CHARGE CARRIERS
-
GENERATION
AND
RECOMBINATION

INJECTION LEVEL

$N_D = 10^{16}$

Equilibrium: $n_{n0} p_{no} = n_i^2 = 10^4 \times 10^{16} = 10^{20}$

Low injection level (e.g. due to light): $\Delta n = \Delta p << N_D$

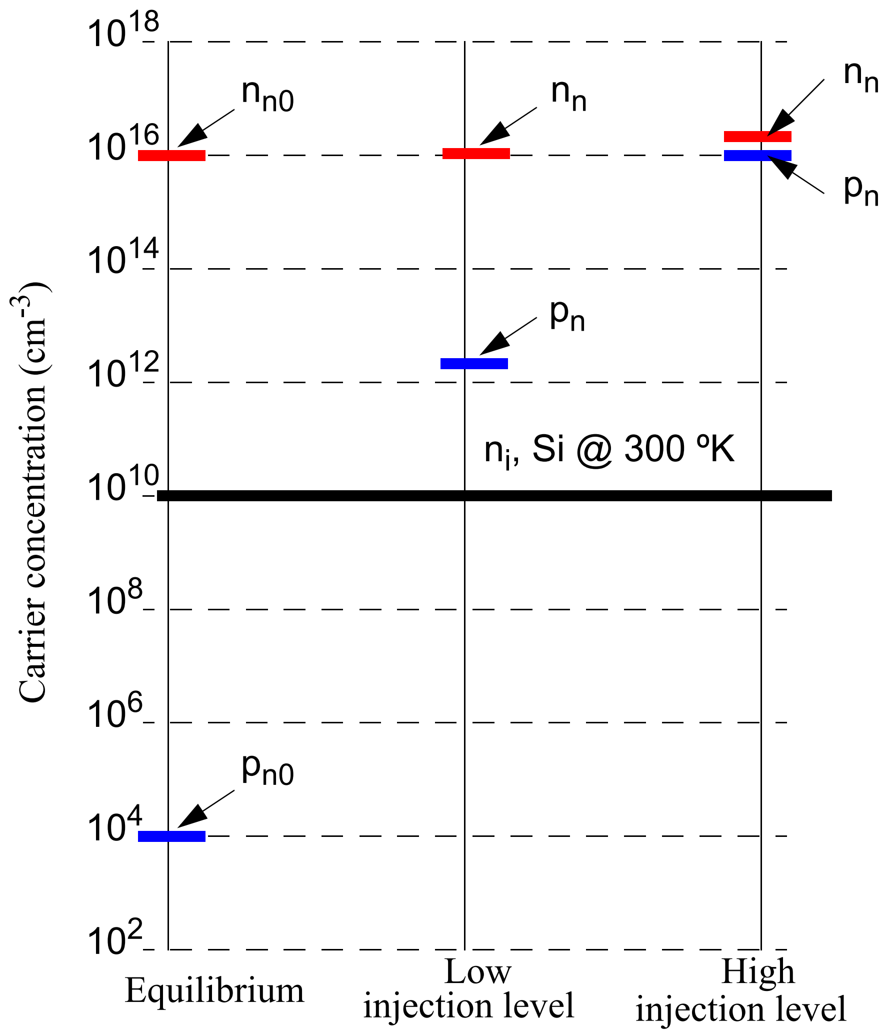
Adding 10^{12} carriers /cm^3

$p_n = 10^4 + 10^{12} \approx 10^{12}$
$n_n = 10^{16} + 10^{12} \approx 10^{16}$

High injection level: $\Delta_n = \Delta_p \approx N_D$ (not relevant)

Adding 10^{16} carriers /cm^3

$p_n = 10^4 + 10^{16} \approx 10^{16}$
$n_n = 10^{16} + 10^{16} = 2 \times 10^{16}$



Ref: Grove

GENERATION / RECOMBINATION

Definitions:
G_{th}: Dark (thermal) generation rate
R: Recombination rate
$U = R - G_{th}$: Net recombination rate
G_L: Generation rate due to absorption of light

Uniformly illuminated semiconductor.
Net change in carrier concentration p_n:

$$\frac{dp_n(t)}{dt} = G_L + G_{th} - R = G_L - U \qquad (2.1)$$

Assuming U is proportional to excess carrier concentration (concentration beyond equilibrium). Equilibrium concentration: p_{n0}. Lifetime: τ_p

$$U = \frac{1}{\tau_p}(p_n(t) - p_{n0}) \qquad (2.2)$$

Combining these gives the differential equation:

$$\frac{dp_n(t)}{dt} = G_L - \frac{p_n(t) - p_{n0}}{\tau_p} \qquad (2.3)$$

Steady state (G_L=U):

$$\frac{dp_n(t)}{dt} = 0$$

$$p_n(t)\big|_{ss} = p_{n0} + \tau_p G_L \equiv p_L \qquad (2.4)$$

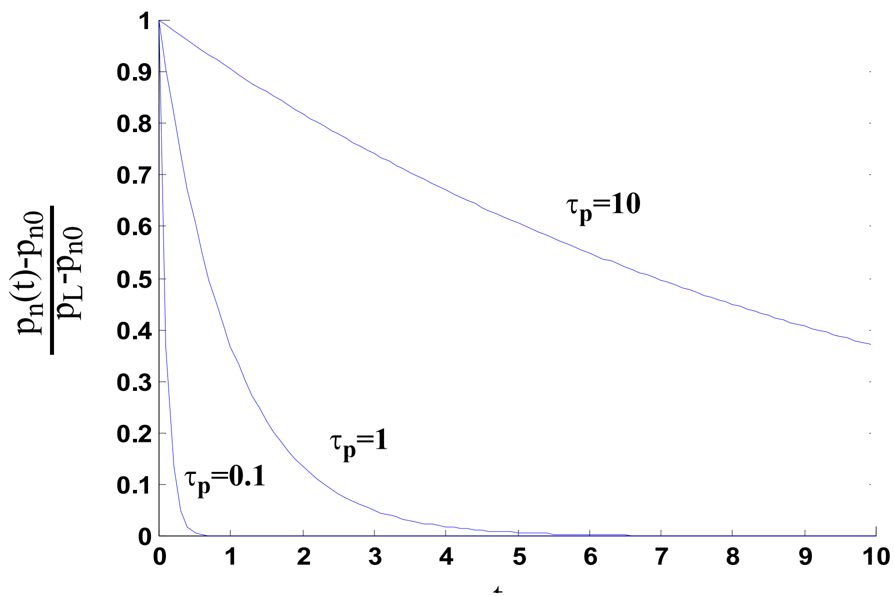
Turning light off:

$$G_L = 0$$

$$\frac{dp_n(t)}{dt} = -\frac{p_n(t) - p_{n0}}{\tau_p}$$

Solving the differential equation. Initial condition $p_n(0)=p_L$:

$$p_n(t) = p_{n0} + (p_L - p_{n0})e^{-t/\tau_p} \qquad (2.5)$$

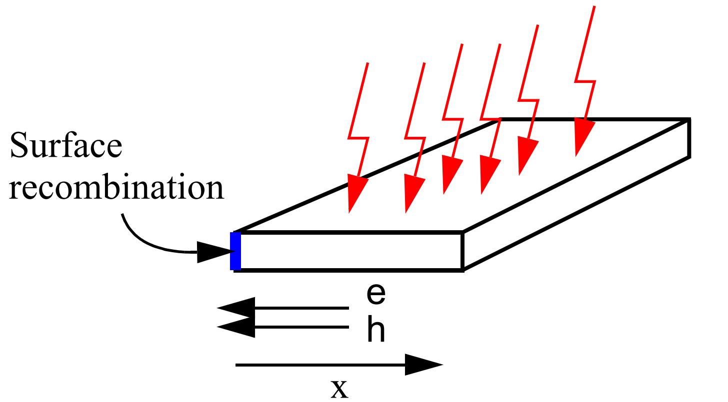


Ref: Grove

Surface recombination:

Charge carriers diffuse towards the surface.
Diffusion Flux:

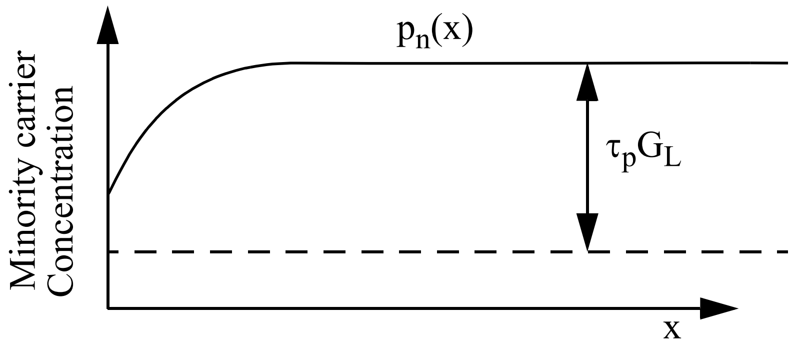
$$F_p = -D_p \frac{\partial p_n(x,t)}{\partial x} \qquad (2.6)$$



Concentration is given by the 'transport equation':

