UiO : Department of Informatics
University of Oslo

INF 5460
Electronic noise – estimates and countermeasures

Lecture no 8 (Mot 5)
Noise in bipolar transistors

We have previously found *Eni*, *En*, and *In* for an amplifier. We will now do this for a bipolar transistor. We will see that the noise is both depending on the operating point (current and voltage), the transistor semiconductor process and layout parameters.

The hybrid-π model

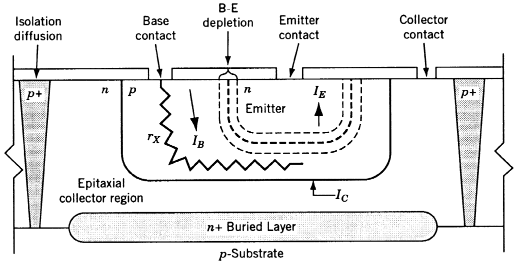


Figure 5-2 Cross-section diffused *npn* transistor.

Before we look at the noise we will look at a simple model for bipolar transistors. This model applies for both npn and pnp.

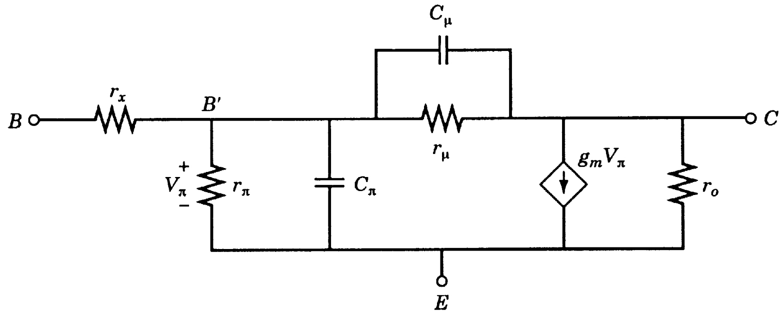


Figure 5-1 Hybrid-π bipolar transistor small-signal model.

Note that in the figure is B the external connection point for the base B while B' is the internal, efficient point that best represents the base.

r_X: is the stray resistance in the base.

r_π: Real part of impedance B'-E (NB: No thermal noise)

c_π: Imaginary part of the impedance B'-E drawn as a capacitance.

$g_m V_\pi$: Current generator that determines Ic.
r_0: Dynamic output resistance of C

r_μ and C_μ: Models the depletion zone between B' and C. Ignored if one wants a simpler low frequency model.

Some well-known expressions:

$$\beta_0 = \frac{I_C}{I_{BE}} = \frac{g_m V_\pi}{V_\pi / r_\pi} = g_m r_\pi$$

Presuppose $r\mu=\infty$ and all $C\approx 0$ (i.e. low-frequency consideration.)

Transconductance:

$$g_m = \frac{qI_C}{kT}$$

NB! Small signal ac-parameters related to a dc-current $\Rightarrow$ Limited validity range. Emitter resistance:

$$r_e = \frac{1}{g_m} \cong \frac{0.025}{I_C}\Omega$$

$r\pi$ in relation to some of the previous mentioned parameters:

$$r_\pi = \frac{\beta_0}{g_m} = \beta_0 r_e$$

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$$

An important value is the gain-bandwidth product f_T. We can modify the expression so that we get:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

f_{hfe} is the "beta-cut-off frequency". This is the frequency where β is 1/√2 of its low frequency value (DC-value) β_0.

$$f_{hfe} = f_\beta \cong \frac{f_T}{\beta_0}$$

Some example values:

r_π=97kΩ	r_0=1.6MΩ
r_x=278Ω	r_μ=15MΩ
g_m=0.0036S	C_μ=4pF
β_0=350	C_π=25pF
f_T=19.8MHz	f_β=56.5kHz

Noise model for the BJT

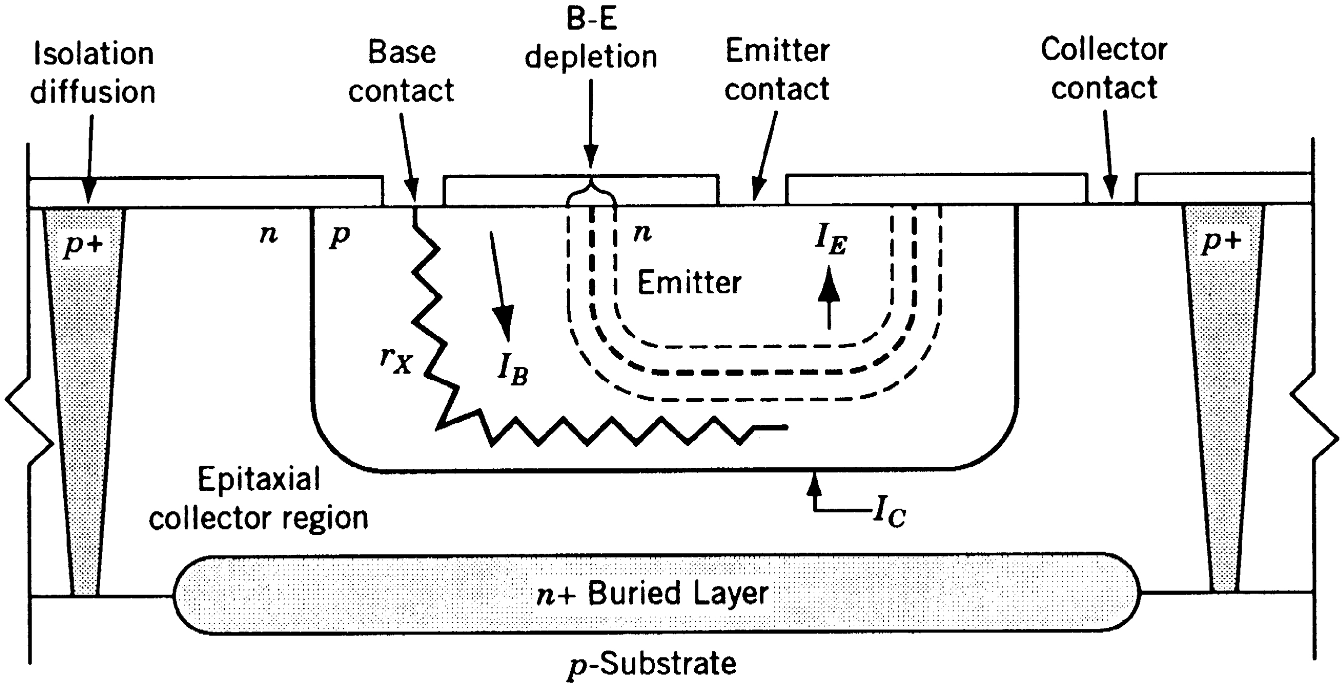


Figure 5-2 Cross-section diffused *npn* transistor.

The resistor r_X provides thermal noise. I_B and I_C provide shot-noise. The current passing through the base-emitter zone closest to the surface will have some flicker noise.

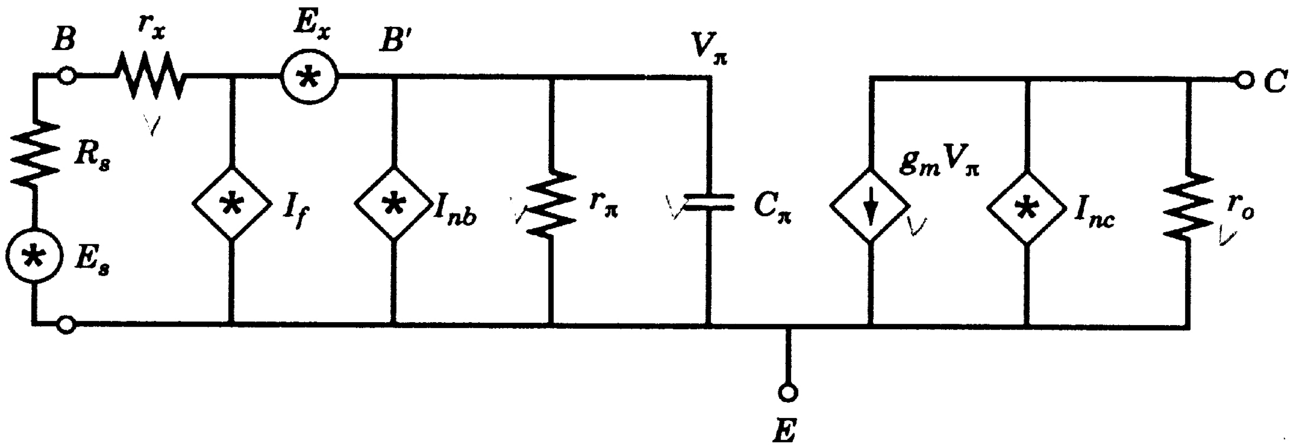


Figure 5-3 Hybrid-π bipolar transistor noise model.

