Synthetic Aperture Radar and Sonar – SAR and SAS

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Some illustrations from Roy Hansen, FFI/UIO
SAR Resolution

- Real aperture beamwidth: 
  \( \theta \approx \frac{\lambda}{D} \)
- Size of footprint: 
  \( L_F \approx \theta \cdot R = \frac{R \cdot \lambda}{D} \)
- Size of synthetic aperture = footprint: 
  \( L_s \approx L_F \)
- Beamwidth of synthetic aperture: 
  \( \theta_s = \frac{1}{2} \cdot \frac{\lambda}{L_s} = \frac{1}{2} \cdot \frac{\lambda}{(R \cdot \lambda/D)} = \frac{D}{2R} \)
  - Factor 2 due to two-way system, transmitter is also focused
- Ground resolution: 
  \( \Delta X_s = R \theta_s = \frac{D}{2} \)

4 Aperture synthesis. The target is in the beam for a time \( T_s = \frac{L_s}{V} \). After phase-correcting the signals, a synthetic antenna pattern is obtained which is equivalent to that of a conventional antenna of length \( 2L_s \).
SAR Resolution

- Ground resolution: \( \Delta X_s = R \theta_s = D/2 \)

- Note:
  - The smaller the real antenna, the better the resolution
  - Resolution is independent of range
  - Why?
    » A small D causes the synthetic aperture to be larger
    » But, small D means energy is spread over larger area, so SNR suffers

- Range resolution: \( \Delta X_r = cT/2 = c/2B \)
  - as in any pulsed system
SAR – Doppler Interpretation

- Doppler equation: \( f_D = 2 \cdot \frac{v}{c} \cdot f_0 \sin \theta \)
- Max Dopplershift: 
  \[ f_D = 2 \cdot \frac{v}{c} \cdot f_0 \sin \theta / 2 \approx 2 \cdot \frac{v}{c} \cdot f_0 \theta / 2 \]
  \[ = \frac{v}{c} \cdot \frac{c}{\lambda} \cdot \frac{\lambda}{D} = \frac{v}{D} \]
- Doppler bandwidth: \( B_D = 2 \cdot f_D \)
- Time resolution: \( t_m = 1 / B_D = D / 2v \)
- Equivalent azimuth resolution: \( X_a = v \cdot t_m = D / 2 \)
- QED!
  Same result as found from aperture-resolution considerations

http://www2.jpl.nasa.gov/basics/bsf12-1.html
SAR – Doppler - Sampling

• Doppler shift is in the range +/- $f_D$
• Proper complex sampling with $PRF > 2f_D = 2v/D$
• Max movement of aperture per pulse: $x = v \cdot T = v/PRF = D/2$
  - No point in having $x < \lambda/2$ so $x \leq \max(D/2, \lambda/2)$
  » Gough & Hawkins, IEEE JOE, Jan 1997 claim that there should be no more than $D/4$ between pulses
    • Element beamwidth/Doppler bandwidth is not easily defined:
      – $D/4$ $\Leftrightarrow$ null-to-null sinc bw
      – $D/2$ $\Leftrightarrow$ 3dB.
    • A question of acceptable level of azimuth ambiguity

- Satellite – the simplest SAR
- Real aperture: \( D = 10 \) m
- Frequency: 5.3 GHz
- Wavelength: \( \lambda = 5.66 \) cm
- Height: \( H = 785 \) km
  - Angle 23 deg => distance \( R = \frac{785}{\cos(23)} \approx 850 \) km
- Real aperture beamwidth: \( \theta = \frac{\lambda}{D} = 0.33^\circ \)
- Real aperture azimuth resolution = synthetic aperture: \( L_s = \frac{\lambda}{D \cdot R} = 4850 \) m
- SAR resolution: \( D/2 = 5 \) m
- \( B = 19 \) MHz => Range res 8m
- Velocity: \( v = 7 \) km/sec

<http://www.sso.admin.ch/Themes/02-Earth_observations/english/e_03_ESA_missions_in_orbit.htm>
ERS-1 SAR image of west coast of Norway, 22 June 1996.

http://marsais.nersc.no/product_wind.html
Sampling considerations

- Fast enough for Doppler $\Leftrightarrow$ no grating lobes: $\text{PRF} > \frac{2v}{D}$
- Simple radar, only one pulse in medium at a time
  
  \[
  \text{PRT} = \frac{1}{\text{PRF}} > \frac{2R}{c}
  \]
  
  i.e. $\frac{2v}{D} < \text{PRF} < \frac{c}{2R}$ or $R < \frac{Dc}{4v}$

- Satellite radar, SAR:
  - $R < \frac{3\times10^8}{(4\times7\times10^3)} = 107$ km
  - Swath width $< R$ but further away than $R$ such as in satellite SAR $\Rightarrow$
    - Several pulses in medium at a time
    - But, no sampling while tx
    - And no sampling during subsatellite echo
Sat. SAR coverage