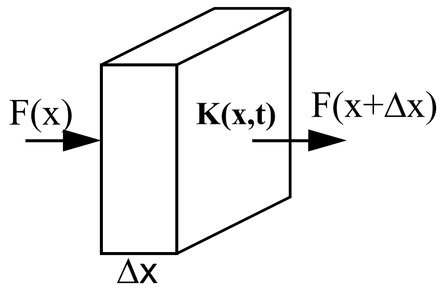
$$\frac{\partial p_n(x,t)}{\partial t} = -\frac{\partial F_p}{\partial x} + G_L - U$$

$$\frac{\partial p_n(x,t)}{\partial t} = D_p \frac{\partial^2 p_n(x,t)}{\partial x^2} + G_L - \frac{p_n - p_{n0}}{\tau_p} \qquad (2.7)$$



Ref: Grove

Transport equation:



$$\Delta x \frac{\partial K}{\partial t} = F(x) - F(x + \Delta x)$$

$$\frac{\partial K}{\partial t} = \left.\frac{F(x) - F(x+\Delta x)}{\Delta x}\right|_{\Delta x \to 0} = \frac{\partial F}{\partial x}$$

Steady state and boundary conditions.

$$\frac{\partial p_n(x,t)}{\partial t} = 0$$

$$p(x = \infty) = p_L = p_{n0} + \tau_{\mathbf{p}} G_L \qquad (2.8)$$

$$D_p \left.\frac{\partial p_n}{\partial x}\right|_{x=0} = s_p[p_n(0) - p_{n0}]$$

Carriers which reach the surface (x=0) recombine there.

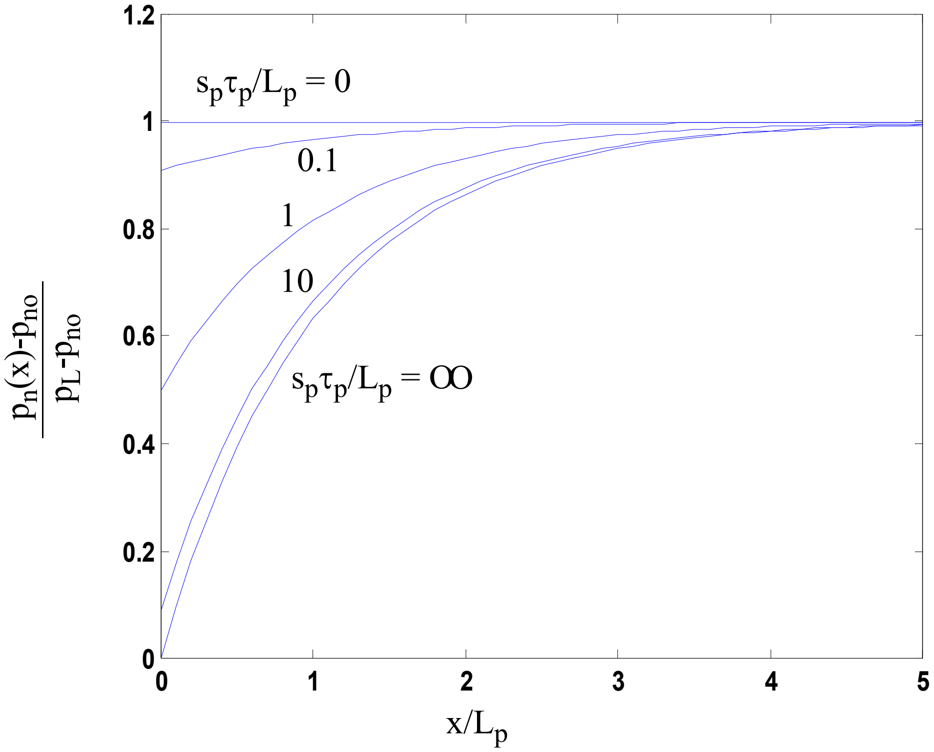
s_p is a proportionality constant, surface recombination rate. Diffusion is proportional to the excess carrier concentration.

Differential equation has the solution:

$$p_n(x) = p_L - (p_L - p_{n0})\frac{s_p\tau_p/L_p}{1 + s_p\tau_p/L_p}e^{-x/L_p} \qquad (2.9)$$

$$L_p \equiv \sqrt{D_p\tau_p}$$

L_p: Diffusion length



Ref: Grove

LATTICE DISLOCATIONS

Dislocations deviates from the perfect periodicity. **The surface is an obvious example.** Energy states in the band gap becomes recombination centres, “stepping stones”. These increases the probability of recombination, i.e. reduce the lifetime of free charge carriers.

Probability of occupied centre at energy level E_T:

$$f(E_T) = \frac{1}{1 + e^{(E_T - E_F)/kT}} \qquad (2.10)$$

k is Boltzmanns constant and T is absolute temperature.

Probability of an unoccupied centre: 1-f

$$v_{th} = \sqrt{3k\frac{T}{m}} \approx 10^5 \frac{m}{s} \qquad \text{Termal velosity}$$

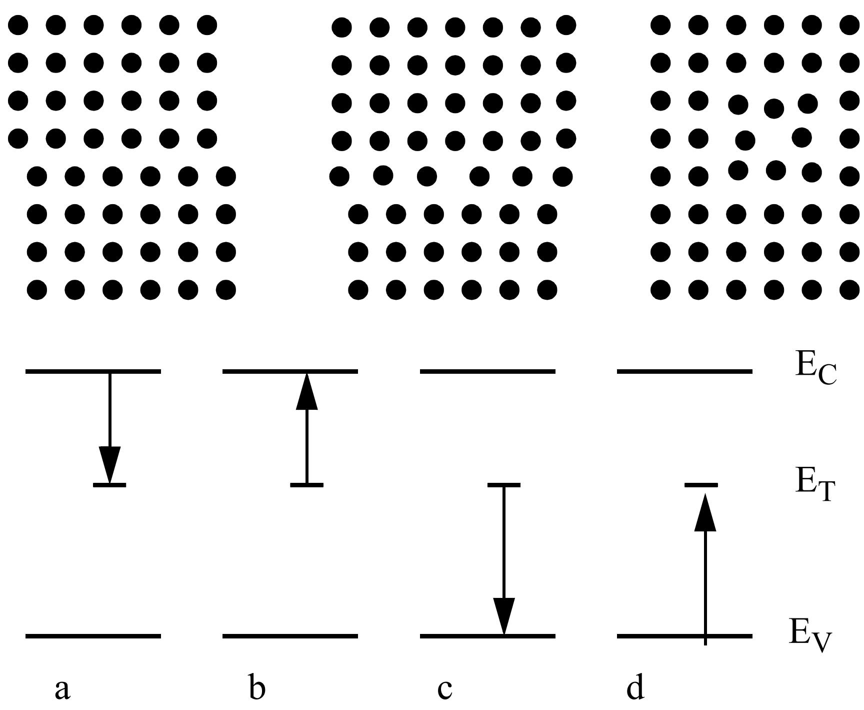
$$\sigma_n \approx 10^{-17} m \qquad \text{capture cross section}$$

Transition rate a: $r_a = v_{th}\sigma_n n N_t(1 - f(E_T))$ (2.11)

Transition rate b: $r_b = e_n N_t f(E_T)$ (2.12)

Transition rate c: $r_c = v_{th}\sigma_p p N_t f(E_T)$ (2.13)

Transition rate d: $r_d = e_p N_t(1 - f(E_T))$ (2.14)



$k = 1.3805 \times 10^{-23}$ J / ºK

Ref: Grove

e_n and e_p are emission probability (depend on the distance to conduction band and valence band respectively)

Emission probability:

At equilibrium, process a and b have equal rate and e_n and e_p can be found by setting $r_a = r_b$:

$$v_{th}\sigma_n n N_t(1 - f(E_T)) = e_n N_t f(E_T)$$

$$e_n = v_{th}\sigma_n \frac{1 - f(E_T)}{f(E_T)} n = v_{th}\sigma_n e^{(E_T - E_F)/kT} N_c e^{-(E_C - E_F)/kT}$$

$$e_n = v_{th}\sigma_n N_c e^{-(E_C - E_T)/kT} \qquad (2.15)$$

Corresponding result for e_p:

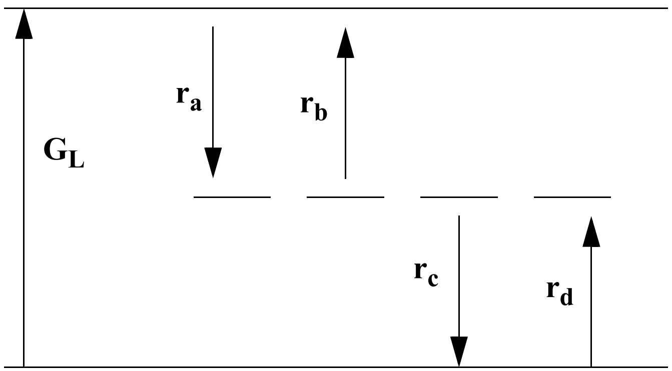
$$e_p = v_{th}\sigma_p N_v e^{-(E_T - E_V)/kT} \qquad (2.16)$$

