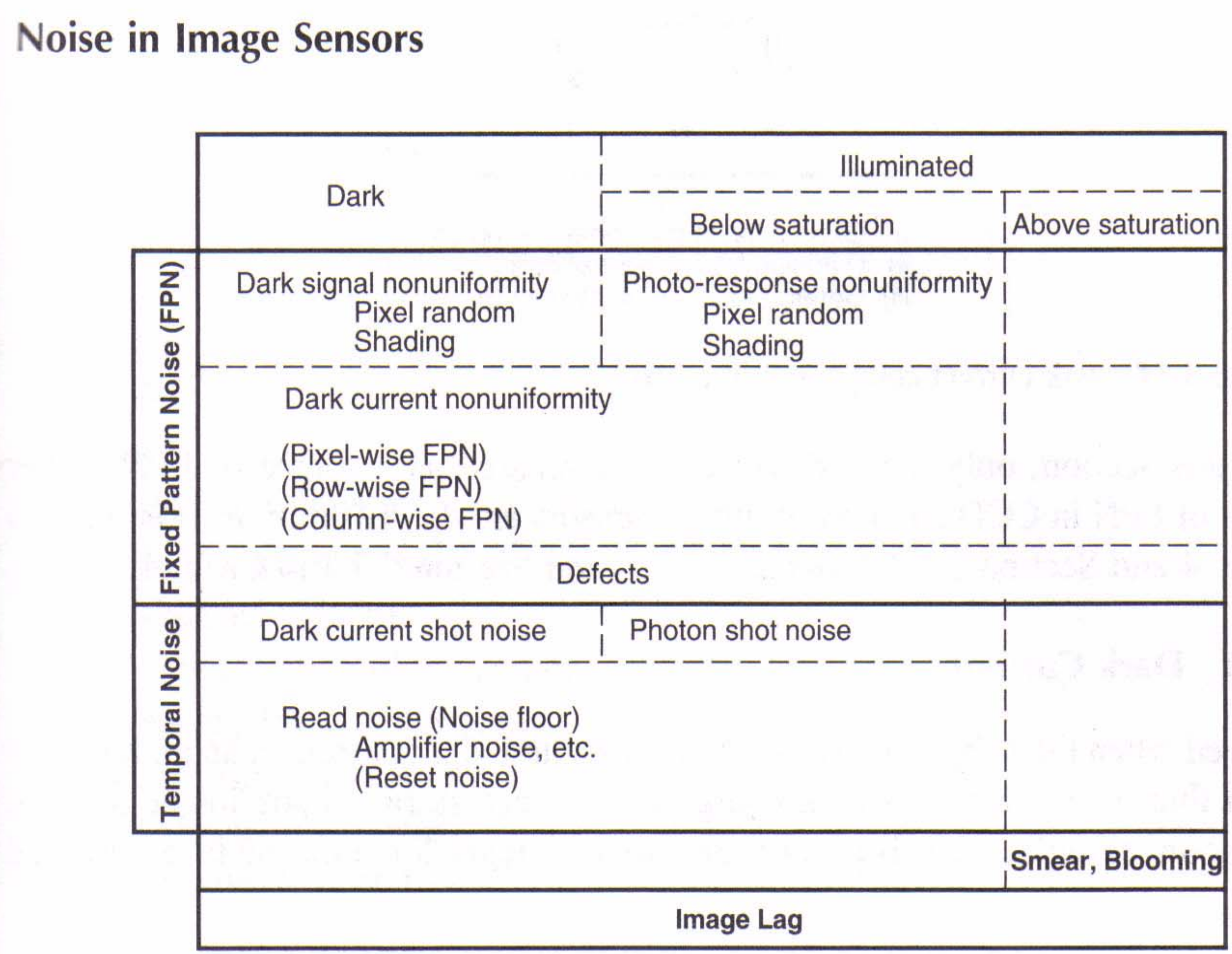
NOISE

Noise Categories

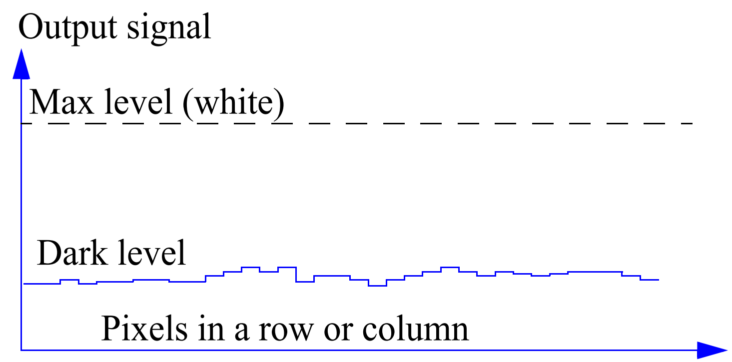


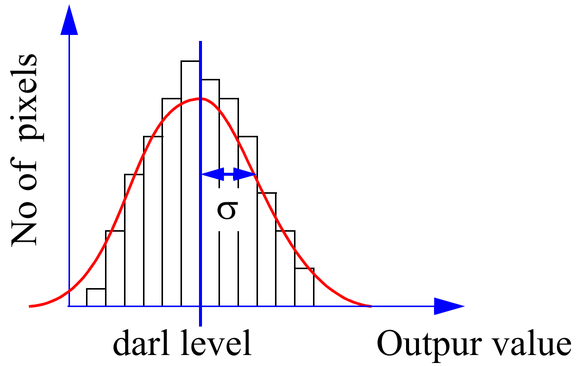
Ref: Nakamura

Fixed Pattern Noise

FPN (fixed pattern noise) in dark images = DSNU (Dark Signal Non-Uniformity)

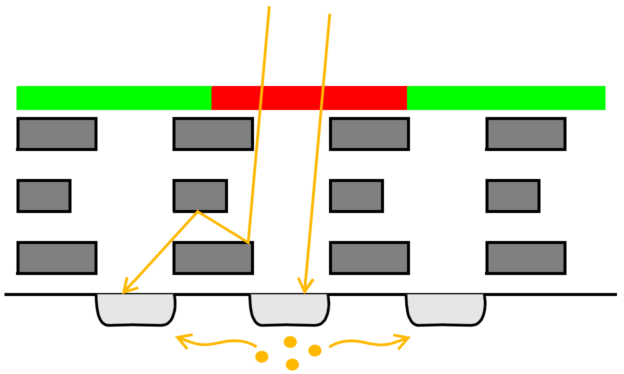
- Variations in offset and gain
  Constant deviations between read out channels offset and gain.
- Pixel wise deviations in dark currents
  (dark signal increases with the integration time)
  Crystal defects
  Surface effects related to the in-pixel transistors.
  Temperature differences across the array (Shading)
- White spots (pixels with very high leakage)
- Defect pixels





FPN (fixed pattern noise) for illuminated images = PRNU (Photo response Non-Uniformity)

- White spots or defect pixels
- Deviations in signal
  Constant with constant exposure
  (Pixel variations)
  Gain deviations in read out chain.
- Shading
  Optical shading
  Angle of incoming light (chief ray angel)
  Gradient in capacitor values (determines the gain)
- Crosstalk
  Light that hits neighbouring pixel
  Charge carrier that diffuse from neighbouring pixel.

