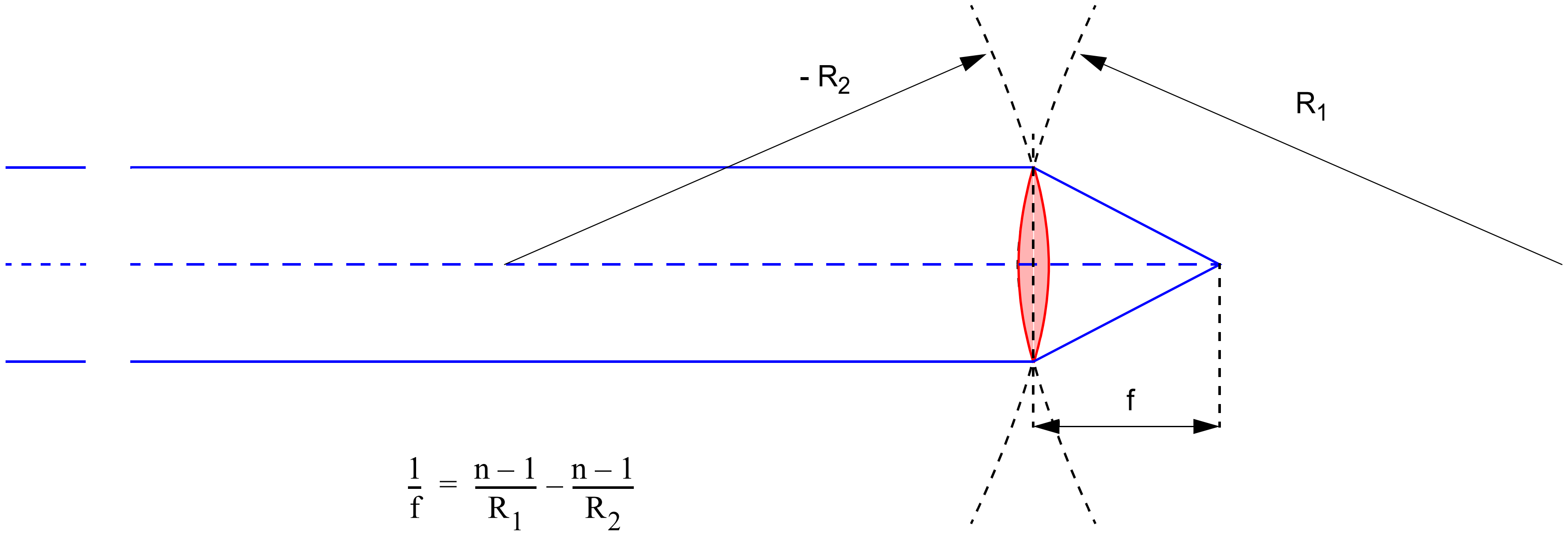
OPTICS

Focal length

A thin lens: A object in 'infinite' distance from the lens, such that parallel beams form the object enter the lens, is focused in the focal point.
The focal length, f, is the distance lens - focal point.