Ref: Grove

ILLUMINATED SEMICONDUCTOR

Uniform light and steady state *
Electrons enter and leave the conduction band at the same rate:

$$\frac{dn_n}{dt} = G_L - (r_a - r_b) = 0$$



Holes enter and leave the valence band at the same rate.:

$$\frac{dp_n}{dt} = G_L - (r_c - r_d) = 0$$

No accumulation of electrons in the recombination centres:

$$r_a - r_b = r_c - r_d$$

Solve for the occupation factor f, using the expressions for the processes a, b, c, d [(2.11), (2.12), (2.13), (2.14)]
It follows the Fermi-model **

$$f(E_T) = \frac{n\sigma_n + \sigma_p N_v e^{-(E_T - E_V)/kT}}{\sigma_n\left(n + N_c e^{-(E_C - E_T)/kT}\right) + \sigma_p\left(p + N_v e^{-(E_T - E_V)/kT}\right)} \qquad (2.17)$$

Ref: Grove

* *Equilibrium = steady state with out external influence*

** *Cannot use the expression for Fermi energy because this is based on equilibrium But we can use the model.*

We have from chapter 1:

$$n = N_c e^{-(E_C - E_F)/kT} \qquad n = n_i e^{(E_F - E_i)/kT}$$

$$p = N_v e^{-(E_F - E_V)/kT} \qquad p = n_i e^{(E_i - E_F)/kT}$$

$$n_i^2 = N_v N_c e^{-E_G/(kT)}$$

E_F is valid at equilibrium only, but the model can be used for E_T and can be written as:

$$f(E_T) = \frac{n\sigma_n + \sigma_p n_i e^{(E_i - E_T)/kT}}{\sigma_n\left(n + n_i e^{(E_T - E_i)/kT}\right) + \sigma_p\left(p + n_i e^{(E_i - E_T)/kT}\right)} \tag{2.18}$$

Replacing f(E) in the individual processes, get the net recombination rate: $U = r_a - r_b = r_c - r_d$

$$U = \frac{\sigma_n \sigma_p v_{th} N_t [pn - n_i^2]}{\sigma_n\left(n + N_c e^{-(E_C - E_T)/kT}\right) + \sigma_p\left(p + N_v e^{-(E_T - E_V)/kT}\right)} \tag{2.19}$$

Alternatively:

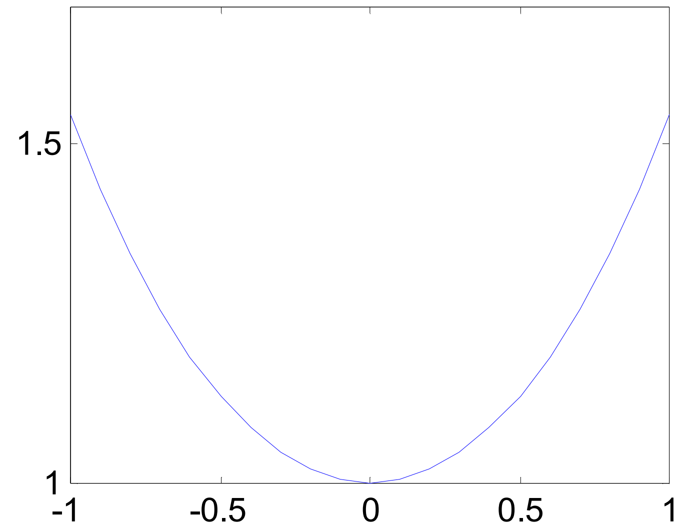
$$U = \frac{\sigma_n \sigma_p v_{th} N_t [pn - n_i^2]}{\sigma_n\left(n + n_i e^{(E_T - E_i)/kT}\right) + \sigma_p\left(p + n_i e^{(E_i - E_T)/kT}\right)} \tag{2.20}$$

(Recall that $pn = n_i^2$ in equilibrium only)

Ref: Grove

Special case: $\sigma_p = \sigma_n = \sigma$

$$U = v_{th}\sigma N_t \frac{pn - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_T - E_i}{kT}\right)} \qquad (2.21)$$



$pn-n_i^2$ is the deviation from equilibrium and the driving force for recombination

Maximum recombination rate when $E_T = E_i$,
i.e. E_T has the value in the middle of the energy gap.

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Ref: Grove

We have the expression for net recombination: (2.20)

$$U = \frac{\sigma_n \sigma_p v_{th} N_t [pn - n_i^2]}{\sigma_n\left(n + n_i e^{(E_T - E_i)/kT}\right) + \sigma_p\left(p + n_i e^{(E_i - E_T)/kT}\right)}$$

Knowing that at low level injection in N-type semiconductor

$$n_n \gg p_n$$

$$n_n \gg n_i e^{|E_T - E_i|/(kT)}$$

we can simplify (2.20) for N-type such that:

$$U \approx \frac{\sigma_n \sigma_p v_{th} N_t [p_n n_n - n_i^2]}{\sigma_n n_n} = \sigma_p v_{th} N_t [p_n - p_{n0}] \quad (2.22)$$

(Recall that both n_i and p_{n0} represent equilibrium).

Ref: Grove

Using (2.2):

$$U = \frac{p_n - p_{n0}}{\tau_p},$$

Expression for minority carriers in N-type (holes):

$$\tau_p = \frac{1}{\sigma_p v_{th} N_t} \quad (2.23)$$

Similar for P-type:

$$U \approx \sigma_n v_{th} N_t [n_p - n_{p0}] \quad (2.24)$$

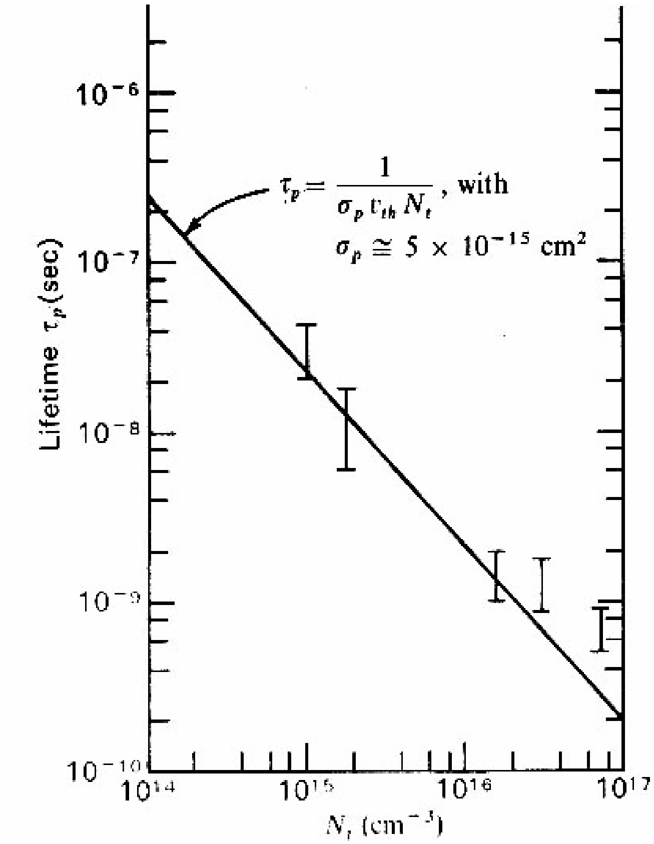
and lifetime for minority carriers (electrons):

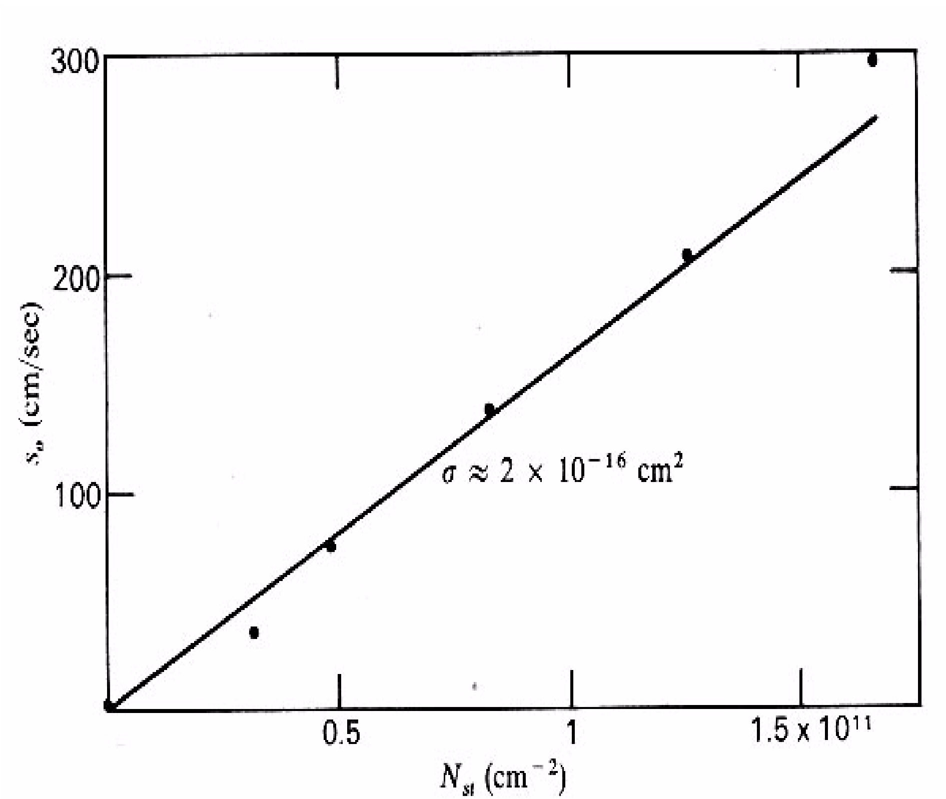
$$\tau_n = \frac{1}{\sigma_n v_{th} N_t} \quad (2.25)$$

Given these conditions, the recombination rate independent on the majority carrier concentration. That is, the minority carrier concentration determines the recombination rate.