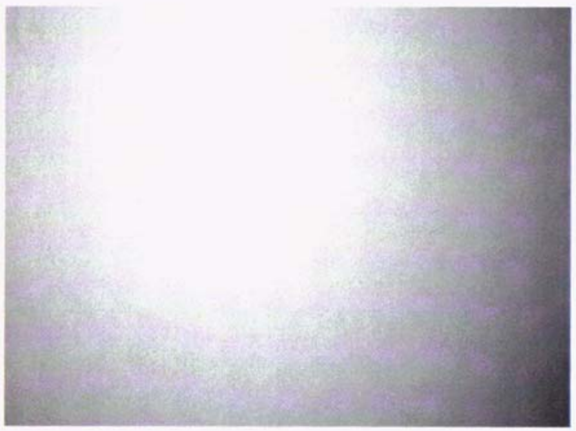


Example on Shading (exaggerated)

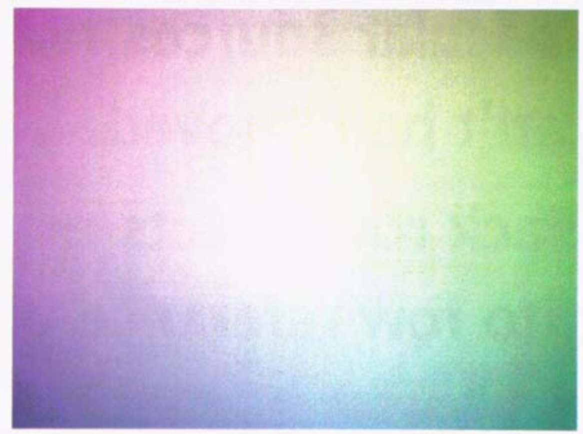
Vignetting and color shading

<u>Sources of shading</u>

- Lens vignetting
- IR filter CRA dependence
- Pixel angular response / asymmetry



Lens vignetting: color planes share a common center



Color shading: color planes have different centers (because of pixel asymmetry)

Ref: Rick Baer (Aptina)

TEMPORARY NOISE

Photon noise

- m individual, independent and random events
- Average rate (number of events per time): $\overline{n}$ probability of n events within a given time interval.

Poisson distribution:

$$P(n) = \frac{(\bar{n})^n}{n!} e^{(-\bar{n})} \qquad \textbf{(5.1)}$$

Mean value of the m independent events:

$$\overline{n_m} = \sum_n n_m P(n) \qquad \text{(m=1, 2, 3,...)} \qquad \textbf{(5.2)}$$

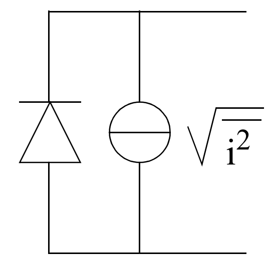
It can be shown that in Poisson distributions, the variance is given by:

$$\sigma^2 = \overline{n^2} - (\bar{n})^2 = \bar{n} \qquad \textbf{(5.3)}$$

Mean value of number of photons that hit the sensor is n.
The rms noise (variance in number of photons) is the square root of the mean value.

[ref.: Ziel/King]

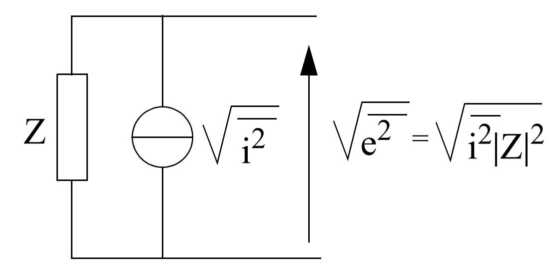
Shot Noise in the Diode



Variation in number of electrons emitted over a PN-junction (Schottky's theorem).
Gives Poisson distribution

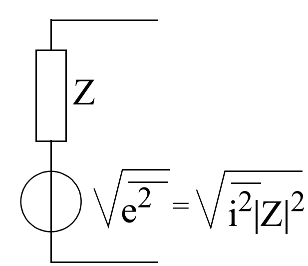
Determines the single event as q and $q\bar{n}$= I (current).
For frequencies much lower than the transit time, the spectral density becomes (energy densisty):

$$S_i(0) = 2qI$$



An average current I in a narrow frequency band Δf is equivalent with a noise current generator $(2qI\Delta f)^{1/2}$ in parallel with the diode.

$$\overline{i^2} = 2qI\Delta f \quad \textbf{(5.4)}$$



Ref.: Ziel

Thermal noise

The random variable is a sum of a large number of independent statistic variables (pulses).
The pulse height X (discrete values) is normal distributed:

$$P(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(X-\bar{X})^2/2\sigma^2}$$

In a continuous probability density function:

$$dP(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\bar{x})^2/2\sigma^2} dx \qquad \textbf{(5.5)}$$

The central limit theorem, that is when n is large: Independent variables: X_1, X_2, X_3,..., X_n

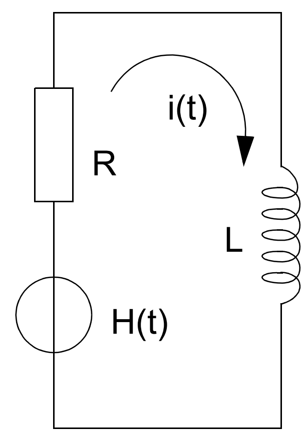
- Equal probability density function
- Equal expectation value
- Equal variance:
- $Y = \sum_{i=1}^{n} X_i$ normal distributed for large n
- $\bar{Y} = n\overline{X_1}$
- $\sigma_y^2 = n\sigma_1^2$

Thermal noise in resistance

The carriers thermal movements results in voltage pulses at the terminals of the resistor. Each pulse is short and has equal probability. Based on the central limit theorem, normal distribution is considered. Because all single pulse has equal probability, all frequencies has equal probability as well. Therefore we get a flat frequency spectrum $S_H(f) = S_H(0)$, white noise.
The average free energy in a system in equilibrium is $^3/_2$ kT (k=Boltzmanns constant, T=absolute temperature).
The average free energy of the current in the loop (one-dimensional system) is kT/2.

$$\frac{L}{2}i^2 = \frac{kT}{2} \qquad \textbf{(5.6)}$$

Voltage around the loop: $H(t) = Ri + L\frac{di}{dt}$ written as fourier series:



$$H(t) = \sum_n \alpha_n e^{j\omega_n t} \qquad i(t) = R\sum_n \beta_n e^{j\omega_n t} \qquad \frac{di}{dt} = L\sum_n j\omega_n\beta_n e^{j\omega_n t}$$

$$\alpha_n = (R + j\omega_n L)\beta_n \qquad \beta_n = \alpha_n\frac{1}{(R + j\omega_n L)}$$

System function: $1/(R+j\omega_n L)$

Integrating the density function S over all frequencies to get the power value of the current:

$$\overline{i^2} = \int_0^\infty S_i(f)df = \int_0^\infty \frac{S_H(0)}{R^2 + (2\pi f_n)^2 L^2}df = \frac{S_H(0)}{2\pi RL}\operatorname{arc}\mathbf{tan}\frac{2\pi L}{R}f_n\Big|_0^\infty = \frac{S_H(0)}{4RL}$$

Replacing i^2 solved form (5.6) yields:(5.6)