Now we have four noise sources for the transistor. In addition we have the noise in the source. r_π and r_0 represent real parts of impedances without having thermal noise. To simplify, C_μ and r_μ have been omitted i.e. we look at frequencies below $f_T/\sqrt{\beta}$. Above this frequency some of the noises become correlated and we will have more noise than the model indicates.

Thermal noise in r_x: $$E_x^2 = 4kTr_x$$

Shot-noise due to I_B and I_C are, respectively:

$$I_{nb}^2 = 2qI_B$$ and $$I_{nc}^2 = 2qI_C$$

The flicker noise can be expressed by:

$$I_f^2 = \frac{K I_B^{\gamma}}{f^{\alpha}}$$

Often alpha can be set to one and *K* can be replaced by $2qf_L$ where f_L is a corner frequency typically in the range 3kHz to 7MHz. We will then get:

$$I_f^2 = \frac{2qf_L I_B^{\gamma}}{f}$$

The noise voltage as a result of this current can be found by multiplying it with the resistance it will go through: r_x. However experimental data shows that the effective r_x in this context is less. We therefore create a new r'_x which is ca. $r_x/2$.

$$E_f^2 = \frac{2qf_L I_B^{\gamma} r'^2_x}{f}$$

Equivalent input noise

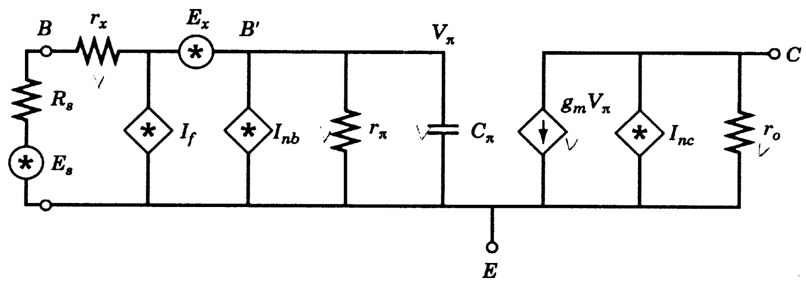


Figure 5-3 Hybrid-π bipolar transistor noise model.

Method:
1) Noise at output
2) Gain
3) Equivalent noise on the input = output noise/gain

1) Noise at the output:

First we find the noise at the output. If C (the output) is short circuited to E (i.e. no external contribution) we will have: $I_{no}^2 = I_{nc}^2 + \left(g_m E_\pi\right)^2$

Here is $E\pi$ the noise voltage between B‘ and E. After substituting the value for $E\pi$ we get:

$$I_{no}^2 = I_{nc}^2 + g_m^2\left[\frac{\left(E_x^2 + E_s^2\right)Z_\pi^2}{\left(r_x + R_s + Z_\pi\right)^2} + \frac{\left(I_{nb}^2 + I_f^2\right)Z_\pi^2\left(r_x + R_s\right)^2}{\left(r_x + R_s + Z_\pi\right)^2}\right]$$

2) Gain:

$$I_O = g_m V_\pi = \frac{g_m V_s Z_\pi}{r_x + R_s + Z_\pi} \qquad K_t = \frac{I_o}{V_s} = \frac{g_m Z_\pi}{r_x + R_s + Z_\pi}$$

3) Equivalent noise at the Input

$$E_{ni}^2 = \frac{I_{no}^2}{K_t^2} = E_x^2 + E_s^2 + \left(I_{nb}^2 + I_f^2\right)\left(r_x + R_s\right)^2 + \frac{I_{nc}^2\left(r_x + R_s + Z_\pi\right)^2}{g_m^2 Z_\pi^2}$$

We put in for the noise voltages and noise currents and let Δf = 1:

$$E_{ni}^2 = 4kT\left(r_x + R_s\right) + 2qI_B\left(r_x + R_S\right)^2 + \frac{2qf_L I_B^\gamma\left(r'_x + R_s\right)^2}{f} + \frac{2qI_C\left(r_x + R_S + Z_\pi\right)^2}{g_m^2 Z_\pi^2}$$

At low frequencies the last term may be written as:

$$\frac{2qI_C\left(r_x + R_S + r_\pi\right)^2}{\beta_0^2}$$

and at higher frequencies up towards $f_T/\sqrt{\beta_0}$ as

$$\frac{2qI_C\left(r_x + R_S + \dfrac{1}{\omega C_\pi}\right)^2}{\dfrac{g_m^2}{\omega^2 C_\pi^2}} \cong 2qI_C\left(r_x + R_S\right)^2\left(\frac{f}{f_T}\right)^2$$

Divided into a low-frequency component and a high frequency we get:

$$E_{ni}^2 = 4kT\left(r_x + R_s\right) + 2qI_B\left(r_x + R_S\right)^2 + \frac{2qI_C\left(r_x + R_S + r_\pi\right)^2}{\beta_0^2}$$

$$+ \frac{2qf_L I_B^\gamma\left(r'_x + R_S\right)^2}{f} + 2qI_C\left(r_x + R_S\right)^2\left(\frac{f}{f_T}\right)^2$$

En and In for bipolar transistors

En:

We find E_n by setting $R_s=0$ in the expression for the equivalent input noise.

$$E_n^2 = 4kTr_x + 2qI_B r_x^2 + \frac{2qI_C r_\pi^2}{\beta_0^2} + \frac{2qf_L I_B^\gamma r'^2_x}{f} + 2qI_C r_x^2 \left(\frac{f}{f_T}\right)$$

Since $r_\pi=\beta_0 r_e$ and $r_x^2 \ll \beta_0 r_e^2$ we can simply to

$$E_n^2 = 4kTr_x + 2qI_C r_e^2 + \frac{2qf_L I_B^\gamma r'^2_x}{f} + 2qI_C r_x^2 \left(\frac{f}{f_T}\right)^2$$

In:

We find I_n by letting R_s be high. We divide all terms by R_s^2 and let R_s go against ∞.

$$I_n^2 = 2qI_B + \frac{2qI_C}{\beta_o^2} + \frac{2qf_L I_B^\gamma}{f} + 2qI_C \left(\frac{f}{f_T}\right)^2$$

Since $I_C/\beta_0^2 \ll I_B$ the first term will dominate the second term which thus can be removed. We will then have:

$$I_n^2 = 2qI_B + \frac{2qf_L I_B^\gamma}{f} + 2qI_C \left(\frac{f}{f_T}\right)^2$$

Example

Find E_{ni}^2 for a 2N4250 transistor where I_C=1mA, R_S=10kΩ, Δf=10Hz, f_C=1kHz

Noise in the source resistance can be calculated:

$$E_t^2 = 4kTR_S = 4 \cdot 1.38 \cdot 10^{-23} Ws/K \cdot 300K \cdot 10k\Omega = 1.65 \cdot 10^{-16} Ws\Omega$$

For further calculation, we use:

$$E_{ni}^2 = \left(E_t^2 + E_n^2 + I_n^2 R_S^2\right)\Delta f$$

Based on the information in the book, we have two alternatives for how we can find E_n and I_n: From reading the figures or by calculation.

1) Reading values from the figure:

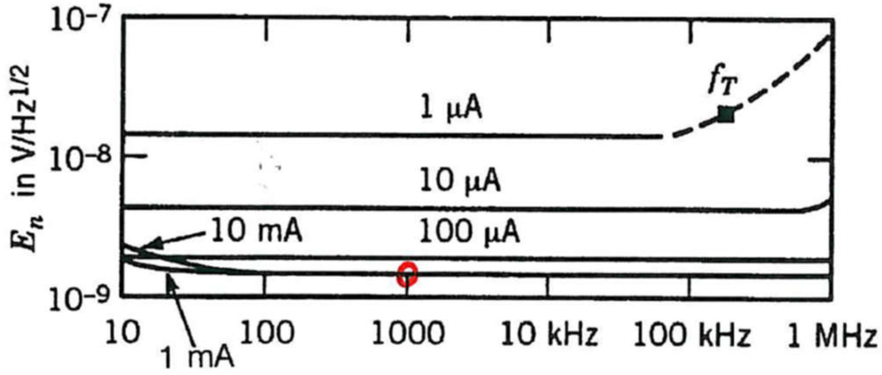
In Figure 5-9 we find E_n to about 2nV/√Hz and I_n to about 1pA/√Hz for 1mA. Substituted in the equation above we get E_{ni} to be about 51.4nV.