EXAMPLES ON THE ORIGIN OF RECOMBINATION CENTRES

- Impurities from other groups than III and V in the periodic table gives energy states in the band gap.
- Surface states due to the lattice non-regularity.
  Approximately atoms/area (~10^{15} cm^{-2}). Lower density on oxidized surface (~10^{11} cm^{-2})





Ref: Grove

THE DIODE AS PHOTO SENSOR

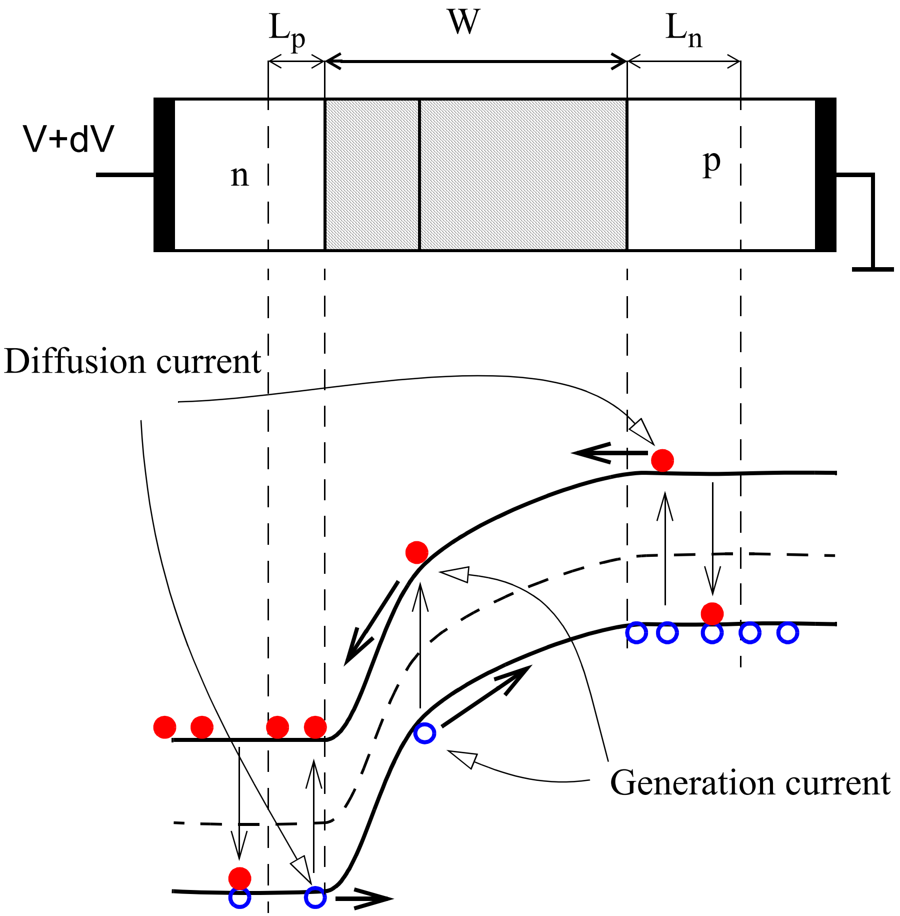
CURRENT - VOLTAGE CHARACTERISTICS

There are two sources of reverse biased current:

- Generation current
  Carriers generated in the depletion region. The field sweeps carriers out of the depletion region, electrons to the n-region and holes to the p-region.
- Diffusion current
  Carriers generated outside the depletion region, but within a diffusion length from the depletion region. Minority carriers diffuse to the edge of the depletion region and swept across by the field.

Carriers that are swept across becomes majority carriers.

There are (almost) zero free carriers in the depletion region and therefore low probability for recombination there.



GENERATION CURRENT:

Carriers are quickly driven out of the depletion region.

- $pn << n_i$
- No recombination
- $V_R >> kT/q$

Using (2.20), recombination = generation.

Generation rate is therefore ($p,n<<n_i$):

$$U = \frac{-\sigma_n \sigma_p v_{th} N_t n_i^2}{\sigma_n n_i e^{(E_T - E_i)/kT} + \sigma_p n_i e^{(E_i - E_T)/kT}} \equiv -\frac{n_i}{2\tau_0} \quad (2.26)$$

$$\tau_0 \equiv \frac{\sigma_n e^{(E_T - E_i)/kT} + \sigma_p e^{(E_i - E_T)/kT}}{2\sigma_n \sigma_p v_{th} N_t}$$

For $\sigma_p = \sigma_n = \sigma$:

$$U = -\frac{\sigma v_{th} N_t n_i}{2\cosh\left(\frac{E_i - E_T}{kT}\right)} \quad (2.27)$$

Generation current (dark current):

$$I_{gen} = q|U|WA_j \quad (2.28)$$

$$I_{gen} = \frac{1}{2} q \frac{n_i}{\tau_0} W A_j \quad (2.29)$$

A_j is the cross section of the depletion region

We see that “step stones” close to E_i gives the largest contribution to the generation current.

Ref: Grove

DIFFUSION CURRENT:

We apply the differential equation (2.7) for surface recombination on minority carriers.

- The concentration at the edge is 0 due to the field which sweeps the carriers across the junction.
- Steady state, no variation with time.

$$D_p \frac{\partial^2 n_p}{\partial x^2} + G_L - \frac{n_p - n_{p0}}{\tau_n} = 0$$

Far from the depletion region (2.4):

$$n_p(\infty) = n_{p0} + \tau_n G_L$$

At the edge of the depletion region:

$$n_p(0) = 0$$

Solution:

$$n_p(x) = (n_{p0} + \tau_n G_L)(1 - e^{-x/L_n}) \quad (2.30)$$

L_n= diffusion length for the electron in the p-region.

Diffusion current (differentiate):

$$I_{diff,\,n} = -q\left(-D_n \frac{dn_p}{dx}\bigg|_{x=0}\right) A_j = qD_n \frac{(n_{p0} + \tau_n G_L)}{L_n} A_j \quad (2.31)$$

Corresponding for holes in n-region:

$$I_{diff,\,p} = qD_p \frac{(p_{n0} + \tau_p G_L)}{L_p} A_j \quad (2.32)$$

No injection (no light):

$$I_{diff,\,n} = qD_n \frac{n_{p0}}{L_n} A_j = qD_n \frac{n_i^2}{N_A L_n} A_j$$

$$I_{diff,\,p} = qD_p \frac{p_{n0}}{L_p} A_j = qD_p \frac{n_i^2}{N_D L_p} A_j \quad (2.33)$$

Ref: Grove

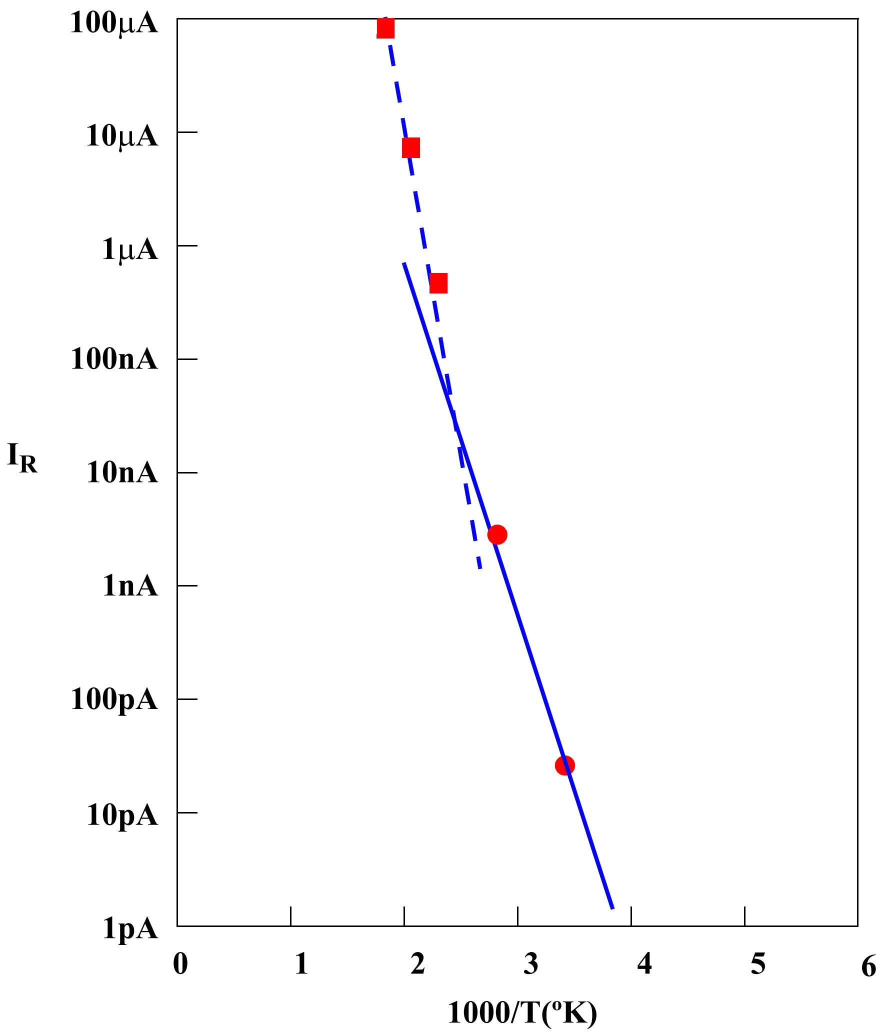
REVERSE CURRENT VS. TEMPERATURE

V_R= 1V

Circles: Generation current
Squares:Diffusion current
Solid line:Temperature dependence of n_i.
Dotted line:Temperature dependence of n_i^2

Generation current is proportional to n_i,
Diffusion current is proportional to n_i^2

- Low temperature:
  The generation current dominates
- High temperature:
  The diffusion current dominates.