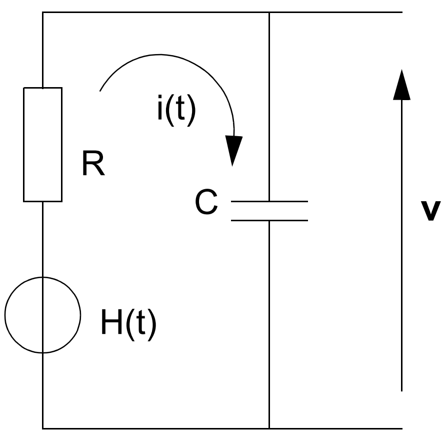
$$S_H(0)\Delta f = \overline{v_R^2} = 4kTR\Delta f \qquad \textbf{(5.7)}$$

The Langevin Method
Ref.: Ziel

Thermal Noise across a Capacitor

Average free energy for the circuit in equilibrium:

$$\frac{1}{2}C\overline{v^2} = \frac{1}{2}kT$$



The circuit has the transfer function:

$$\frac{v_C(f)}{v_H(f)} = \frac{1}{1 + j2\pi fRC}$$

Power value:

$$v_C^2(f) = S_H(f)\frac{1}{1 + 4\pi^2 f^2 R^2 C^2} \qquad \textbf{(5.8)}$$

The voltage source is a resistor with white noise $S_H(f) = S_H(0)$.
Integrating over all frequencies with:

$$\overline{v_C^2} = S_H(0)\int_0^\infty \frac{1}{1 + 4\pi^2 f^2 R^2 C^2}df = \frac{S_H(0)}{2\pi RC}\operatorname{arc}\mathbf{tan}(2\pi RCf)\Big|_0^\infty = \frac{S_H(0)}{4RC} \qquad \textbf{(5.9)}$$

where $S_H(0) = 4kTR$, such that:

$$\overline{v_C^2} = \frac{4kTR}{4RC} = \frac{kT}{C}$$

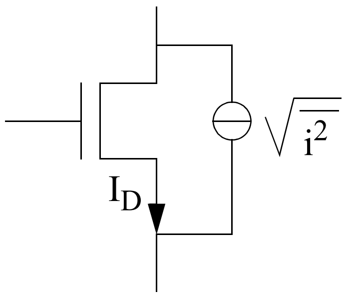
$$\overline{q_C^2} = \overline{v_C^2}C^2 = kTC \qquad \textbf{(5.10)}$$

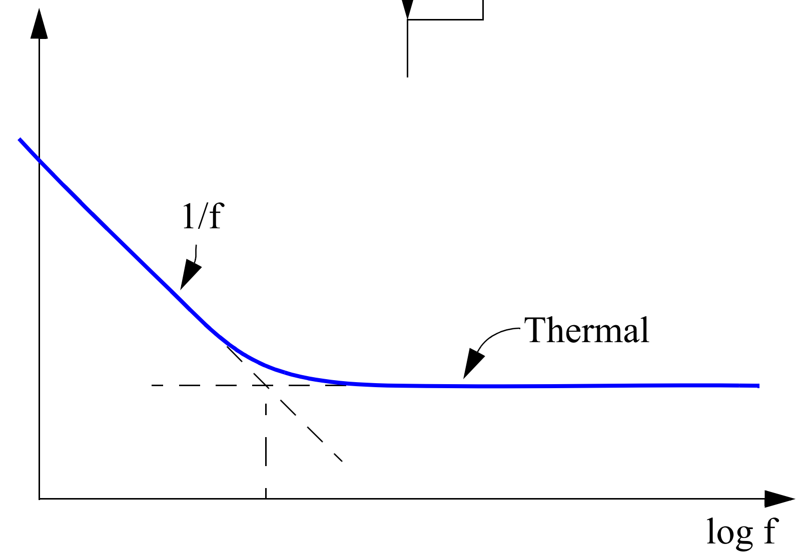
Ref.: Ziel

Noise in MOS-transistors

Two components

- Thermal, white,
  carriers in thermal/statistic movements.
- 1/f
  Frequency dependent.
  Decreases with frequency as $1/f^{\alpha}$
  $\alpha \sim 1$



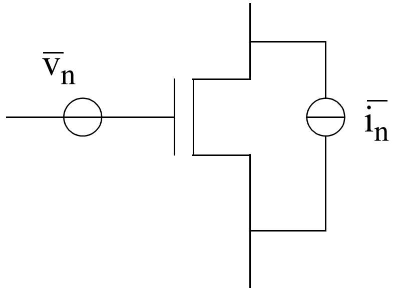


Thermal

Transistor in saturation

$$\overline{\Delta i_t^2} = 4kT\frac{2}{3}g_m\Delta f \quad \textbf{(5.11)}$$

$$\overline{\Delta v_g^2} = \frac{\overline{\Delta i_t^2}}{g_m^2} = 4kT\frac{2}{3}\frac{1}{g_m}\Delta f \qquad (\alpha = 2/3) \quad \textbf{(5.12)}$$



Transistor in linear operation region

$$\overline{\Delta v_t^2} = 4kT\frac{1}{g_{ds}}\Delta f \quad \textbf{(5.13)}$$

$$\overline{\Delta i_t^2} = \frac{\overline{\Delta v_t^2}}{R^2} = \overline{v_t^2}g_{ds}^2 = 4kTg_{ds}\Delta f \quad \textbf{(5.14)}$$

1/f Noise

- Fluctuation of free carrier density in the channel due to “traps” in the interface Si/SiO_2 and the close oxide.
- Fluctuation in the carrier mobility due to interaction with fonons and free path length.
- It can be shown (Ziel) that the statistic interaction has a wide range of time constants which gives the 1/f spectrum.

Empiric expression for 1/f noise (ref. Tsividis):

$$\overline{\Delta i_f^2} = \frac{K g_m^2}{C_{ox} W L} \frac{1}{f} \Delta f \qquad \textbf{(5.15)}$$

$$\overline{\Delta v_f^2} = \frac{K_f}{C_{ox} W L} \frac{1}{f} \Delta f \qquad \textbf{(5.16)}$$

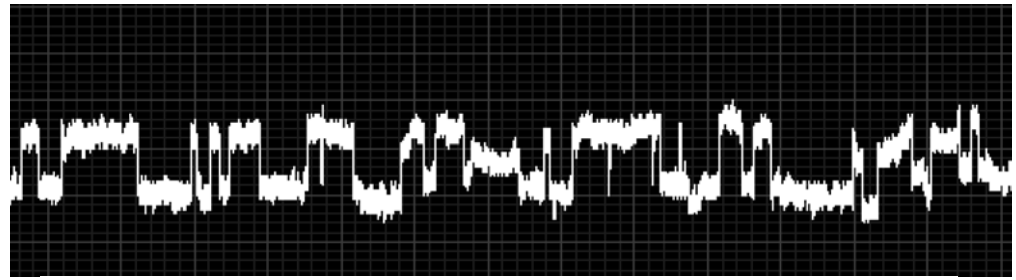
K is a technology dependent constant.