- Solid lines: blind, transmission takes place
- Dotted lines: subsatellite echo
- PRF is usually set so that blind and subsat. echo regions coincide

9 Spaceborne SAR coverage diagram. The solid diagonal lines represent ‘blind regions’ which are invisible to the SAR owing to the fact that the radar cannot receive while transmitting. These regions form boundaries between mutually ambiguous swaths. The dotted lines represent regions which are saturated by strong subsatellite radar returns. The pulse repetition frequency of a system is usually set so that these regions coincide with the blind regions. Also shown on the diagram are the six swaths chosen for the VSAR system.
Aircraft SAR

- Real aperture: D=1 m
- Frequency: 5.3 GHz
- Wavelength: \( \lambda = 5.7 \text{ cm} \)
- Height: H=4-5 km, i.e. \( R = \frac{H}{\cos(30)} \approx 5 \text{ km} \)
- Real aperture beamwidth: \( \theta = \frac{\lambda}{D} = 3.3^\circ \)
- Real aperture azimuth resolution = Synthetic aperture: \( L_s = \frac{\lambda}{D} \cdot R = 285 \text{ m} \)
- SAR resolution: \( D/2 = 0.5 \text{ m} \)
- \( v = 720 \text{ km/hr} = 200 \text{ m/s} \)
- (Some guesses)

- Example: Road in war zone.
- Compare images with some days between
- Look for signs of IEDs (Improvised explosive devices)
Synthetic aperture sonar: Hugin

Height: $R = \text{typ } 20\, \text{m}$, speed: $v = \text{typ } 2\, \text{m/s}$

FFI & Kongsberg Maritime
The SAS challenge

- Max range $R < D \cdot c/4v = 1500 \cdot 0.1/(4 \cdot 2) = 9.4 \text{ m}$ (slide 9)
- Multi-element rx to overcome this range limitation
- $D$ is replaced by $L = ND \Rightarrow$ Phase Center Approximation (PCA) $\Rightarrow$ Range increased by $N$

$$d$$

$L = Nd$

$D = L/2$

$D$ in drawing is $D$ in text!
SAS Geometry (Hugin)

- $R_{\text{max}} = 200 \text{ m} \text{ and } R_{\text{min}} = R_{\text{max}}/10 = 20\text{m}$
Real aperture – synthetic aperture

Real aperture – all rx/tx combinations

SAR

SAS – like seismics
Hugin AUV - HISAS

- Rx: 1.2 m = 32 x 3.75 cm
- Tx: slightly larger than rx element, ~4 cm
- f=70-120, typ 100 kHz, \( \lambda = 1.5 \) cm
- Bandwidth, typ. B=30 kHz
- Synthetic aperture @ range 200 m:
  \[ L_s = \frac{\lambda}{D} \cdot R = \frac{1.5}{4} \cdot 200 = 75 \) m

- Resolution:
  - SAR: \( D/2 = 4/2 = 2 \) cm
  - Range: \( c/2B = 2.5 \) cm
SAR vs SAS

- Criterion for not creating increased sidelobe level:
  - position known to $\lambda/16$

- Satellite ERS-1, $\lambda$=5.7 cm
  - $L_s = 4850$ m, $v = 7$ km/s $=>$ Illumination time = 0.7 sec
  - Must know position within 3.5 mm over 0.7 sec

- Aircraft SAR, $\lambda$=5.7 cm
  - $L_s = 285$ m, $v = 200$ m/s $=>$ Illumination time = 1.4 sec
  - Must know position within 3.5 mm over 1.4 sec

- Sonar Hugin, HISAS $\lambda$= 1.5 cm:
  - $L_s \approx 75$ m (varies approx. 1:10), $v = 2$ m/s $=>$ Illumination time = 38 sec (4 – 38 sec)
  - Must know position within 1 mm over 38 sec!
SAR vs SAS: $c=3 \cdot 10^8$ vs 1500 m/s

- **Motion compensation**: much more severe for sonar as it takes much longer to travel one synthetic aperture => accurate navigation and micronavigation (sub $\lambda$ accuracy)
- More severe **range ambiguity** problem for sonar than radar. Harder to achieve good mapping rate => multielement rx arrays which also can be used for DPCA (displaced phase-center antenna) micronavigation
- **Noise**: SAR – thermal/electronic noise, SAS – noisy medium
- **Medium**: Sonar – multipath, refraction, instability, attenuation; Radar – much more stable, only spherical spreading loss
- Same **range resolution** for smaller bandwidth in SAS than SAR: $\Delta X_r = c/2B$
Imaging modes

- Strip-map ("standard mode")
- Spotlight mode (figure)
- Squint mode

Literature