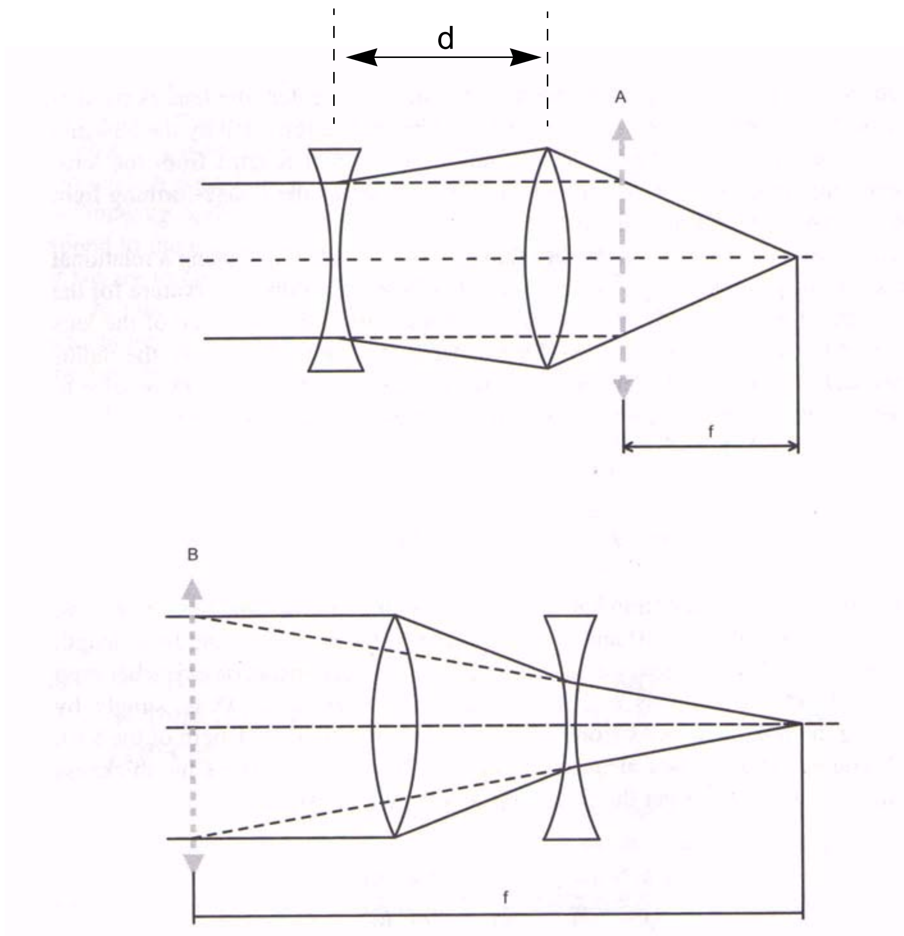
$$\frac{1}{f} = \frac{n-1}{R_1} - \frac{n-1}{R_2}$$