$$E_{ni}^2 = \left[1.6 \times 10^{-16} + \left(2 \times 10^{-9}\right)^2 + \left(10^{-12}\right)^2 \left(10^4\right)^2\right](10)$$

$$E_{ni}^2 = \left[1.6 \times 10^{-16} + 4 \times 10^{-18} + 10^{-16}\right](10)$$

$$E_{ni}^2 = 2.64 \times 10^{-15} V^2$$

$$E_{ni} = 51.4nV$$



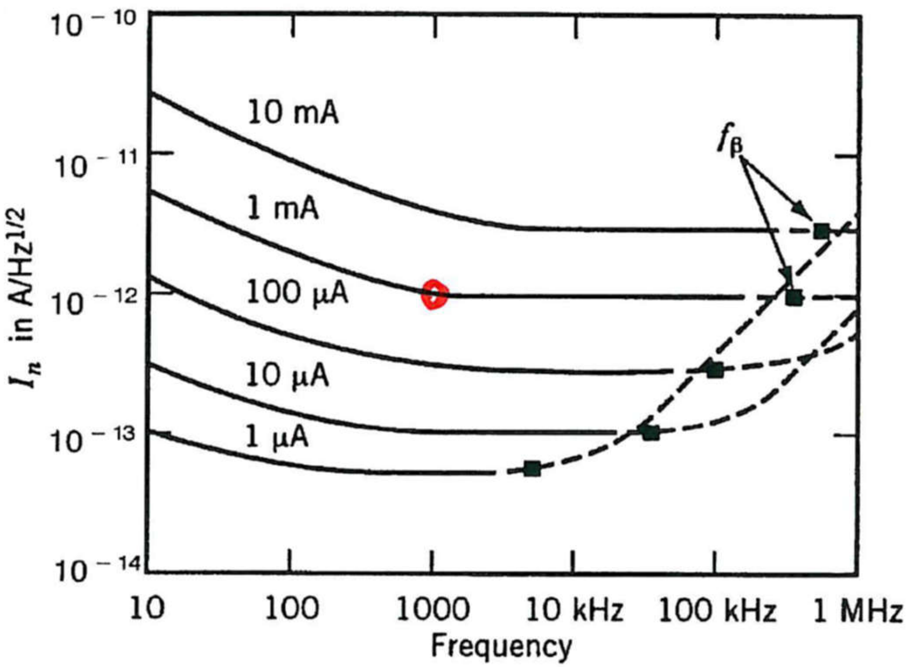


Figure 5-9 E_n and I_n performance of a 2N4250 transistor.

2) Calculation

In addition to the specified values, the values in table 5-1 and the known constants, *re* and *IB* must be calculated. These can be found using the formulas and the other values.

$$E_n^2 = 4kTr_x + 2qI_C r_e^2 + \frac{2qf_L I_B^{\gamma} r'^2_x}{f} + 2qI_C r_x^2 \left(\frac{f}{f_T}\right)^2$$

$$2.70 \cdot 10^{-18} = 2.48 \cdot 10^{-18} + 2.14 \cdot 10^{-19} + 1.39 \cdot 10^{-21} + 1.15 \cdot 10^{-27}$$

$$I_n^2 = 2qI_B + \frac{2qf_L I_B^{\gamma}}{f} + 2qI_C \left(\frac{f}{f_T}\right)^2$$

$$1.19 \cdot 10^{-24} = 8.59 \cdot 10^{-25} + 3.30 \cdot 10^{-25} + 5.11 \cdot 10^{-32}$$

$$E_{ni}^2 = \left(E_t^2 + E_n^2 + I_n^2 R_S^2\right)\Delta f$$

$$2.867 \cdot 10^{-15} = \left(1.65 \cdot 10^{-16} + 2.70 \cdot 10^{-18} + 1.19 \cdot 10^{-16} \cdot \left(10 \cdot 10^3\right)^2\right) \cdot 10$$

$$E_{ni} = 52.8nV$$

Midband noise (=minimum noise)

As we see from the expressions *En* and *In* are frequency-dependent. At low frequencies the 1/f-noise will be substantially while at higher frequencies, we will get an additional frequency dependent part of the shot-noise in the collector. We can talk about a midband where the noise is not strongly frequency dependent, and where other contributions than the frequency-dependent is dominant. The midband indicates in a way the minimum noise level we can achieve.

If we remove the frequency-dependent parts of the expressions for *En* and *In* we get:

$$E_n^2 = 4kTr_x + 2qI_C r_e^2$$

and

$$I_n^2 = 2qI_B$$

<u>When *RS* is low, *En* will dominate and it will be desirable with a small base resistance.</u>
<u>When *RS* is large the *In²RS²*-term will easily dominate. When this is the case it will be important to have a small *IB*. In order to achieve this, the *IC* should be small and β large.</u>

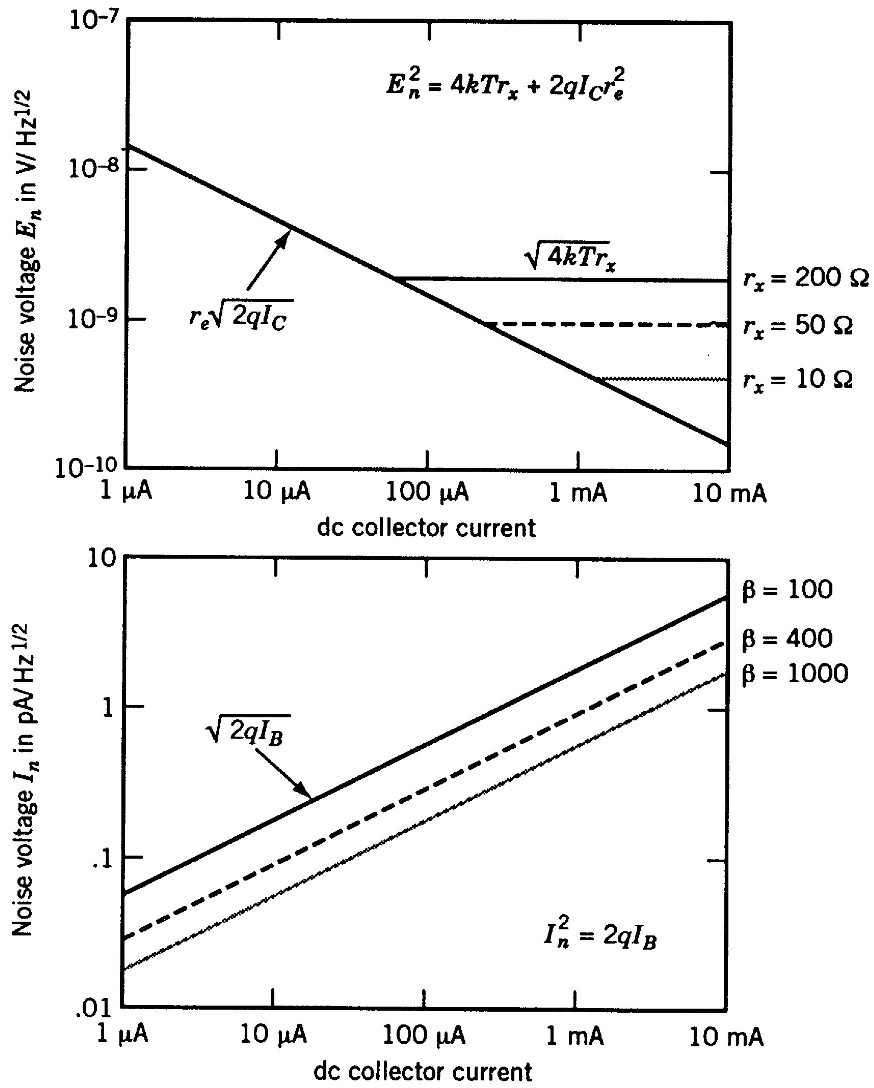


Figure 5-4 Limiting noise voltage and noise current.

(Noise current at the Y-axis in the lower figure!)

Minimizing the noise factor

We found previously the following expressions for optimum noise factor:

$$F_{opt} = 1 + \frac{E_n I_n}{2kT\Delta f}$$

(This can only be achieved when *RS*=*RO*=*En/In*.)

