UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in INF-MAT 4350 — Numerical linear algebra

Day of examination: 4 December 2008

Examination hours: 0900 – 1200

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All 7 part questions will be weighted equally.

Problem 1 Iterative methods

Consider the linear system $Ax = b$ in which

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 7 & 2 \\ 1 & 2 & 4 \end{pmatrix},$$

and $b = (1, 9, -2)^T$.

1a

With $x_0 = (1, 1, 1)^T$, carry out one iteration of the Gauss-Seidel method to find $x_1 \in \mathbb{R}^3$.

1b

If we continue the iteration, will the method converge? Why?

1c

Write a matlab program for the Gauss-Seidel method applied to a matrix $A \in \mathbb{R}^{n,n}$ and right-hand side $b \in \mathbb{R}^n$. Use the ratio of the current residual to the initial residual as the stopping criterion, as well as a maximum number of iterations.

Hint: The function C=tril(A) extracts the lower part of A into a lower triangular matrix C.

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Problem 2 QR factorization

Let

$$A = \begin{pmatrix} 2 & 1 \\ 2 & -3 \\ -2 & -1 \\ -2 & 3 \end{pmatrix}.$$

2a

Find the Cholesky factorization of $A^T A$.

2b

Find the QR factorization of A.

Problem 3 Kronecker products

Let $A, B \in \mathbb{R}^{n,n}$. Show that the eigenvalues of the Kronecker product $A \otimes B$ are products of the eigenvalues of A and B and that the eigenvectors of $A \otimes B$ are Kronecker products of the eigenvectors of A and B.

Problem 4 Matrix norms

Suppose $A \in \mathbb{R}^{n,n}$ is invertible, $b, c \in \mathbb{R}^n$, $b \neq 0$, and $Ax = b$ and $Ay = b + e$. Show that

$$\frac{1}{K(A)} \frac{\|e\|}{\|b\|} \leq \frac{\|y - x\|}{\|x\|} \leq K(A) \frac{\|e\|}{\|b\|},$$

where $\|\cdot\|$ is the Euclidean vector norm in $\mathbb{R}^n$ and $K(A)$ is the condition number of A with respect to the matrix 2-norm.

Good luck!