Ref.: Nakamura

R is the radius of curvature, n is the refractive index

Combinations of lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$



Retrofocus lens: “Shortened” focal length

Telephoto lens: “increased” focal length

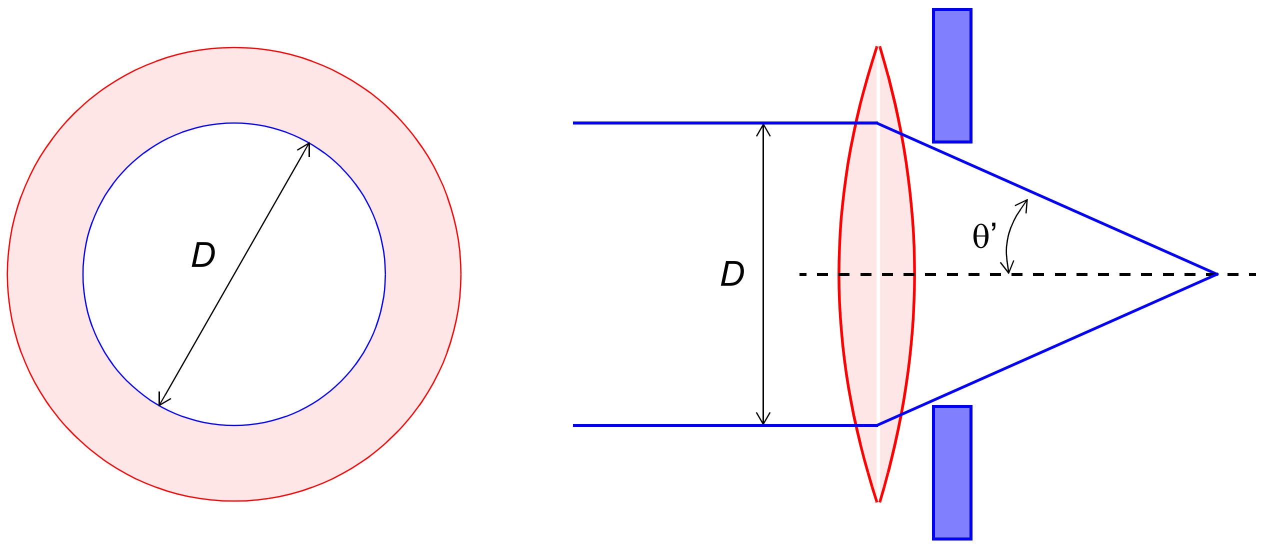
Ref.: Nakamura

Aperture

F-factor, f/# (definition): $$F = \frac{1}{2\sin\theta'} \tag{6.1}$$

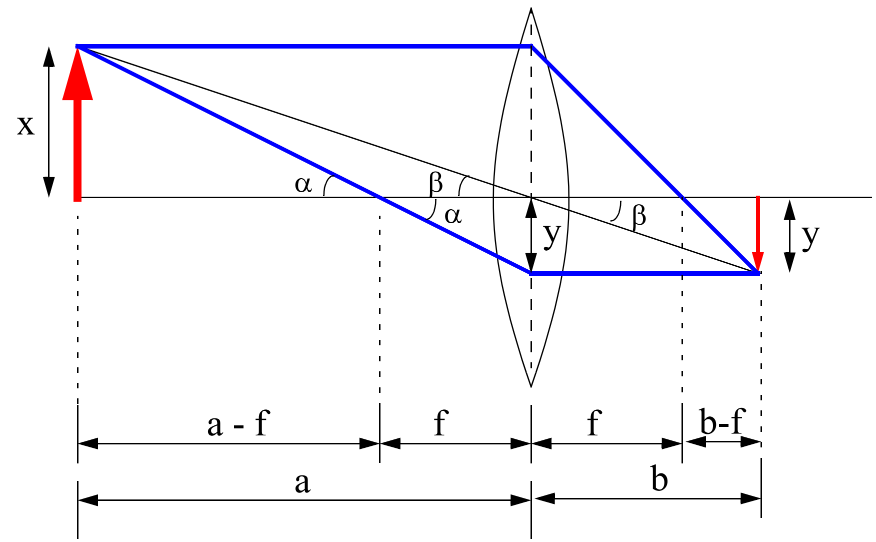
Small angles: $$\sin\theta' \approx \frac{D/2}{f} \Rightarrow F \approx \frac{f}{D} \tag{6.2}$$

Light opening = $\pi\left(\frac{D}{2}\right)^2 = \frac{\pi f^2}{4F^2}$: Light intensity decreases quadratically with increasing f/#



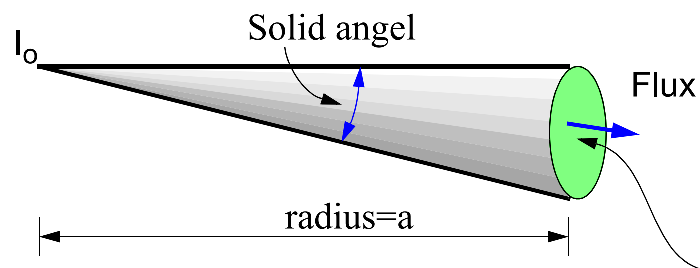
Scaling (one dimension)

$$m = \frac{y}{x} = \frac{b}{a} = \frac{b-f}{f} = \frac{f}{a-f} \rightarrow \frac{1}{f} = \frac{1}{a} + \frac{1}{b} \quad \textbf{(6.3)}$$



Flux through a spheric area is given by the intensity I_o, multiplied width the solid angle. Flux from an object with area A_o becomes proportional with I_oA_o.

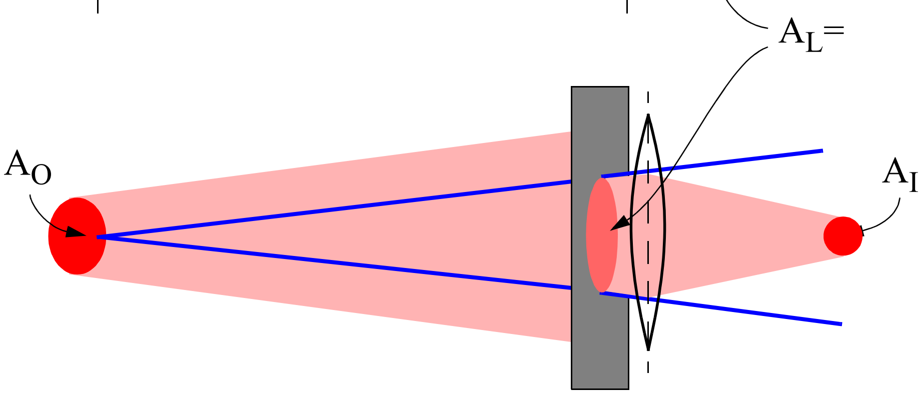
The solid angle is given by spheric area divided by the square of radius a of the sphere.
Small angles: area ~ aperture.