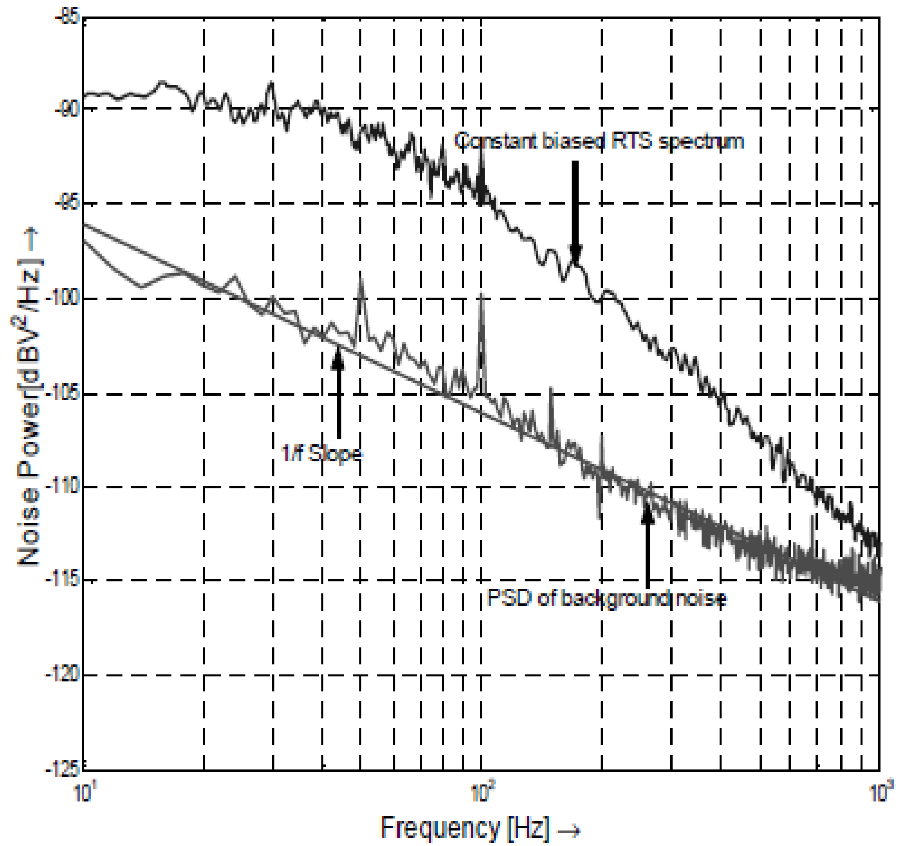
SPICE model: $\overline{i_f^2} = \frac{(KF) I_D^{AF}}{C_{ox} L^2} \frac{1}{f}$ where KF and AF (~1) is SPICE parameters

Random Telegraph Signals (RTS)

As MOS devices becomes smaller, capture of single charge carriers becomes more dominant than capturing av many carriers that creates the typical 1/f noise.

According to the theory, the mechanism similar, but capture and release of single carriers make the drain current to flip between 2 distinct values (in addition to the thermal noise).

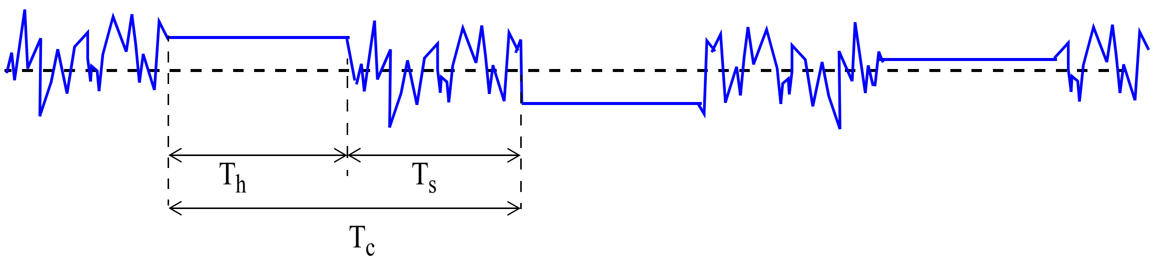


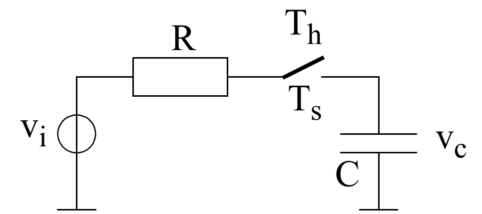


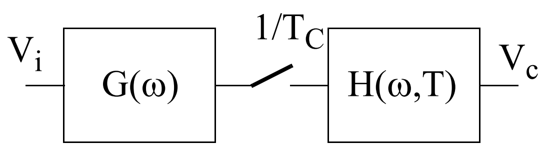
Images:

J.S. Kolhatkar et al., Separation of Random Telegraph Signals from *1/f* Noise in MOSFETs under Constant and Switched Bias Conditions

Noise during sampling







When the switch (T) is conducting:
Fourier transform of the transfer function

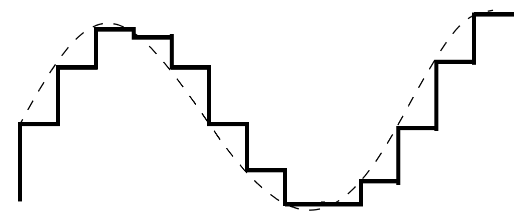
$$G(\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau} \qquad \textbf{(5.17)}$$

Fourier transform of the sampled signal (from signal processing theory):

$$X(j\omega) = \frac{1}{T_c}\sum_k V_i(j\omega)G(j\omega)\left(j\omega_a + j\frac{2\pi k}{T_c}\right) \quad \textit{[ref. Oppenheim]} \qquad \textbf{(5.18)}$$

$V_i(j\omega)$*$G(j\omega)$ corresponds to the noise described in (5.8).

Zero order hold funtion:



Output form zero order hold-element:

$$x_f(t) = \sum_n X(nT_h)[u(t - nT_h) - u(t - (n+1)T_h)]$$

Ref.: Unbehauen/Vittoz

The LaPlace Transform

$$X_f(s) = \sum_n X_n(nT_h)\left[\frac{e^{-snT_h}}{s} - \frac{e^{-s(n+1)T_h}}{s}\right] = \sum_n X_n(nT_h)\left[\frac{e^{-snT_h}}{s}(1 - e^{-sT_h})\right]$$

$$= \frac{1 - e^{-sT_h}}{s}\sum_n X_n(nT_h)[e^{-snT_h}] = \frac{1 - e^{-sT_h}}{s}X_n(s)$$

Transfer function of the hold element:

$$H_h(s) = \frac{X_n(s)}{X_f(s)} = \frac{1 - e^{-sT_h}}{s}$$

In the fourier version s=jω

$$H_h(j\omega) = H_h(s)\Big|_{s = j\omega} = \frac{1 - e^{-j\omega T_h}}{j\omega} = \frac{e^{j\omega T_h/2} - e^{-j\omega T_h/2}}{j\omega e^{j\omega T_h/2}(T_h/2)}\frac{T_h}{2}$$

$$H_h(j\omega) = T_h\frac{\sin(\omega T_h/2)}{\omega T_h/2}e^{-j\omega T_h/2} \quad \textbf{(5.19)}$$

Frequency response to the signal from the S/H system

$$V_c(j\omega) = X(j\omega)H_h(j\omega)\,\textbf{(5.18)} \quad \textbf{(5.20)}$$

Properties of the Laplace transform:

$$L[u(t)] = \frac{1}{s}$$

$$L[u(t - T)] = \frac{1}{s}e^{-sT}$$

Transform of a sampeled sequense:

$$F^*(s) = L[f^*(t)]$$

$$= \sum_{n = -\infty}^{\infty} f(nT)e^{-snT}$$

Consider the base component in the Fourier transform X(jω) of the sampeled signal.