Ref: Grove

Light intensity

Illuminance (light flux) has the unit [Lux]=[Lumen/m2]
vs.
Irradiance (power) has the unit [W/m2

Wave length	Photopic conversion	
λ (nm)	lm/W	W/lm
380	0.027	37.03704
390	0.082	12.19512
400	0.270	3.70370
410	0.826	1.21065
420	2.732	0.36603
430	7.923	0.12621
440	15.709	0.06366
450	25.954	0.03853
460	40.980	0.02440
470	62.139	0.01609
480	94.951	0.01053
490	142.078	0.00704
500	220.609	0.00453
507	303.464	0.00330
510	343.549	0.00291
520	484.930	0.00206
530	588.746	0.00170
540	651.582	0.00153
550	679.551	0.00147
555	683.000	0.00146
560	679.585	0.00147

Wave length	Photopic conversion	
λ (nm)	lm/W	W/lm
570	650.216	0.00154
580	594.210	0.00168
590	517.031	0.00193
600	430.973	0.00232
610	343.549	0.00291
620	260.223	0.00384
630	180.995	0.00553
640	119.525	0.00837
650	73.081	0.01368
660	41.663	0.02400
670	21.856	0.04575
680	11.611	0.08613
690	5.607	0.17835
700	2.802	0.35689
710	1.428	0.70028
720	0.715	1.39860
730	0.355	2.81690
740	0.170	5.88235
750	0.082	12.19512
760	0.041	24.39024
770	0.020	50.00000

Sensor efficiency

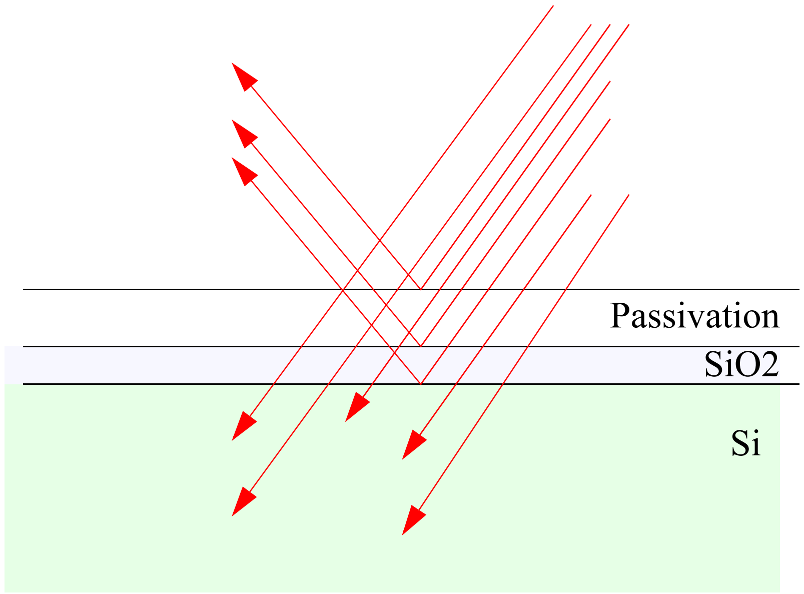
- Photons with energy larger than the bandgap can generate e-h pair.
- Some of the photons are reflected at the surface and do not contribute.
- Some of the photons are reflected at the silicon oxide surface or Passivation surface and do not contribute.
- Some of the generated charge carriers that are collected recombine fast, and are therefore lost.

Reflection factor:

$$R = \frac{\text{Reflexed intensity}}{\text{Incoming intensity}} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad (2.34)$$

where n_1 and n_2 is the refractive index to the interfacing materials.
Example: SiO_2 (n=1.45) and Si (n=4) result in R=22%

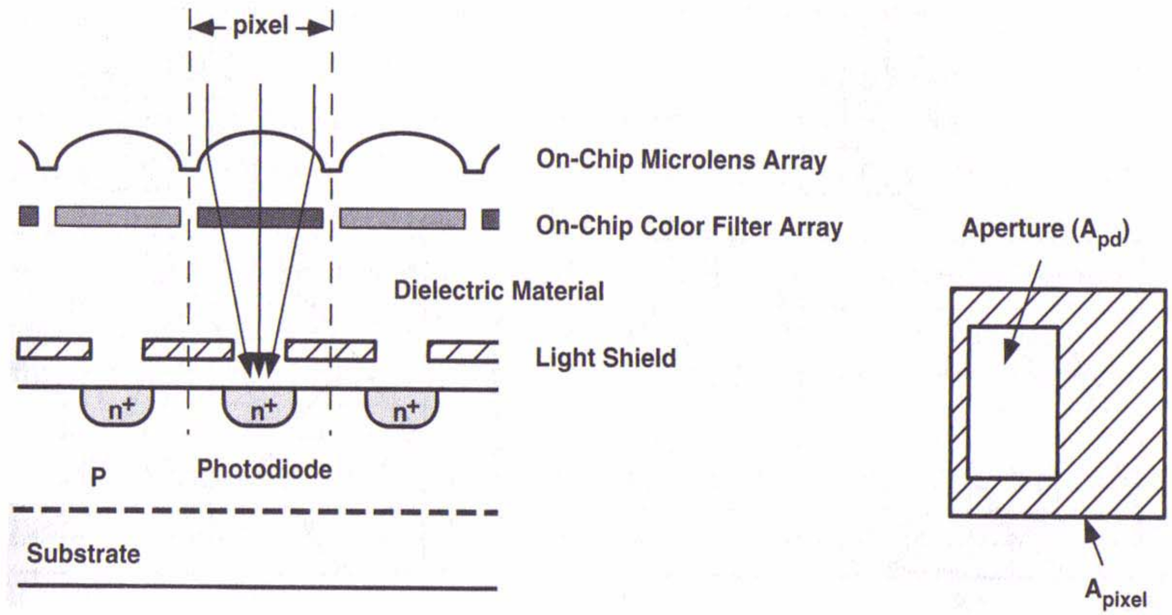
Efficient light = incoming light x (1-R)



Fill Factor

Ratio photosensitive area to total pixel area

$$FF = (A_{PD}/A_{pixel})100\%$$



(a) (b)

Photon energy

Plank and black body radiation:

Atoms in a heated body behaves like harmonic oscillators,
each oscillator can absorb or emit energy, in an amount proportional to its frequency:

$$E = h\nu \qquad (2.35)$$

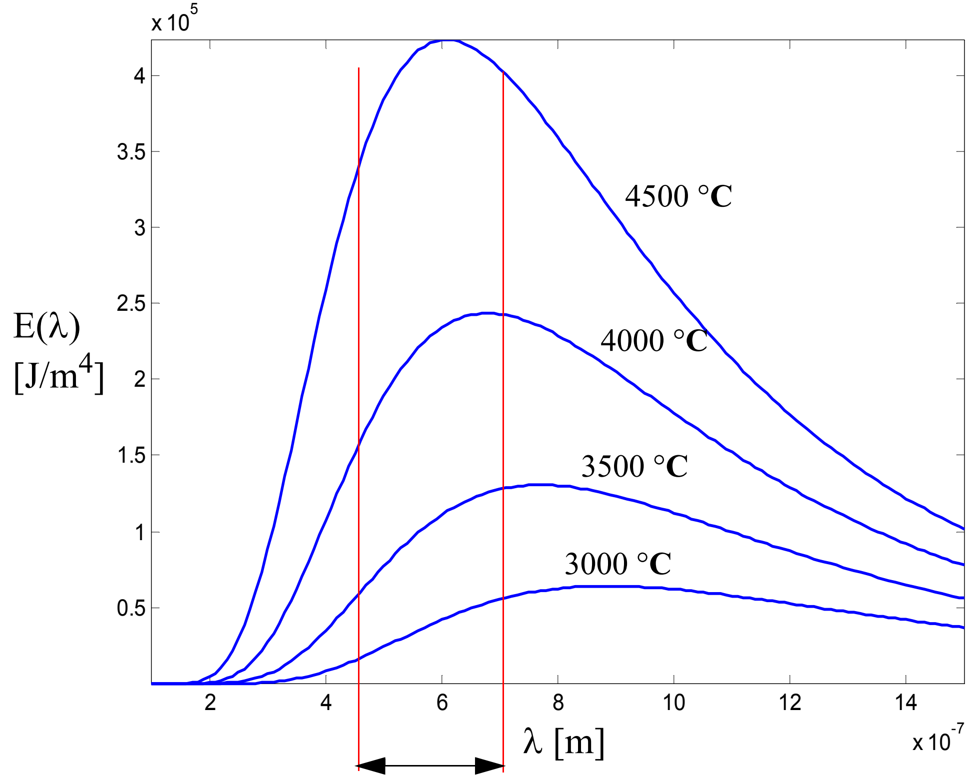
where ν is the frequency,
h is Plancks constant: 6.626 x 10^{-34} Js

The energy is quantized:

$$E_n = nh\nu$$

where n is a positive integer: Number of photons.