We insert the frequency independent terms we found for *En* and *In* and get:

$$F_{opt} = 1 + \sqrt{\frac{2r_x}{\beta_0 r_e} + \frac{1}{\beta_0}}$$

To achieve low noise we must:

- Reduce r_x
- Increase β_0
- Reduce I_C ($r_e \sim 1/I_C$)

Normally you will achieve the lowest noise when the collector current is less than 100µA. If the collector current is very small, we are left with:

$$F_{opt} = 1 + \frac{1}{\sqrt{\beta_0}}$$

Optimal *Rs*:

The optimum condition above presumes that *Rs=R0=En/In*. We insert the expressions for *En* and *In* and get:

$$R_0 = \sqrt{\frac{0.05\beta_0 r_x}{I_C} + \frac{(0.025)^2 \beta_0}{I_C^2}}$$

We see that reduced *Ic* requires larger *Rs*!

When the base resistance can be neglected, then we have:

$$R_0 \cong \frac{0.025\sqrt{\beta_0}}{I_C}$$

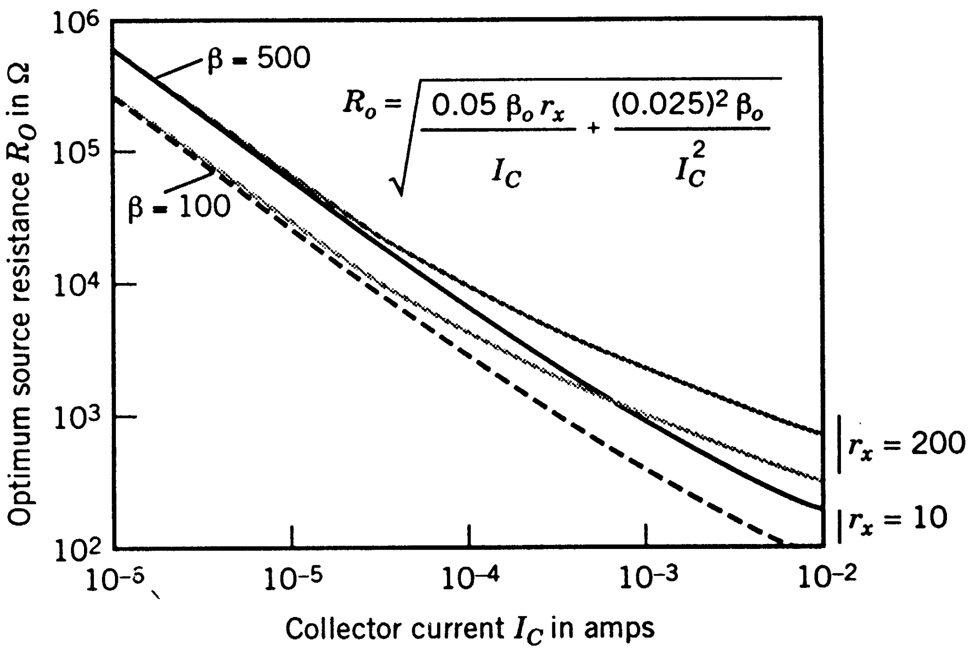


Figure 5-5 Graph of optimum source resistance versus I_C.

The frequency range dominated by 1/f-støy (i.e. low frequencies)

At low frequencies the flicker (1/f)-noise will dominate. We return to our original expression for E_n and I_n and retain only the flicker noise. We will then have:

$$E_n^2 = \frac{2qf_L I_B^\gamma r'^2_x}{f^\alpha}$$

and

$$I_n^2 = \frac{2qf_L I_B^\gamma}{f^\alpha}$$

These are the same with the exception of the resistance $r'_x{}^2$.

Optimal R_S will in this case be:

$$R_S = R_0 = E_n / I_n = r'^2_x$$

We see here that R_S is independent of all other values than $r'x^2$.

In this frequency range, we have:

$$F_{opt} = 1 + \frac{qf_L I_B^\gamma r'_x}{kTf^\alpha}$$

How to achieve low noise in this frequency band?

Small r_x'

Small I_C (and thus small I_B).

This provides good conditions also in the frequency range we discussed earlier. However it does not ensure good high frequency qualities.

Operation Conditions and Noise

Equivalent input noise is expressed by:

$$E_{ni}^2 = E_t^2 + E_n^2 + I_n^2 R_S^2$$

In figure 5-4 we saw that *En* declined with growing *Ic* while *In* grows with growing *Ic*. Knowing this, one can expect that noise will be larger for low and high *Ic* and have a minimum in the middle. (Thermal noise in the source (*Et*) will not be affected by *Ic*.) Since the contribution from *In* scale with *Rs* the minimum value will move with *Rs*.

Since *In* is growing with growing *Ic*, growing *Rs* will give that the minimum point moves towards lower *Ic*. This can be seen in the figure below where the horizontal axis is the current while the curves for some selected resistors are outlined.

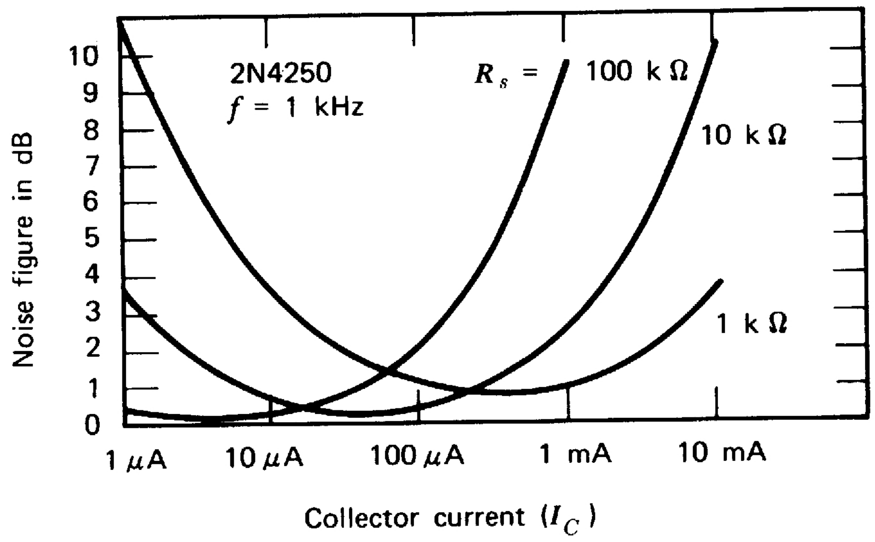


Figure 5-6 Effect of collector current and source resistance on noise figure.

In the curve below we switch and let the horizontal axis be the resistance while the curves for some selected currents are outlined.

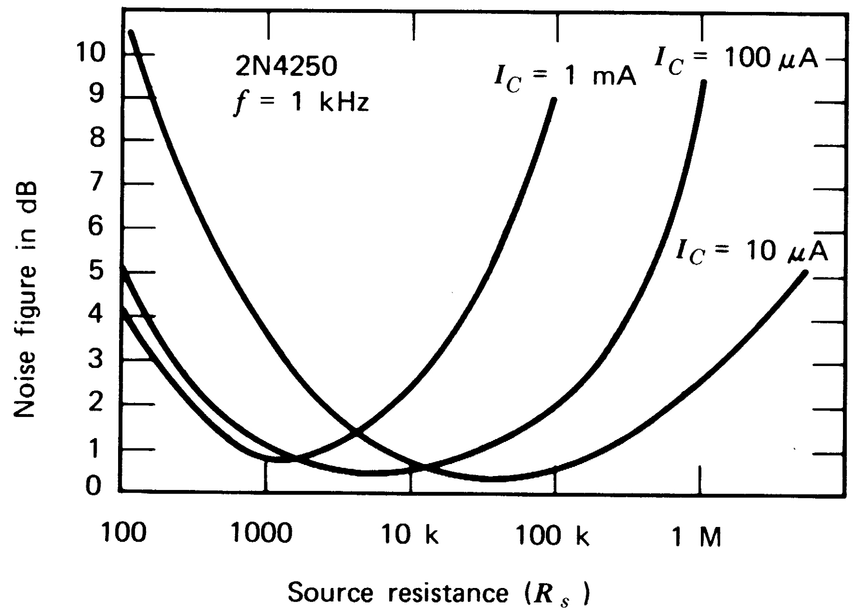


Figure 5-7 Noise figure variation with source resistance versus collector current.

Conclusion:

- For a given *Rs* there is a minimum noise.
- For a given *Ic*, there is a minimum noise.
- Optimal *Rs* decreases with increasing IC.

The figure shows an alternative way to present the curves. The six images are six different frequencies. For a given frequency a combination of *Rs* and *Ic* should be found that provides the greatest "1dB-area" around the point of the selected *Rs* and *Ic*.