Intensity at the image plane, the sensor, can be written as

$$I_i = I_oA_o\frac{A_L}{a^2}\frac{1}{A_I} \approx I_o\frac{\pi D^2}{4}\frac{1}{a^2m^2} = \frac{I_o\pi D^2}{4(1+m)^2f^2}$$

Where D is the diameter of the aperture.
$m^2 = A_i / A_o$ is the ratio image area to the object area.

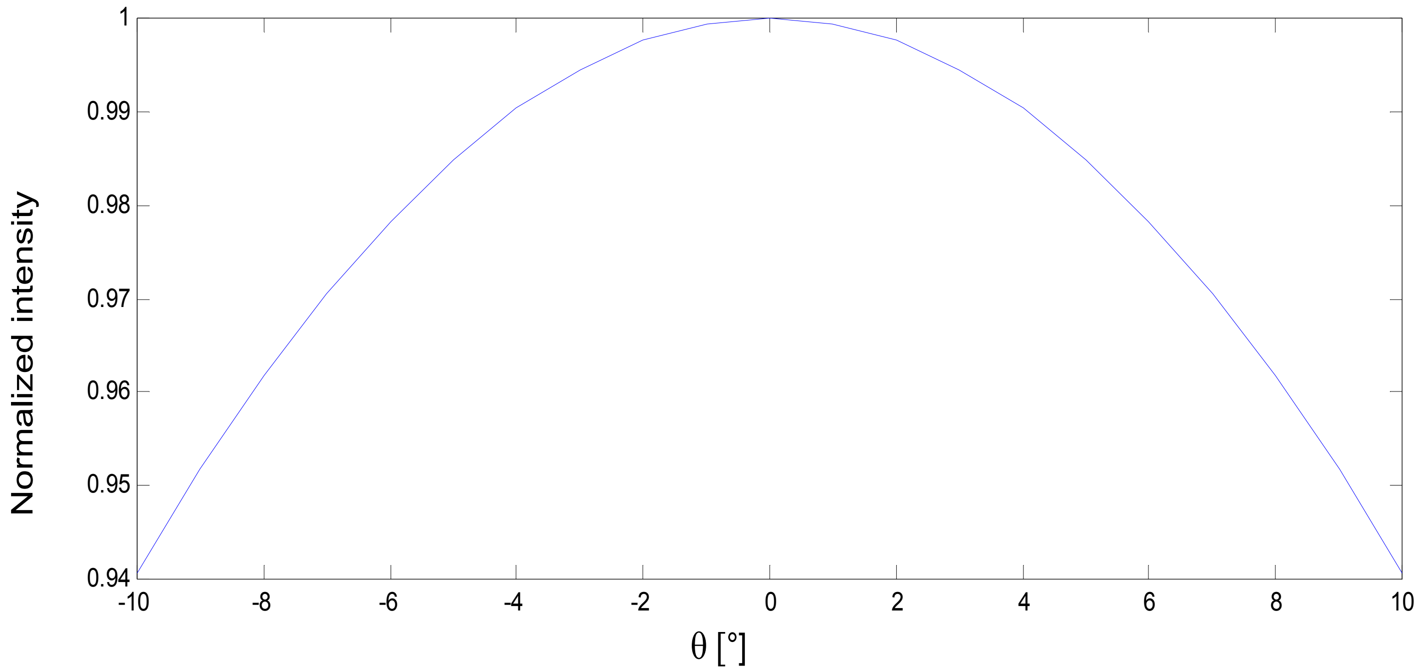


Typically, m << 1 and the illuminance at the image plane can be written as

