi}{\partial t} = \frac{1}{c_0^2} \frac{\partial}{\partial t} \left[ (\nabla \phi)^2 + \frac{1}{c_0^2} \frac{\partial \phi}{\partial t} \right]. \tag{11}
\]

- Aanonsen, Barkved, Tjetta, Tjetta: Distortion and harmonic generation in the nearfield of a finite amplitude sound beam, JASA, 1984

• Notice:
  - \( \Phi \) is velocity potential
  - Squaring on r.h.s. implies nonlinearity, \( B/A \) is nonlinearity coefficient
  - R.h.s: 1. term = local generation, 2. term is cumulative effect
  - \( D \) is viscous absorption term, i.e. \( \text{water} \Rightarrow \text{attenuation} \propto \omega^2 \)

Simplified equations for simulations

• Westerveld equation (1963)
  - Right-hand side: 1. term (local term) is dropped
  - \( \text{OK}\) away from source (quasi-plane wave)

• KZK-equation (Khoklov-Zabolotskaya-Khoklov, 1969, 1971)
  - Weak nonlinearity: Dissipation and nonlinearity cause slow changes of the beam in space
  - For a directed sound beam where variations across beam are more rapid than along the beam
  - Bergen code: [http://folk.uib.no/nmajb/Bergencode.html](http://folk.uib.no/nmajb/Bergencode.html)

• Burgers’ equation (1948)
  - Like KZK
  - \( + 1-D \) = plane waves = no diffraction
  - \( v \) = fluid velocity
  - \( \beta = 1+2B/A \)
  - \( b \) – related to viscous absorption
Nonlinearity parameter

<table>
<thead>
<tr>
<th>Material</th>
<th>B/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood</td>
<td>6.1</td>
</tr>
<tr>
<td>Brain</td>
<td>6.6</td>
</tr>
<tr>
<td>Fat</td>
<td>10</td>
</tr>
<tr>
<td>Liver</td>
<td>6.8</td>
</tr>
<tr>
<td>Muscle</td>
<td>7.4</td>
</tr>
<tr>
<td>Water</td>
<td>5.2</td>
</tr>
</tbody>
</table>

- Wikipedia

Non-linear acoustics

c, varies with the particle displacement, u, or pressure p:

\[ c(t) = c_0 + \frac{B}{2A} u(t) = c_0 + \frac{B}{2A} \frac{p(t)}{\rho \gamma} \]

- \( p_0(t) \) = pressure = \( p_0 + p(t) \)
- \( p_0 = 1 \) atmosphere
- \( p(t) \) = applied pressure variation (= ”signal”)
Nonlinearity and plane wave

- A plane wave in water,
- Initial amplitude: 2 MPa (20 atmospheres)
- Frequency of 3.5 MHz
- Propagates for 100 mm.
- Starts to deform immediately,
- Peak-to-peak amplitude and power decrease only slowly, following the usual exponential attenuation of water.
- Beyond 35 mm, however, a shock wave has formed, and the amplitude decreases relatively rapidly.
- By 100 mm, the amplitude has halved, and 80% of the beam's power has been lost.
- Generated by the "Bergen code" written at the University of Bergen in Norway.
- [http://www.bath.ac.uk/~pyscmd/acoustics/nonlin.htm](http://www.bath.ac.uk/~pyscmd/acoustics/nonlin.htm)
Harmonic vs intermodulation distortion

Transmit $f_1$ and $f_2$

1. Harmonic distortion $2f_1, 2f_2, 3f_1, 3f_2, ...$
2. Intermodulation distortion $f_1-f_2, f_1+f_2, 2f_1-f_2, ...$

1. Harmonic imaging:
   - Transmit $f$
   - Generate $2f, 3f, 4f, ...$
     » Usually $2f$ is the most important one

2. Parametric sonar, parametric sound source:
   - Transmit $f_1$ and $f_2$
   - Use difference frequency $f_1-f_2$.

1. Harmonic imaging

- Positive effect on images:
  - 2. harmonic beam is narrower => better resolution
  - Is not generated in sidelobes of 1. harmonic beam => less sidelobes
  - Is generated inside medium => avoids some of the reverberations from chest wall

- Negative effect:
  - 2. harmonics attenuates faster => less penetration

Whittingham, 2007
Circular symmetric (1-D) simulation - J.F. Synnevåg
Burgers equation (Christopher & Parker k-space)

Focus 60 mm, f=2.275 MHz

Liver
2 harm
2a. Parametric sonar

- Topas: Kongsberg Defense & Aerospace
- Parametric sub-bottom profilers
- Low frequency sound generation due to non-linear interaction in the water column from two high intensity sound beams at higher frequencies.
- The resulting signal has a high relative bandwidth (~80%), narrow beam profile
- Penetration ~100 m, 150 ms

<table>
<thead>
<tr>
<th></th>
<th>TOPAS PS18</th>
<th>TOPAS PS40</th>
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<tbody>
<tr>
<td>Secondary frequency</td>
<td>0.5-6kHz</td>
<td>1-10kHz</td>
</tr>
<tr>
<td>Primary frequencies</td>
<td>15-21 kHz</td>
<td>35-45 kHz</td>
</tr>
<tr>
<td></td>
<td>or 30-42 kHz</td>
<td>or 70-90 kHz</td>
</tr>
<tr>
<td>Source levels</td>
<td>Secondary: 208</td>
<td>Secondary: 207</td>
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<tr>
<td></td>
<td>Primary: 242/225 dB</td>
<td>Primary: 241/226 dB</td>
</tr>
<tr>
<td>Hor. resolution</td>
<td>&lt;5 x 5 deg</td>
<td>3 x 6 deg</td>
</tr>
<tr>
<td>Signatures</td>
<td>CW, Chirp, Ricker</td>
<td></td>
</tr>
</tbody>
</table>
2b. Parametric audio sound source

- Non-linear interaction
- Holosonics: Audio Spotlight
- American Technology Corporation: HyperSonic Sound technology:
  - [http://www.atcsd.com/site/content/view/13/104/](http://www.atcsd.com/site/content/view/13/104/)

Mad Labs: Audio Spotlight

- Youtube demo: [http://www.youtube.com/watch?v=veDk2Vd-9oQ&feature=related](http://www.youtube.com/watch?v=veDk2Vd-9oQ&feature=related)
- Mad Labs from the National Geographic Channel presents the Audio Spotlight, focused loudspeaker technology, 3 min 12 sec
- See [http://www.audiospotlight.com](http://www.audiospotlight.com) for more.
Array Processing Implications

• Nonlinearity may create new frequencies that were not present in the source
  – Harmonics
  – Intermodulation: [Sum]/Difference frequencies

• Harmonics: harmonic (octave) imaging in medical ultrasound
• Difference frequency: Parametric sonar, directive audio source