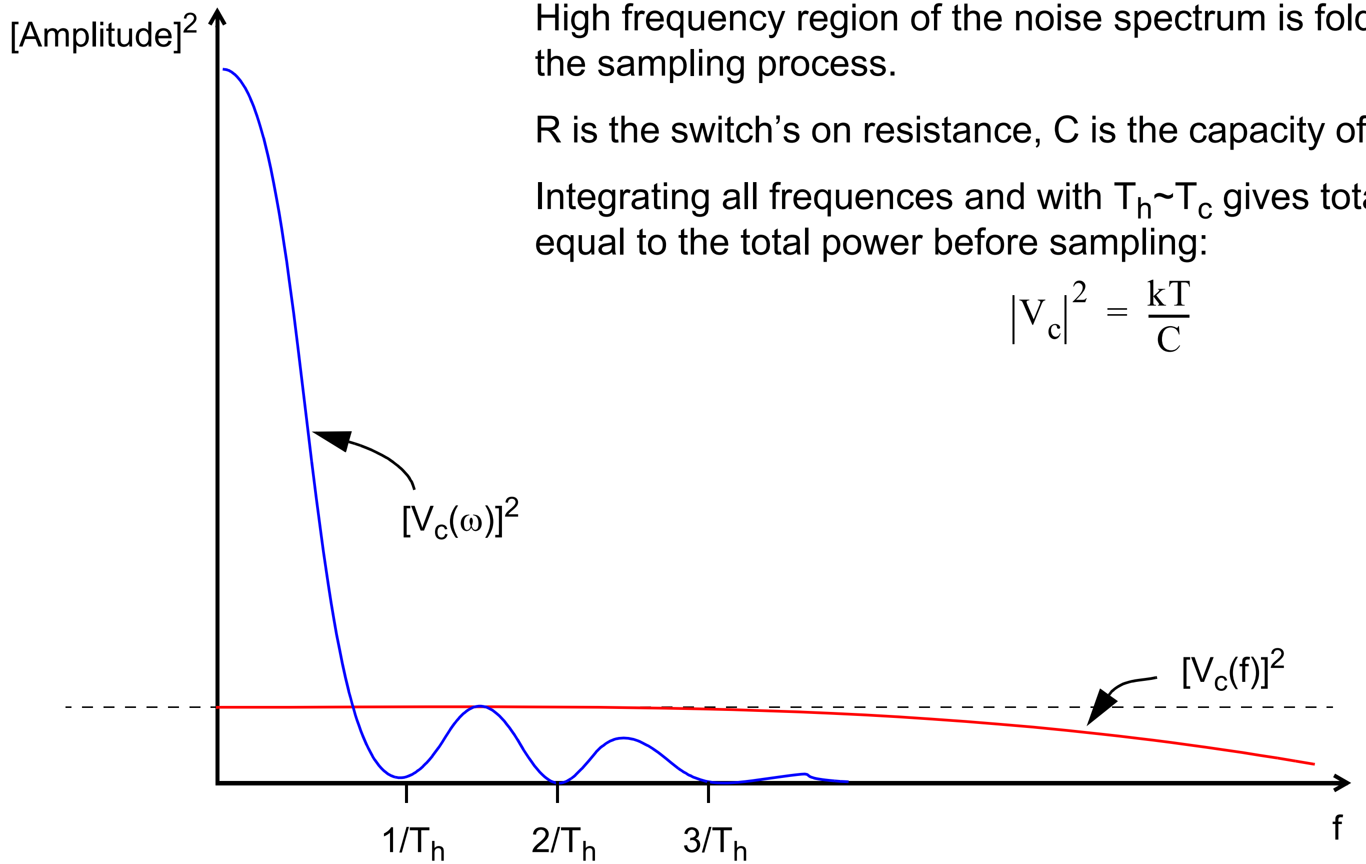
$$|V_c(\omega)|^2 = 4kTR\frac{1}{1+\omega^2R^2C^2}\left[\frac{T_h}{T_c}\frac{\sin(\omega T_h/2)}{\omega T_h/2}\right]^2 \quad \textbf{(5.21)}$$

High frequency region of the noise spectrum is folded down during the sampling process.

R is the switch's on resistance, C is the capacity of the sampling capacitor.

Integrating all frequences and with T_h~T_c gives total noise power equal to the total power before sampling:

$$|V_c|^2 = \frac{kT}{C} \quad \textbf{(5.22)}$$



Correlated double sampling

High pass function - attenuates low frequency noise.

Sample function where the 2nd sample is subtracted form the 1st sample:

$$x(n, \tau_d) = x(n)[\delta(t - nT_h) - \delta(t - nT_h - \tau_d)] \quad \textbf{(5.23)}$$

Simplify by regarding the double sampler only:

$$p_\delta(t) = \delta(t) - \delta(t - \tau_d) \quad \textbf{(5.24)}$$

Fourier transform:

$$P_\delta(j\omega) = 1 - e^{-j\omega\tau_d} = \frac{e^{j\omega\tau_d/2} - e^{-j\omega\tau_d/2}}{e^{j\omega\tau_d/2}} = 2j\,\mathbf{sin}\left(\frac{\omega\tau}{2}\right)e^{-j\omega\tau_d/2} \quad \textbf{(5.25)}$$

$$|P_\delta(j\omega)| = 2\,\mathbf{sin}\left(\frac{\omega\tau}{2}\right) \quad \textbf{(5.26)}$$

For $\omega\tau$ << 1:

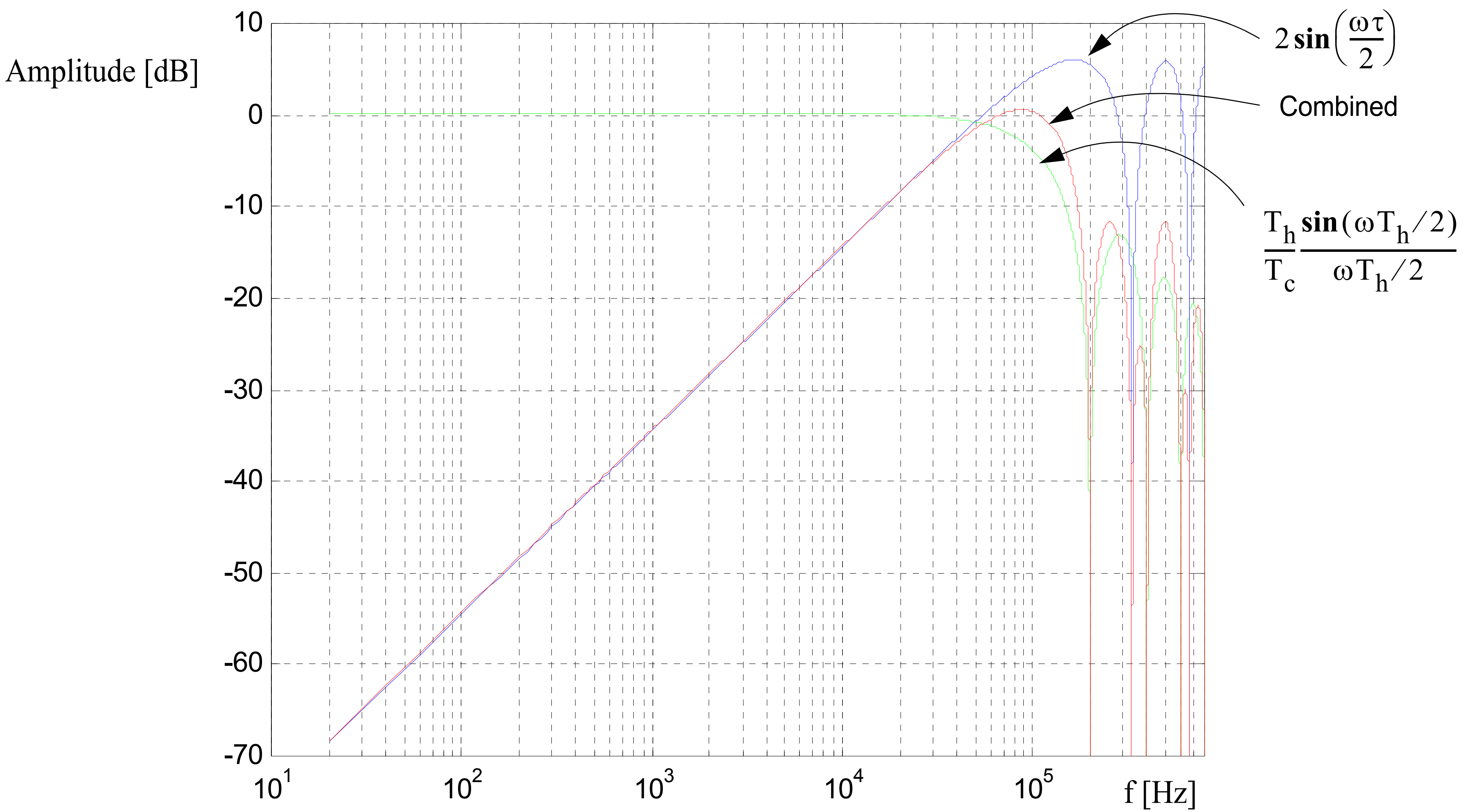
$$|P_\delta(j\omega)| = \omega\tau_d \quad \textbf{(5.27)}$$