Photon wavelength $\lambda = c/\nu$



Visible light: 450nm-700nm

$$E(\nu) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

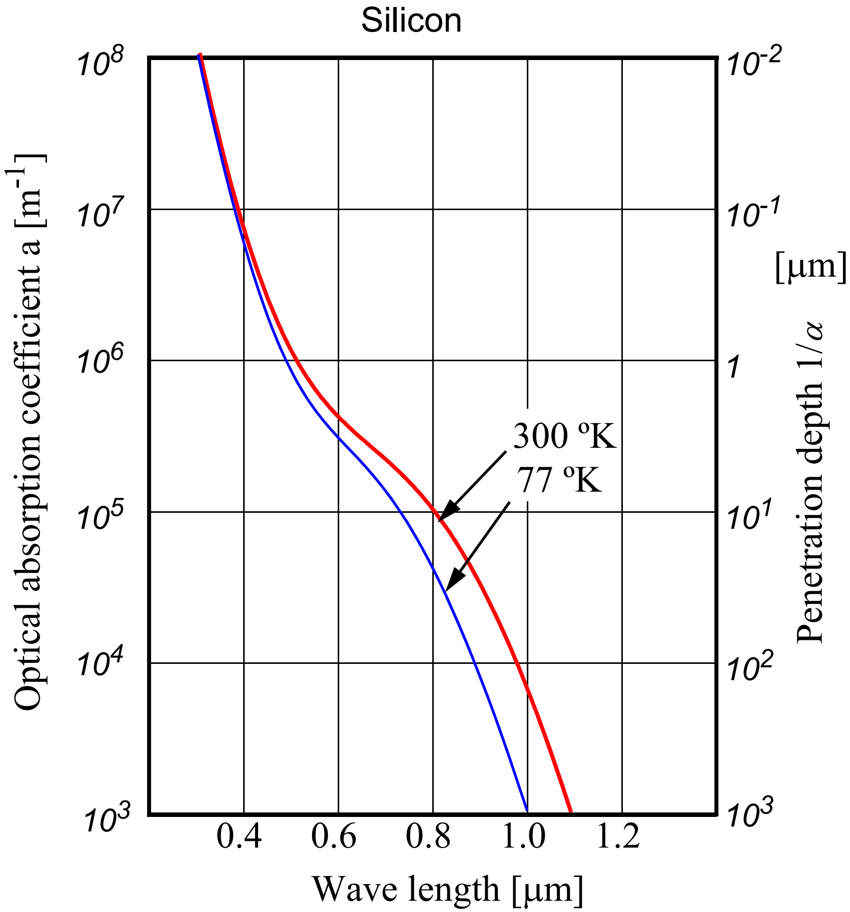
Ref. Alonso-Finn

Absorption

The flux of photons, with energy higher than the band gap, decreases as the photons are absorbed and e-h pair are generated. Thus, the photon flux, $\Phi(x)$, decreases with the penetration depth.

$$d\Phi = \alpha\Phi dx$$

$$\Phi_{ph}(x) = \Phi_0 e^{-\alpha x} \qquad (2.36)$$



Absorption coefficient α depends on the energy, i.e. is larger for shorter wave lengths.

Photons in the blue range of the spectrum

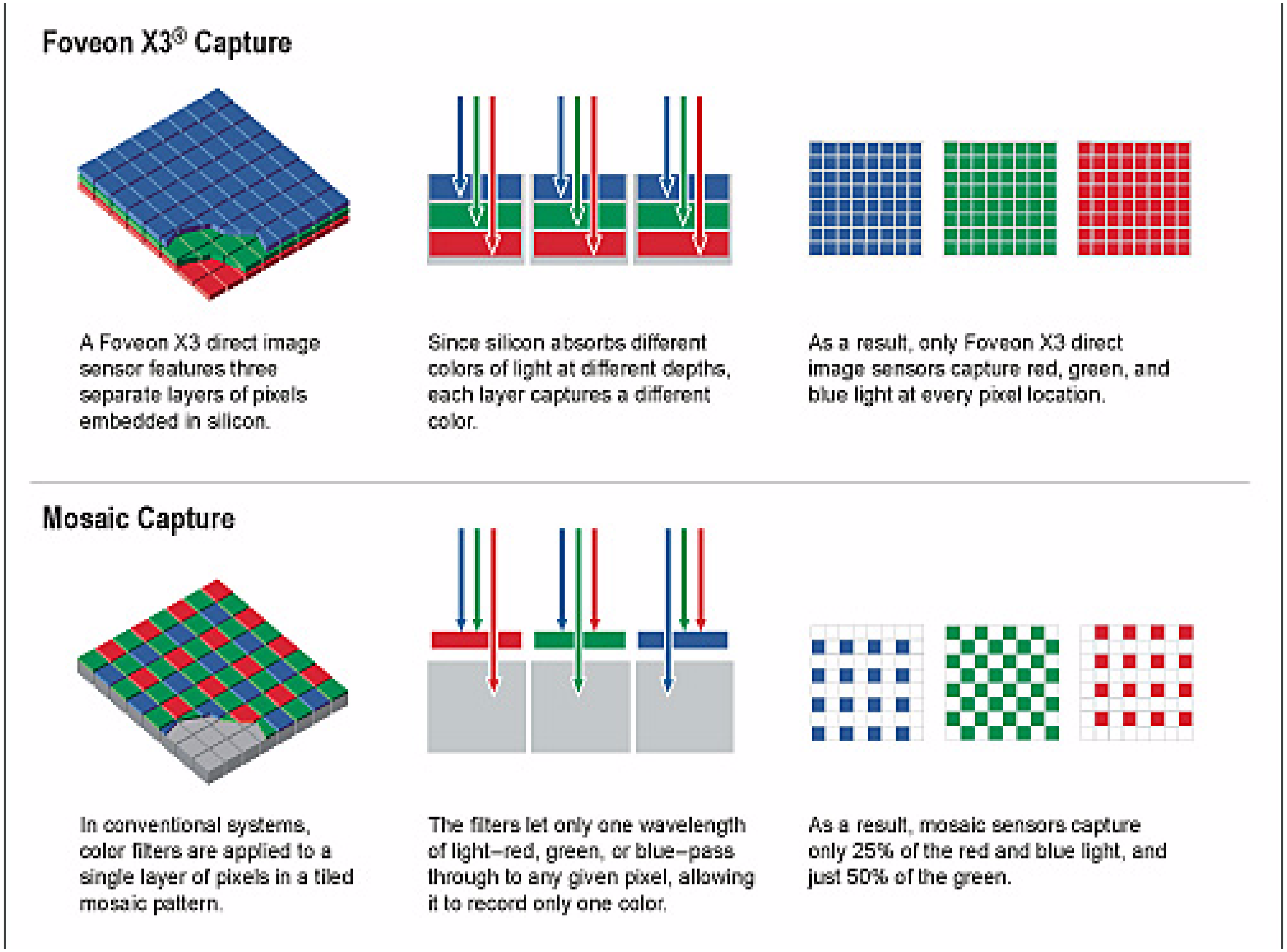
- Short wave length - high energy
- High probability of e-h pair generation.
- Easily absorbed
- Short penetration depth.

Photons in the red range of the spectrum

- Long wave length - low energy
- Low probability of e-h pair generation.
- Passes more material before absorption take place.
- Long penetration depth.