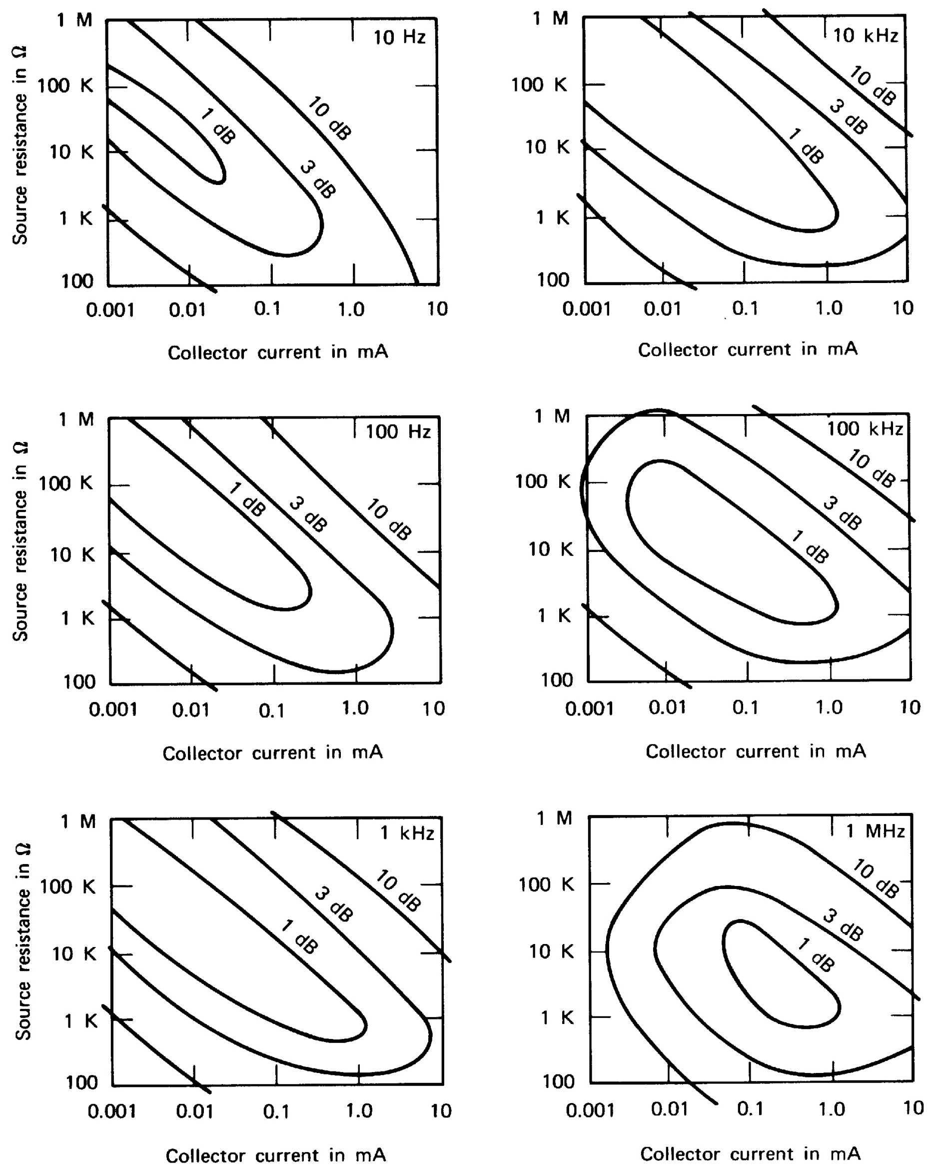


Figure 5-8 Contours of constant narrowband noise figure.

Often it will be useful to consider the noise as a function of frequency.

At low frequencies increases the noise (with reduction in frequency) as *1/√f* while for high frequencies it is proportional to *f*. In the middle area the noise is flat and the curves have their minimum as discussed earlier.

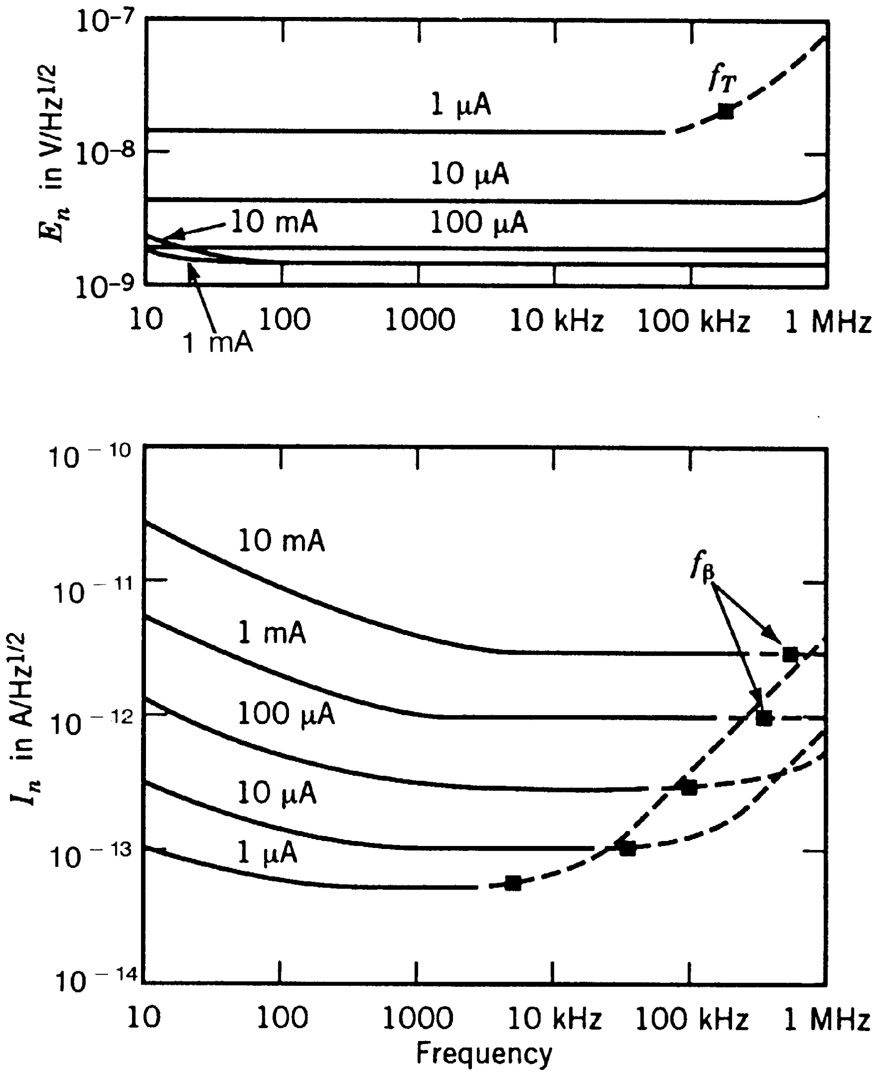


Figure 5-9 E_n and I_n performance of a 2N4250 transistor.

Popcorn noise

Observed in: tunnel diodes, diode transitions, film resistors, transistors and integrated circuits.

The spectral density of the effect of this noise is

$$1/f^{\alpha}$$

where α is between 1 and 2.