$$F[\delta(t)] = 1$$

$$F[\delta(t - T)] = e^{-j\omega T}$$

Ref.: Unbehauen/Oppenheim

Frequency response for double sampling and hold element

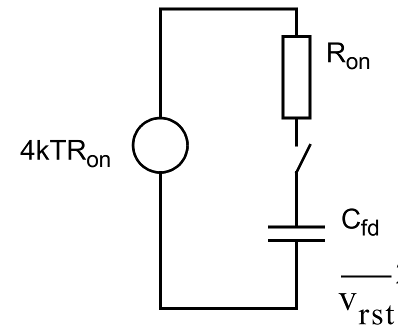


Reset Noise

At the end of the reset period (hard reset) the reset transistor operates similar to a resistor.

It is a R-C circuit that gives a total noise of kT/C

Double sampling reduces this noise significantly.

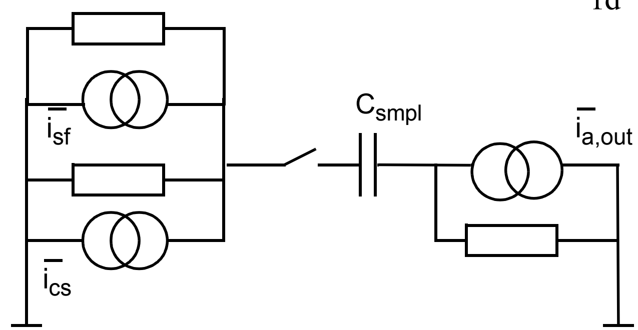


$$\overline{v_{rst}}^2 = \frac{kT}{C_{fd}}$$

Sampling noise

Noise current from the source follower transistor, current source and noise current from the amplifier is added to the noise generated by the switch’s on resistance.

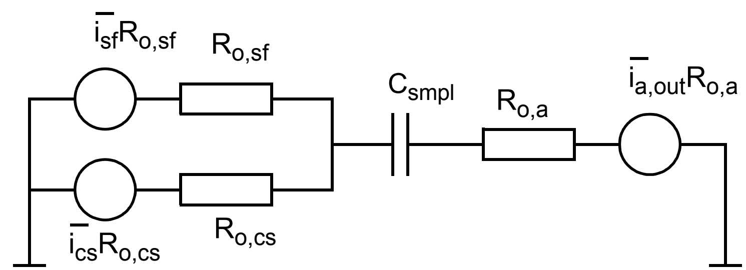
Every noise current with a given spectral distribution can be written as a noise voltage with its spectral distribution in series with an output resistance. This voltage is shaped in the sampling process as shown on the previous pages.



Read-out noise

The voltage on the sampling capacitor is read by the amplifier.

The amplifier noise is added to the reset and sampling noise, which again is shaped by the ADC sampler.

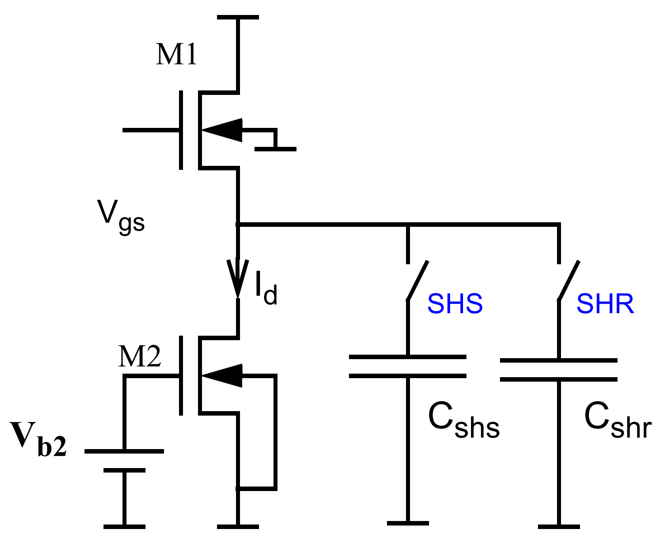


Noise considerations in the Source Follower

Thermal noise

Noise bandwidth: Assuming rectangular shaped frequency spectrum with all its power inside the rectangle.
Normalized white noise: $|H(j\omega)|=1$

$$\Delta f = \frac{1}{|H(j\omega)_{max}|^2} \int_0^{\infty} \frac{|H(j\omega)|^2}{(\omega RC)^2 + 1} d\omega \Bigg|_{|H(j\omega)| = 1} = 2\pi \int_0^{\infty} \frac{1}{(2\pi RCf)^2 + 1} df = \frac{\pi}{2RC}$$



Equivalent thermal noise voltage at the input:

$$\overline{\Delta v_{th}^2} = 4kT\frac{2}{3}\frac{1}{g_m}\Delta f$$

Assuming a large part of the noise as uncorrelated and band limited by SF's output resistance $1/g_m$ and load capacitance $C_{sh} = C_{shr} = C_{shs}$

$$\overline{v_{th,\ shr}^2} \approx 2\left(4kT\frac{2}{3}\frac{1}{g_m}\frac{g_m}{2C_{sh}}\pi\right) = \frac{8\pi}{3}\frac{kT}{C_{sh}} \qquad \textbf{(5.28)}$$

Flicker noise

$$\overline{\Delta v_f^2} = \frac{K_f}{C_{ox}WL}\frac{1}{f}\Delta f$$

Flicker noise has a large low frequency part, which is attenuated by double sampling:

Total noise in differential signal (equivalent input noise at the SF input):