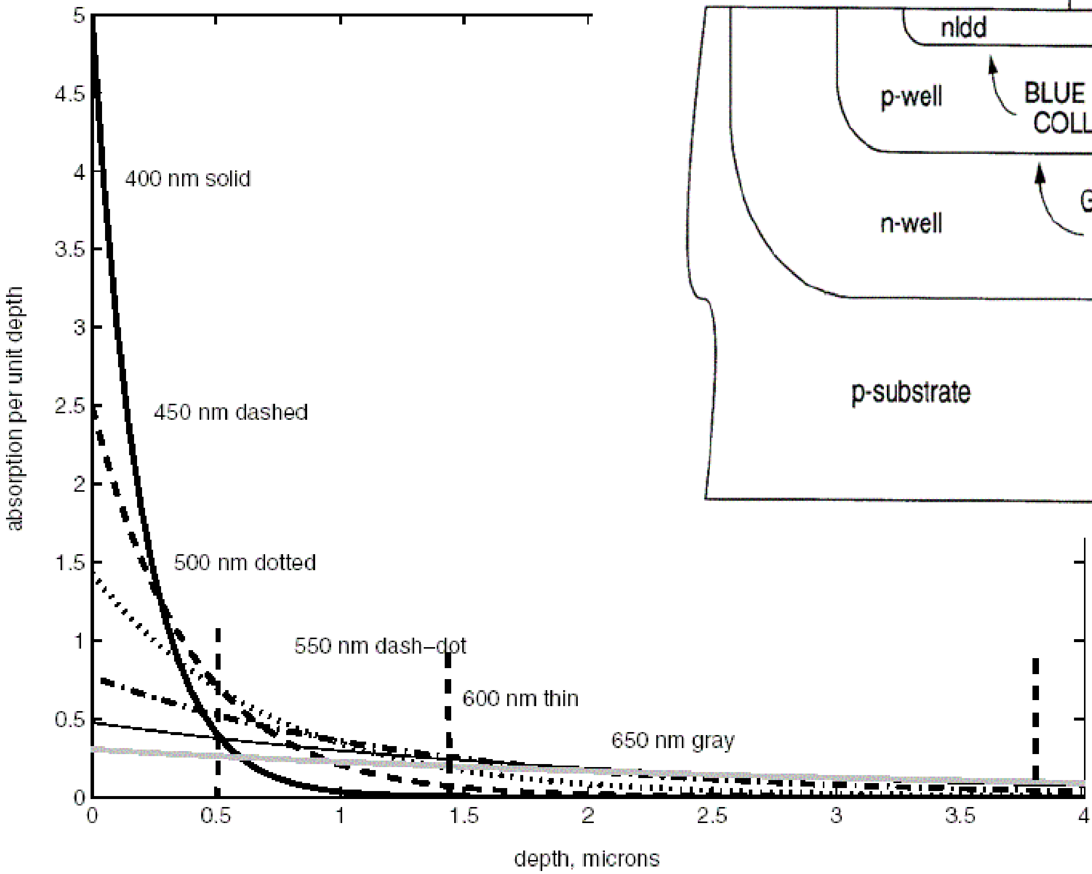
Ref. Sze

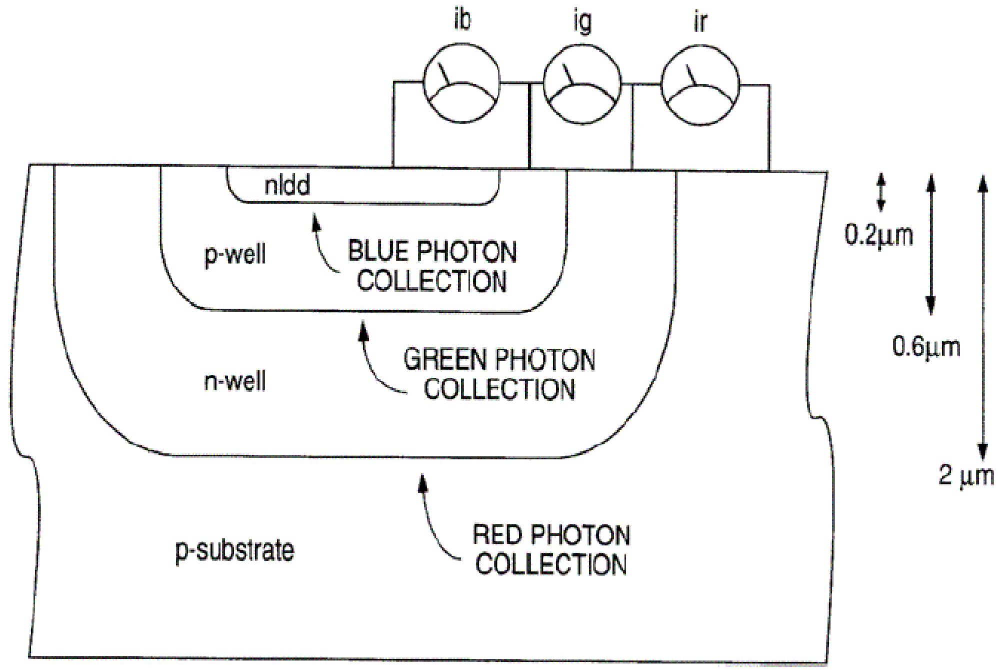
Foveon principle



www.foveon.com

Foveon (cont.)


absorption per unit depth
depth, microns
400 nm solid
450 nm dashed
500 nm dotted
550 nm dash–dot
600 nm thin
650 nm gray
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


ib
ig
ir
nldd
p-well
BLUE PHOTON COLLECTION
n-well
GREEN PHOTON COLLECTION
p-substrate
RED PHOTON COLLECTION
0.2µm
0.6µm
2 µm

Response limits

The responsivity has a lower limit.

The band gap must be exceeded (excitation of electrons).
Lower limit is given by the wave length. Apply (2.35):

$$h\nu \geq E_g \qquad \frac{hc}{\lambda} \geq E_g$$

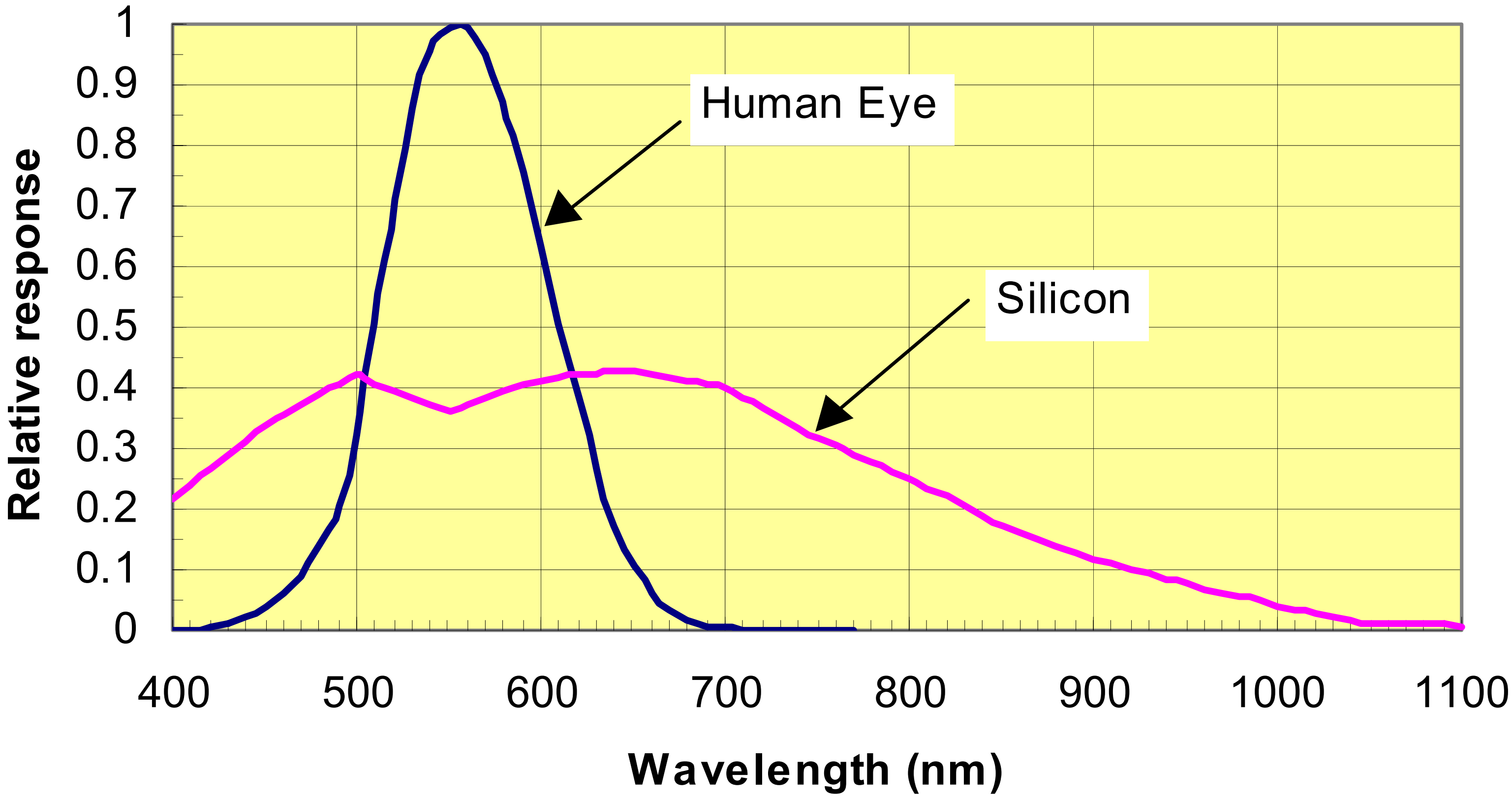
$$E_{g,\,Si} = 1{,}11\ \text{eV} \qquad 1\ \text{eV} = 1{,}602 \cdot 10^{-19}\ \text{J}$$

$$\lambda_{max} = \frac{hc}{E_g} = \frac{6{,}626 \cdot 10^{-34}[\text{Js}]3 \cdot 10^{8}[\text{m/s}]}{1{,}11 \cdot 1{,}602 \cdot 10^{-19}[\text{J}]} = 1{,}06\ \mu\text{m} \qquad (2.37)$$

The Responsivity has an upper limit given by the penetration depth.