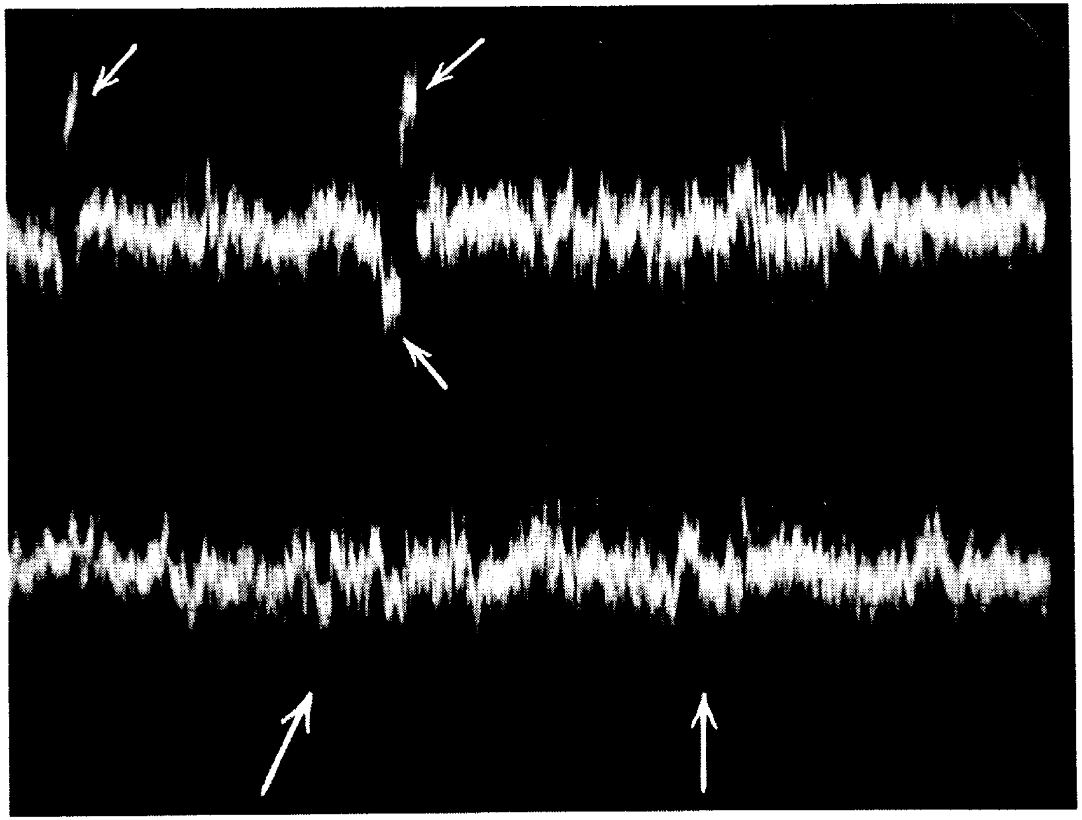


Figure 5-11 "Popcorn noise" is shown in the oscilloscope traces. The top trace is considered to represent a moderate level of this noise. The bottom trace is a low level. Some devices exhibit popcorn noise with five times the amplitude shown in the top trace. Horizontal sensitivity is 2 ms/cm.

In a normal pn-junction is the pulses maximum a few dozen micro ampere, and with a length of few micro seconds.

A simple popcorn current noise generator can be modelled as:

$$I_{bb}^2 = \frac{K'}{f^2}$$

where *K'* is a dimension constant with Ampere as designation.

A more accurate expression is:

$$I_{bb}^2 = \frac{KI_B}{1 + \pi^2 f^2 / 4a^2}$$

where *K* is a constant with Ampere per Hertz as designation and the constant *a* represents the number of bursts per. seconds.

Decomposition of r_x

The base resistance r_x can be divided into two parts:

i) from the contact (metal) to the nearest base-emitter junction and ii) the effective resistance for the distribution of base current along the base-emitter junction. The first is named as r_i while the other is named r_a.

1/f noise that is related with the crystal surface shall only be related to r_i while 1/f noise that is related to the active base region shall be related to the entire r_x.
Popcorn noise is related only to r_i.

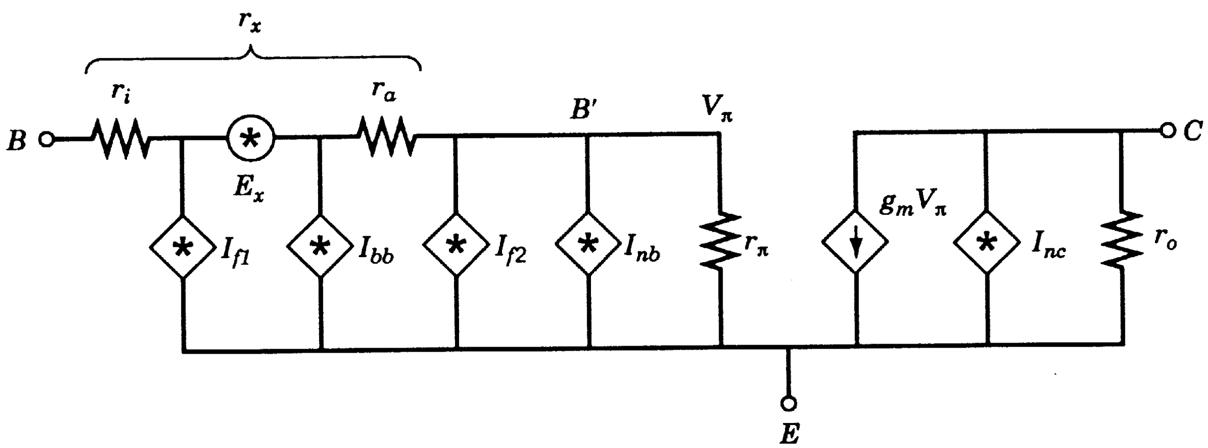


Figure 5-10 Expanded hybrid-π bipolar model with excess noise mechanisms.

A new noise model, where r_x is split up is shown above. Here we have two 1/f-noise sources I_{f1} and I_{f2} and a popcorn noise source I_{bb}.

(These apply to 1Hz bandwidth.)

Shot:	$I_{nb}^2 = 2qI_B$
Shot:	$I_{nc}^2 = 2qI_C$
Thermal:	$E_x^2 = 4kTr_x$
Burst:	$I_{bb}^2 = \dfrac{KI_B}{1+\pi^2 f^2 / 4a^2}$
1/f:	$I_{f1}^2 = \dfrac{K_1 I_B^{\gamma 1}}{f}$
1/f:	$I_{f2}^2 = \dfrac{K_2 I_B^{\gamma 2}}{f}$

Measurement of popcorn noise.

Popcorn noise is primarily a problem for low frequencies in the audio area.

The form below shows a way to measure this noise. Threshold voltage V_R must be chosen so that the thermal noise does not trigger. This also means that the lowest values of popcorn-noise are not measured.

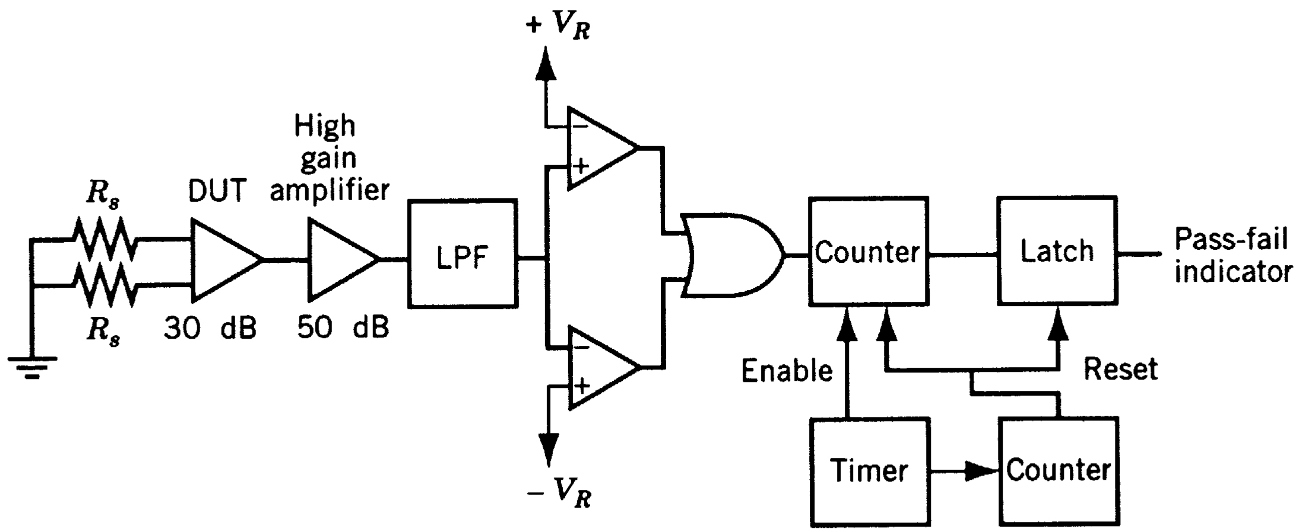


Figure 5-12 Block diagram of system for measuring popcorn noise.