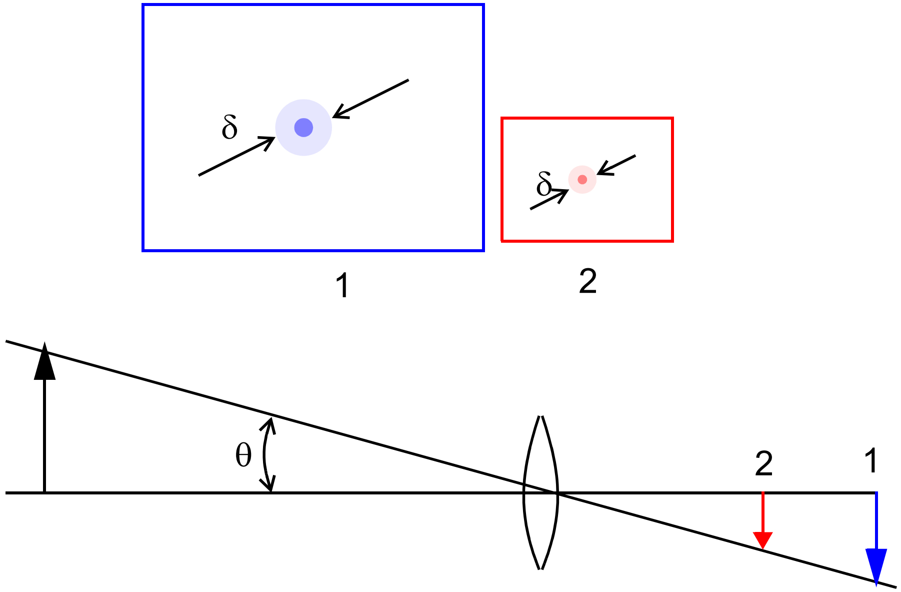
$$I_i \approx I_o\frac{\pi D^2}{4f^2} = I_o\frac{\pi}{4f^2/D^2} = I_o\frac{\pi}{4(F)^2} \quad \textbf{(6.4)}$$

Uniformity

The light will generally decrease form the centre to the periphery and follow a $\cos^4\theta$ shape (this is compensated in some lenses).



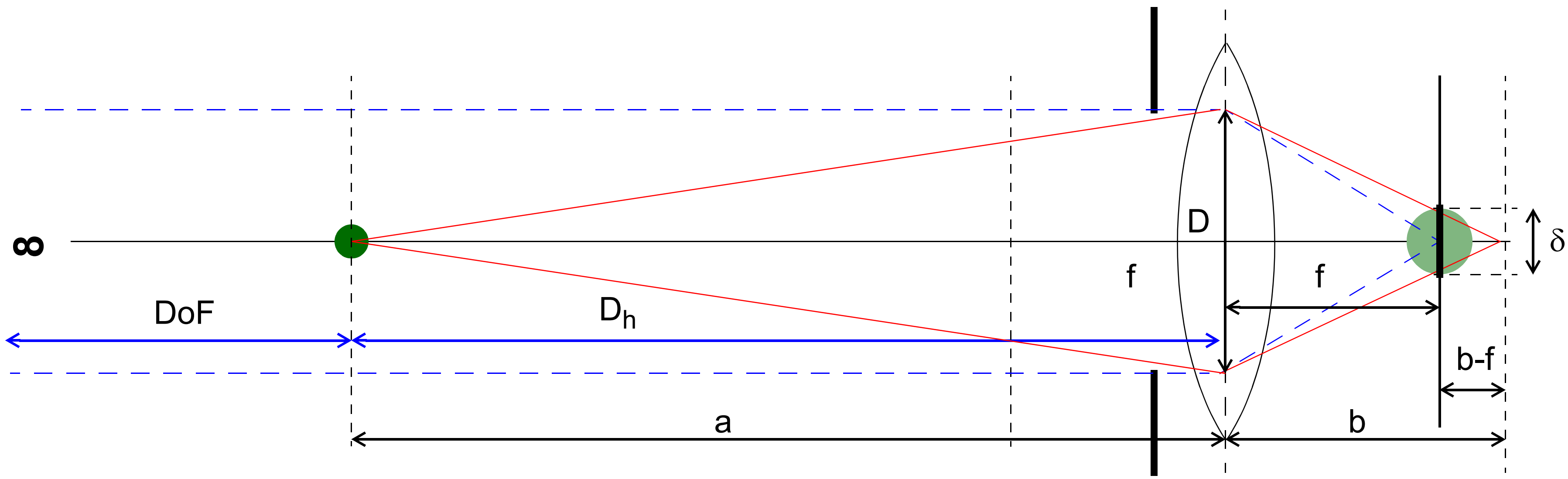
Circle of confusion and scaling with sensor size.