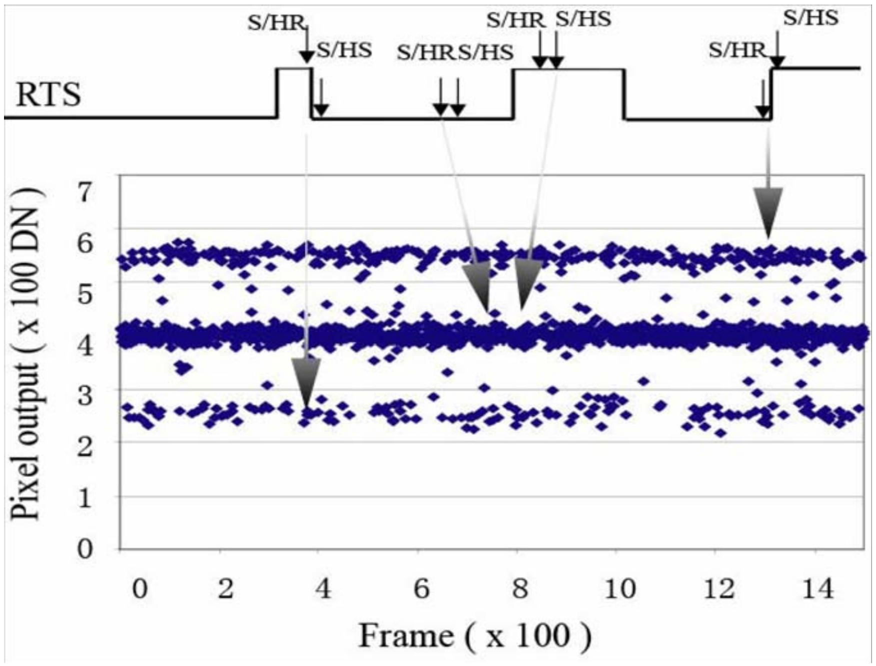
$$\overline{v_{1/f}^2} = 2\int \frac{K_f}{C_{ox}WL}\frac{1}{f}\frac{1}{(2\pi fRC)^2+1}(1-\cos(2\pi f\tau))df \qquad \textbf{(5.29)}$$

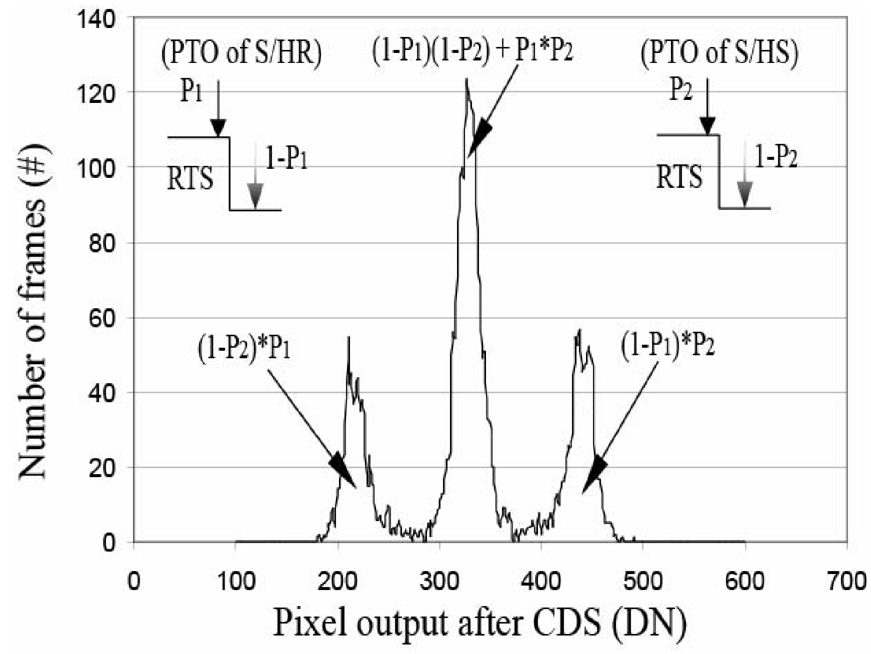
The last term is from (5.26)

$$|P_\delta(j\omega)|^2 = \left[2\sin\frac{\omega\tau}{2}\right]^2 = 4\left(1-\left(\cos\frac{\omega\tau}{2}\right)^2\right) = 4\left(1-\frac{1}{2}-\frac{1}{2}\cos(\omega\tau)\right) = 2(1-\cos(\omega\tau))$$

RTS noise

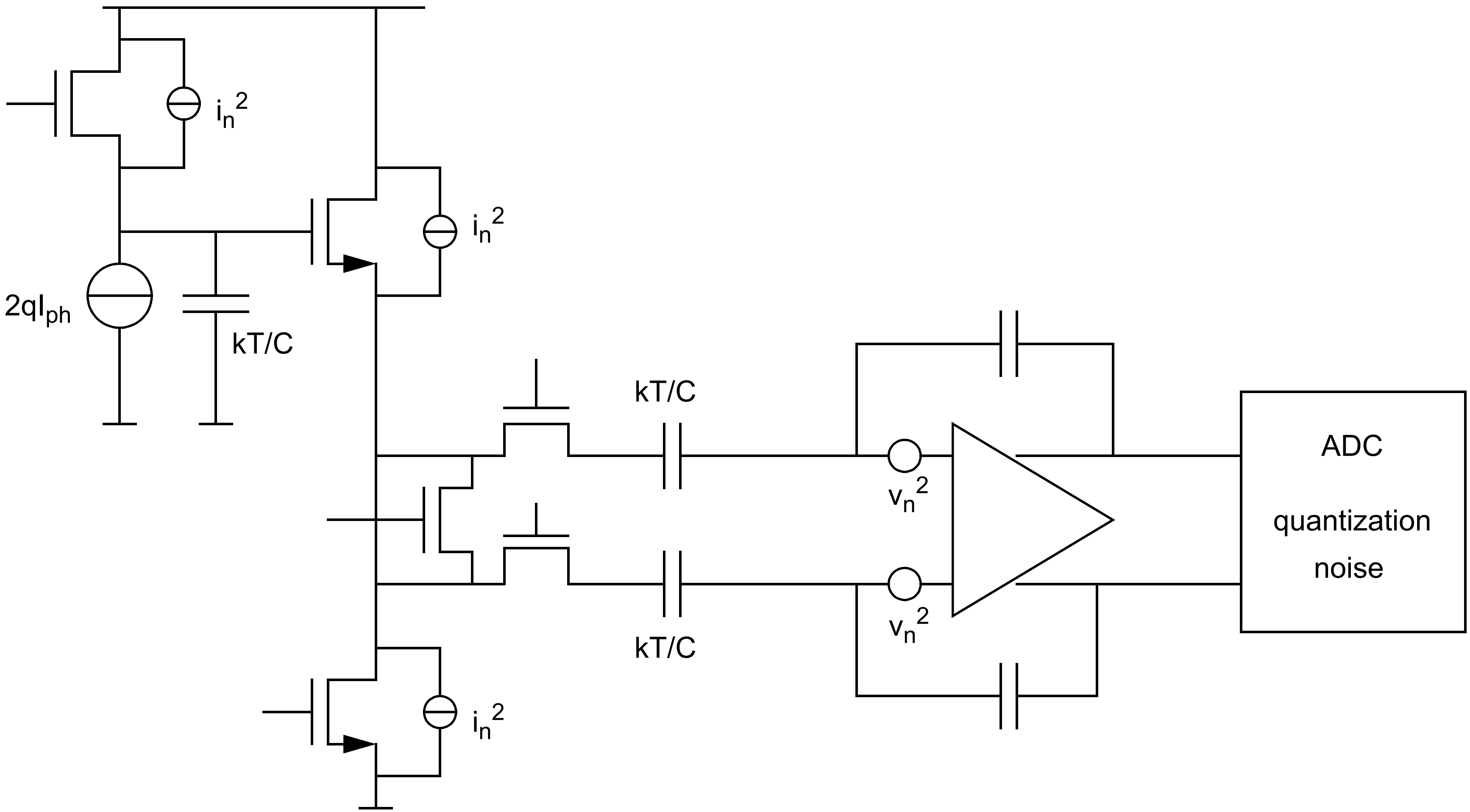
Combined with the Correlated Double sampling, three distinct levels are created





Images: Xinyang Wang et al: Random Telegraph Signal in CMOS Image Sensor Pixels.