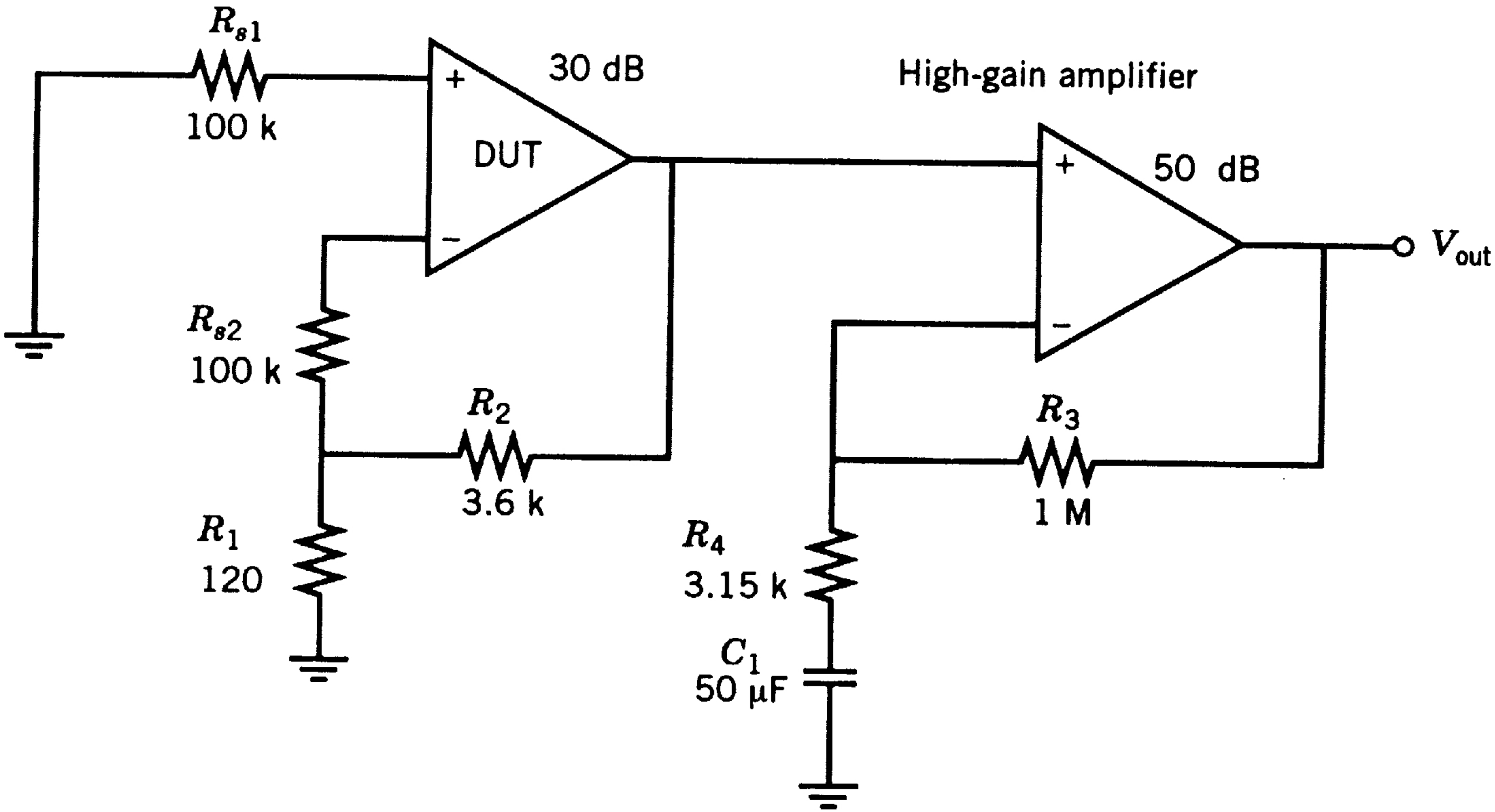


Figure 5-13 Amplifiers used in system for measuring popcorn noise.

Flicker noise and reliability

It turns out that the size of the flicker noise in a component gives a good indication of the component's condition. Comparing two similar components, one could assume that the one with most flicker noise is the least reliable and have the shortest life. Measurement of flicker noise and measurement of the change in flicker noise will thus be able to say something about a system.

Reverse voltage and noise

If the reverse voltage over the base-emitter junction passes the breakdown voltage the transistor characteristic will change. β0 will decline somewhat while the 1/f-noise will increase dramatically. The change will depend on the size of the reverse current and how long it is present.

By adding a large forward current the damage may partially be corrected as shown above.
(1E4s = 02:46:40,
1E5s = 27:46:40)

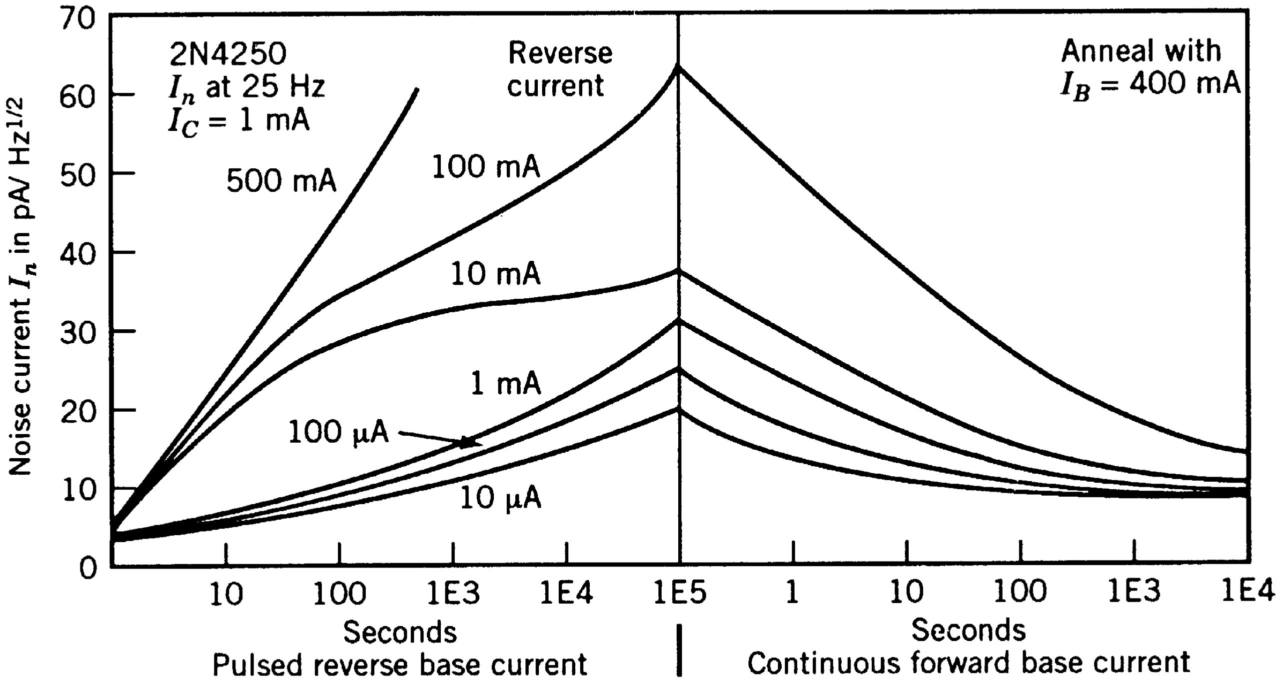


Figure 5-14 Increase in noise current with avalanching and the decrease resulting from current annealing.

Examples of accidental transgression of reverse voltage # 1

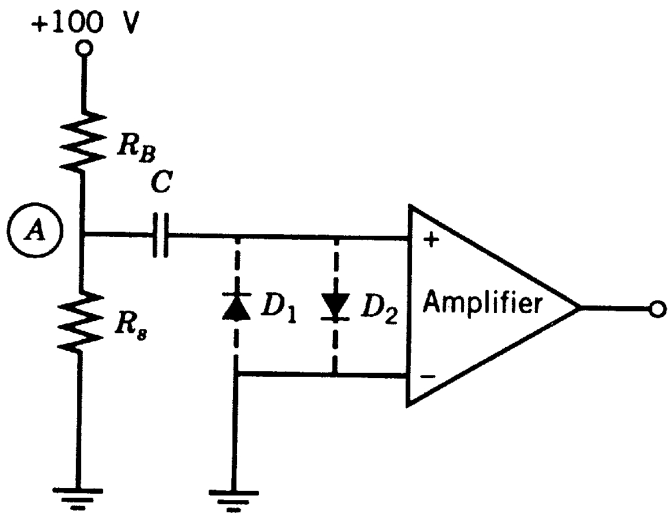


Figure 5-15 Demonstration of avalanching problem.

If point A is short-circuited accidentally or intentionally against the voltage supply or the amplifier is turned off, the charge over C may lead to that the amplifier input transistors gets too large reverse voltage.
Connecting the diodes as shown will prevent that the voltage becomes too large and that the amplifier becomes damaged.