Recombination rate is high at the surface,
where short wave photons are absorbed,
due to high density of energy levels in the band gap at the surface.
(typically N_{ts}=10^{11} s_o ~ 150 results in t_s = 1/s_o ~ 0.7 μs/μm)

Sensitivity vs. wave length



Quantum Efficiency (QE)

One definition [ref. Sze]:
Number of generated electron-hole pair per photon hitting the sensor (pixel).

$$\eta = \frac{I_{ph}/q}{P_{opt}/h\nu} \qquad \left[\frac{\text{elektrons per time unit}}{\text{photons per time unit}}\right] \tag{2.38}$$

Common definition of QE: Number of collected electrons per photon.

Responsivity: The ratio photo current to optical input power [ref. Sze]

$$R = \frac{I_{ph}}{P_{opt}} = \frac{\eta q}{h\nu} = \frac{\eta\lambda}{1{,}24}10^{6} \qquad \left[\frac{A}{W}\right] \tag{2.39}$$

$$\left(\frac{q}{h\nu} = \frac{q\lambda}{hc} = \frac{1{,}602 \cdot 10^{-19}\ \lambda}{6{,}626 \cdot 10^{-34}\ \ 3 \cdot 10^{8}} = \frac{\lambda}{1{,}24 \cdot 10^{-6}}\right)$$

Responsivity is proportional to the wave length.

Conversion gain

C=dQ/dV

$$V = \frac{q}{C} \cdot \text{number of electrons}$$

Voltage per collected electrons is defined as "Conversion Gain":

$$CG = \frac{q}{C} \qquad \left[\frac{\mu V}{e}\right] \tag{2.40}$$

From optical power to voltage:

$$V = CG \cdot QE \cdot t \cdot \frac{P_{opt}}{h\nu} \tag{2.41}$$

Full Well

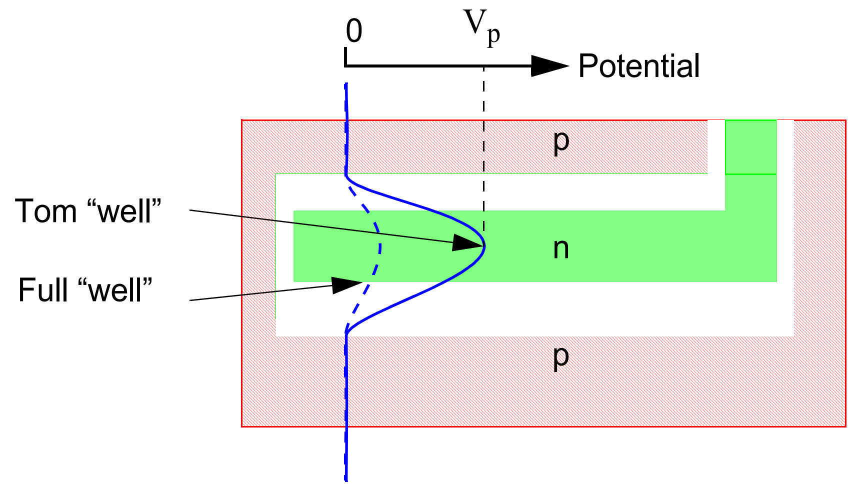
Maximum number of electrons that can be stored in the pixel:

$$N_{sat} = \frac{1}{q} \int_{V_{Reset}}^{V_{max}} C_{PD}(V)dV \tag{2.42}$$

Pinned Photo diode

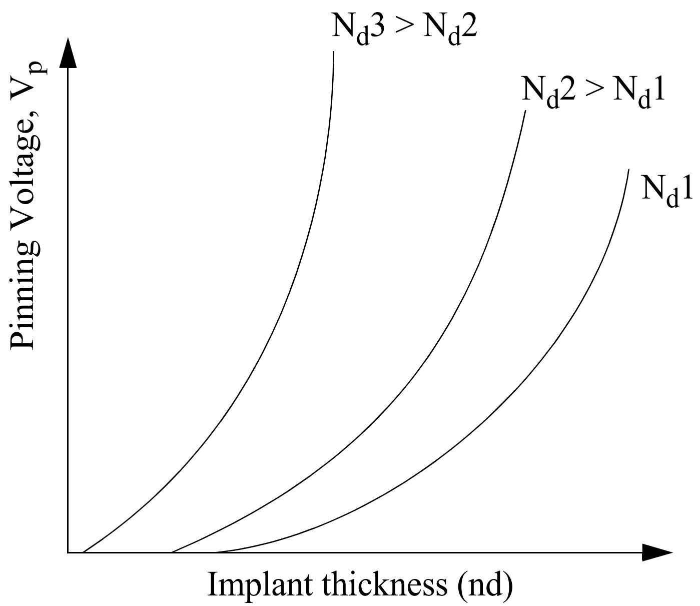
n-region buried in a p-substrate.
Reduces the effect of surface recombination

- Improved response in blue range
- Reduced dark current.



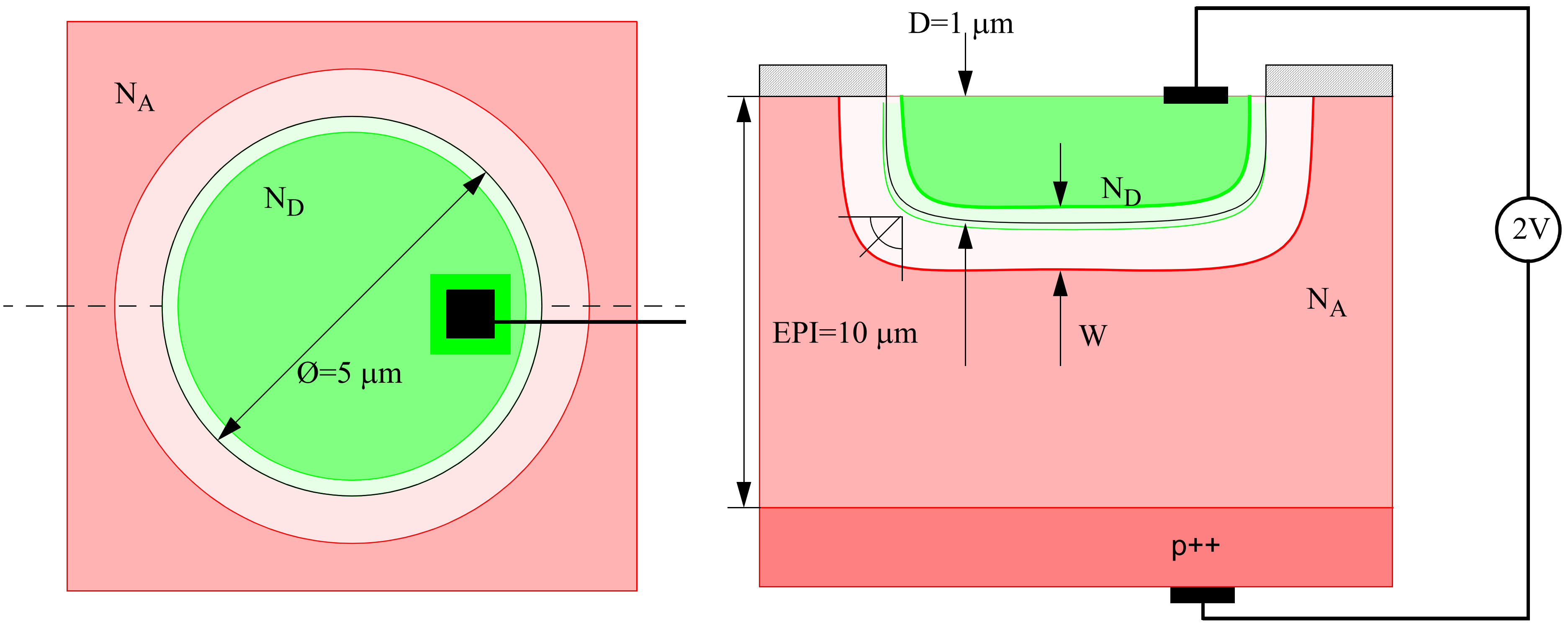
Pinned voltage = the voltage that results in a complete depletion of the n-region.

Pinning voltage depends on doping concentration and implantation range.



Added process step to
standard CMOS process.
Patented: Eastman-Kodak/Motorola (ImageMOS™).

Example 1 - photo diode



Incoming light 550nm (green). Exposure time: 10 ms. External reverse bias V_R = 2V.

- Dark current and signal from a non illuminated sensor?
- Responsivity?
- Required intensity to achieve a signal of 1 V?

Example 1 cont.

Physical data

N_A	10^{16} cm^{-3}	σ_p , σ_n	10^{-15} cm^2 = 10^{-19} m^2	k	1.38 x 10^{-23} J / ºK
N_D	10^{18} cm^{-3}	μ_e	0.135 m^2/Vs	q	1.602 x 10^{-19} C
N_t	3 x 10^{11} cm^{-3}	μ_p	0.048 m^2/Vs	ε_0	8.85 x 10^{-12} C^2 / Nm^2
E_t	E_i			$\varepsilon_{r,Si}$	11.7
η	0.75				
R	0.3 (reflection coefficient)				

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