Noise sources - summary



Comment: Low frequency noise is filtered by double sampling

Equivalent noise reference

Noise voltage referred to the output

$$v_{n,\,out}^2 = (A_v\,CG)^2 q\sigma_{n,\,pix}^2 + v_{n,\,signchain}^2 \quad \mathbf{[V]} \qquad \mathbf{(5.30)}$$

Noise charge, in number of electrons, referred to the input

$$\sigma_{n,\,in}^2 = \frac{v_{n,\,signchain}^2}{q(A_v\,CG)^2} + \sigma_{n,\,pix}^2 \quad \mathbf{[e^-]} \qquad \mathbf{(5.31)}$$

Can be given as equivalent luminance (lux) by including QE and optical loss.

Definitions

Dynamic range - DR

The ratio largest possible signal to the signal that is just at the noise level in one image: Intra scene dynamic range.

$$DR[dB] = 20\mathbf{log}\left(\frac{N_{sat}}{n_{read}}\right) \quad \textbf{(5.32)}$$

Signal to Noise Ratio - SNR

The ratio signal, independent of the level, and the noise.
The Poisson distributed photon noise, $\sigma = \sqrt{N_{sig}}$, is level dependent.

$$SNR[dB] = 20\mathbf{log}\left(\frac{N_{sig}}{n_{tot}}\right) \quad \textbf{(5.33)}$$

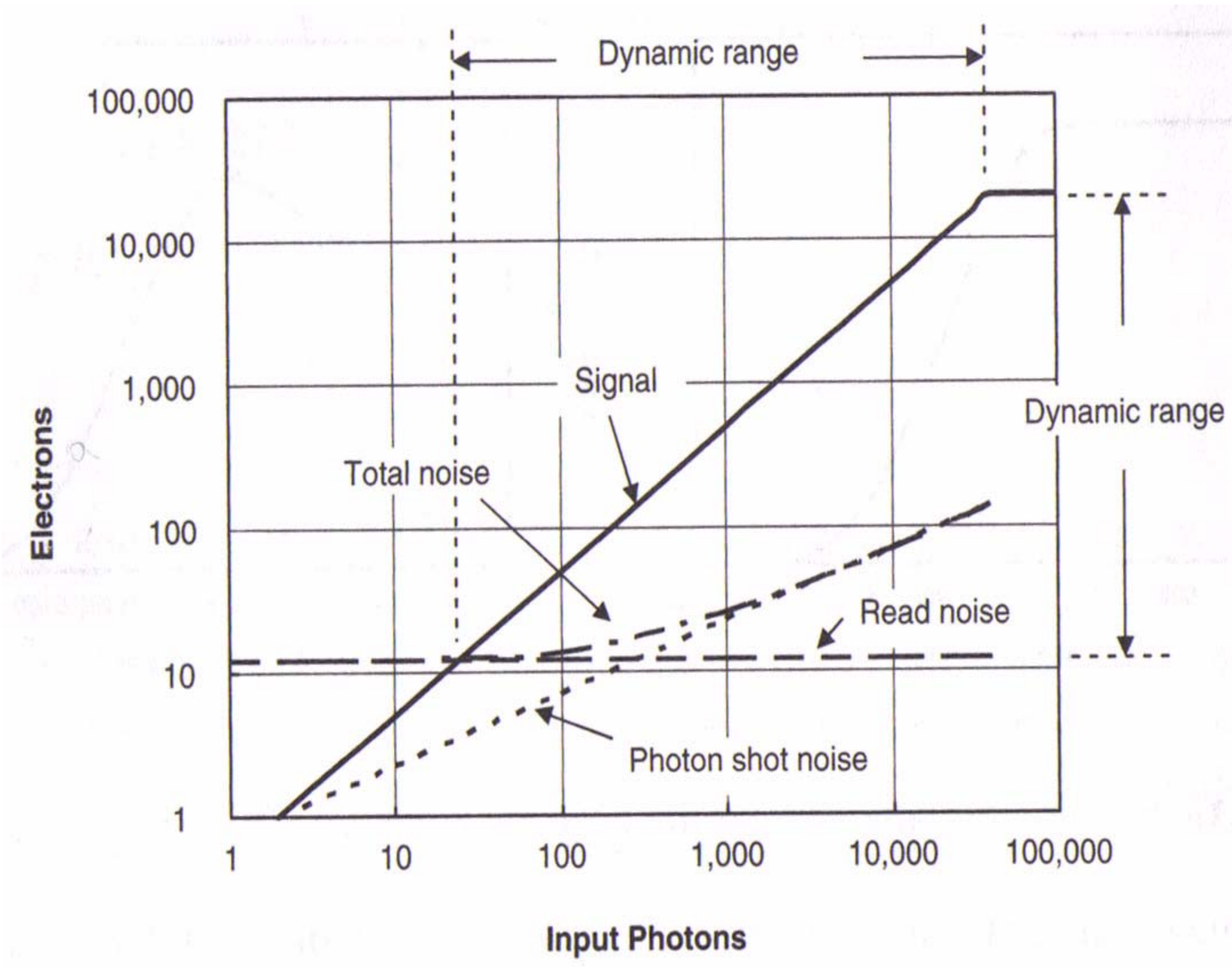
where n_{tot} is read noise plus photon noise.

At large signal levels, the photon noise is dominating.

$$SNR_{high}[dB] = 20\mathbf{log}\left(\frac{N_{sig}}{n_{foton}}\right) = 20\mathbf{log}\left(\frac{N_{sig}}{\sqrt{N_{sig}}}\right) = 20\mathbf{log}\sqrt{N_{sig}} \quad \textbf{(5.34)}$$

and maximum SNR is achieved close to saturation:

$$SNR_{max}[dB] = 20\mathbf{log}\sqrt{N_{sat}} \quad \textbf{(5.35)}$$


Dynamic range
100,000
10,000
1,000
100
10
1
Electrons
Signal
Total noise
Dynamic range
Read noise
Photon shot noise
1
10
100
1,000
10,000
100,000
Input Photons

Reference

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Design of MOS VLSI Circuits for Telecommunications
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Prentice-Hall (1985)

Oppenheim

Digital Signal Processing
Alan V. Oppenheim / Ronald W. Schafer
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Image Sensors and Signal Processing for Digital Still Cameras,
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Example: Read-out chain

- 2 M pixler
- 15 Frames per second
- Exposure time 10 ms

To be discussed:

- The requirements for the speed in
  Source Follower,
  in gain amplifier,
  and ADC
- Required gain settings
- Required ADC resoution