Examples of accidental transgression of reverse voltage # 2

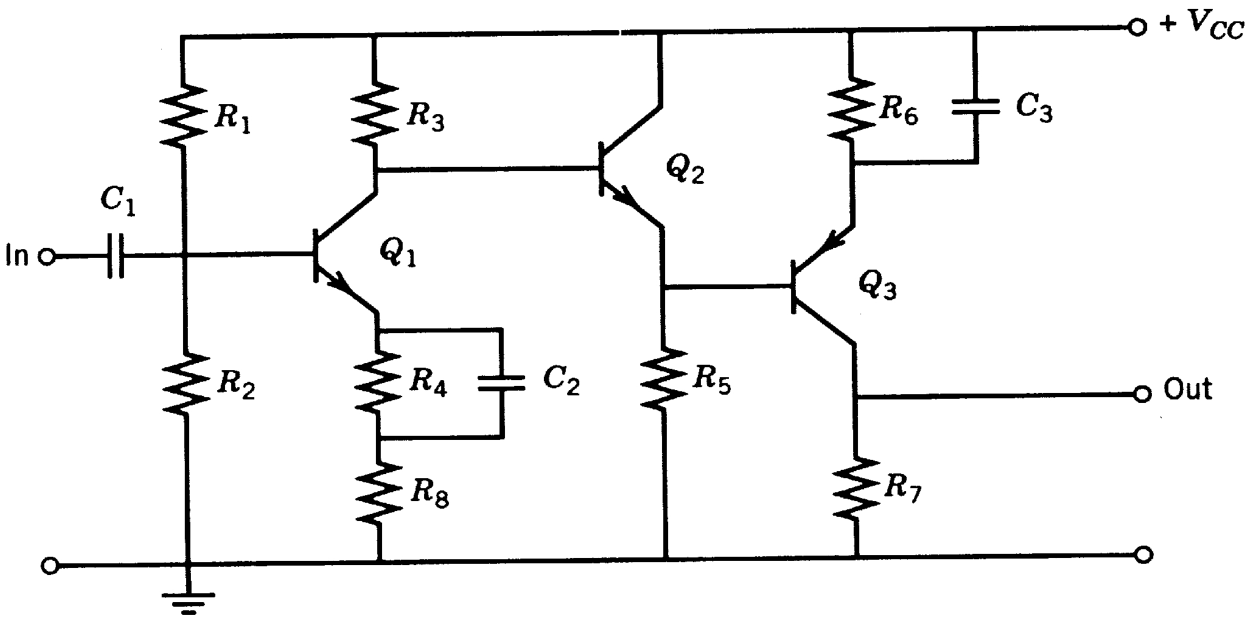
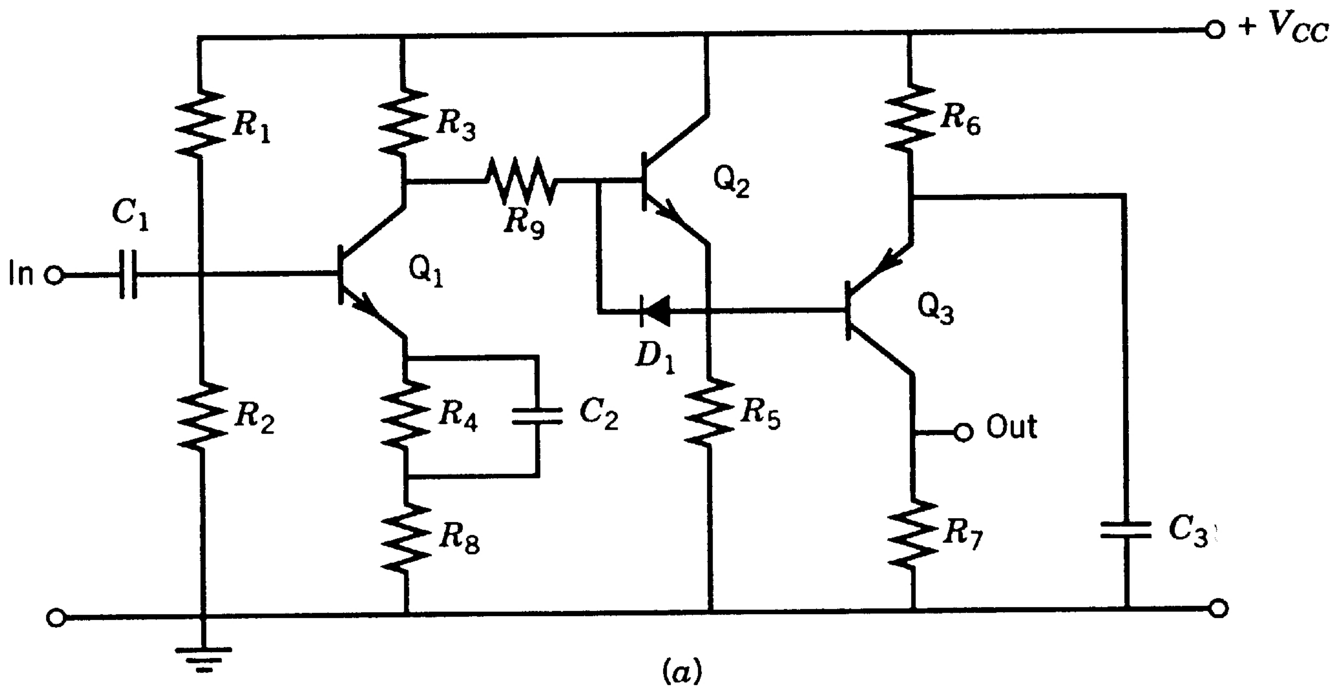


Figure 5-16 Direct-coupled complementary amplifier with single supply.

Before VCC is connected, the voltage across C_2 and C_3 will be zero volts. This will also be the case immediately after VCC is connected to the power supply. We can then consider R_4 and R_6 to be short circuited. Q_1 and Q_3 will carry large current and Q_2's base will have a low voltage while the Q_2' emitter will have high voltage. Hence Q_2 may have a reverse voltage larger than the breakdown voltage and will have changed the properties in a negative direction.

It may be counteracted in several ways. Common to all approaches is that they protect Q_2 at power up and then provide normal function.

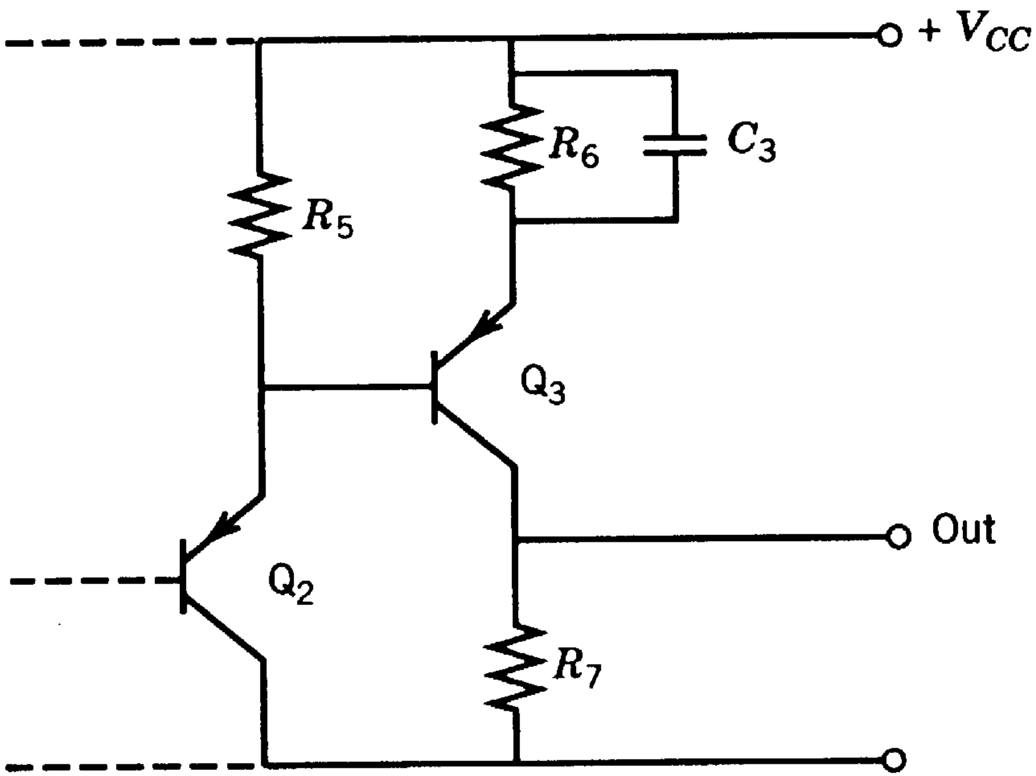


(a)

• C_3 is connected to the ground instead of against *VCC*. At power up the current will now flow through R_6 and not through Q_3. Q_3 will not draw as large current and the Q_2' emitter will remain low.

• The diode D_1 will ensure that the reverse voltage over the Q_2's base-emitter is not too large. At normal operation the diode will have virtually no effect.

If Q_2 can be a *pnp* instead of an *npn* we can have this solution and Q_2 will not be able to be reverse voltage biased.



(*b*)