δ "Circle of confusion": Largest blur circle that will be perceived by the human eye as a point when viewed at a distance of 25 cm.

To keep the field of view (θ) constant, the focal length is reduces with the optical format (size of image plate).

Depth of Field



From triangles above and equation (6.3), D_h = a, and equation (6.2)

$$\frac{\delta}{D} = \frac{b-f}{b} = \frac{f}{D_h - f}$$

gives the "Hyper focal length" :

$$D_h = \frac{D}{\delta} f\left(1 + \frac{\delta}{D}\right) \approx \frac{D}{\delta} f = \frac{f^2}{F\delta} \tag{6.5}$$

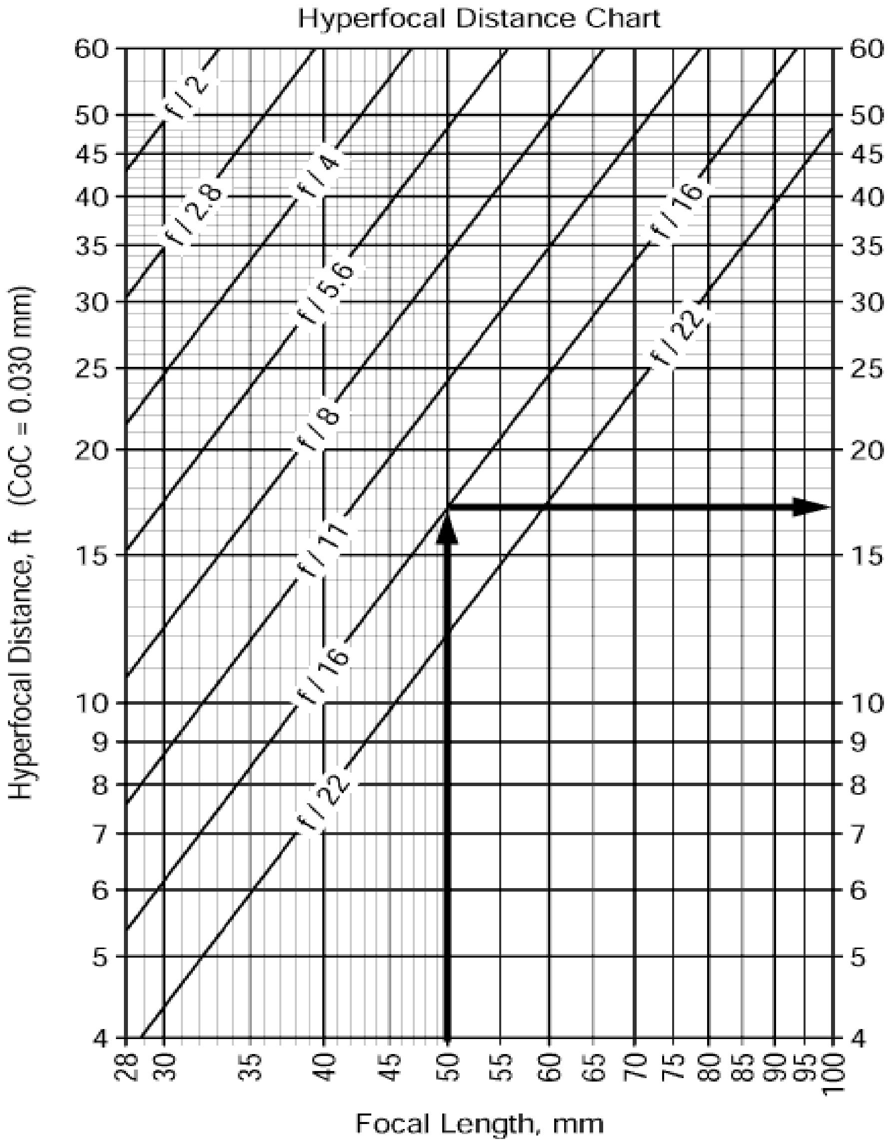
Δ= b-f becomes "Depth of focus":

$$\Delta = \frac{\delta}{D} b \approx \frac{\delta}{f} Ff = \delta F \tag{6.6}$$

Examples

TABLE 2.1
Comparison of Depth between 35-mm Film Camera and DSCs (F2.8)

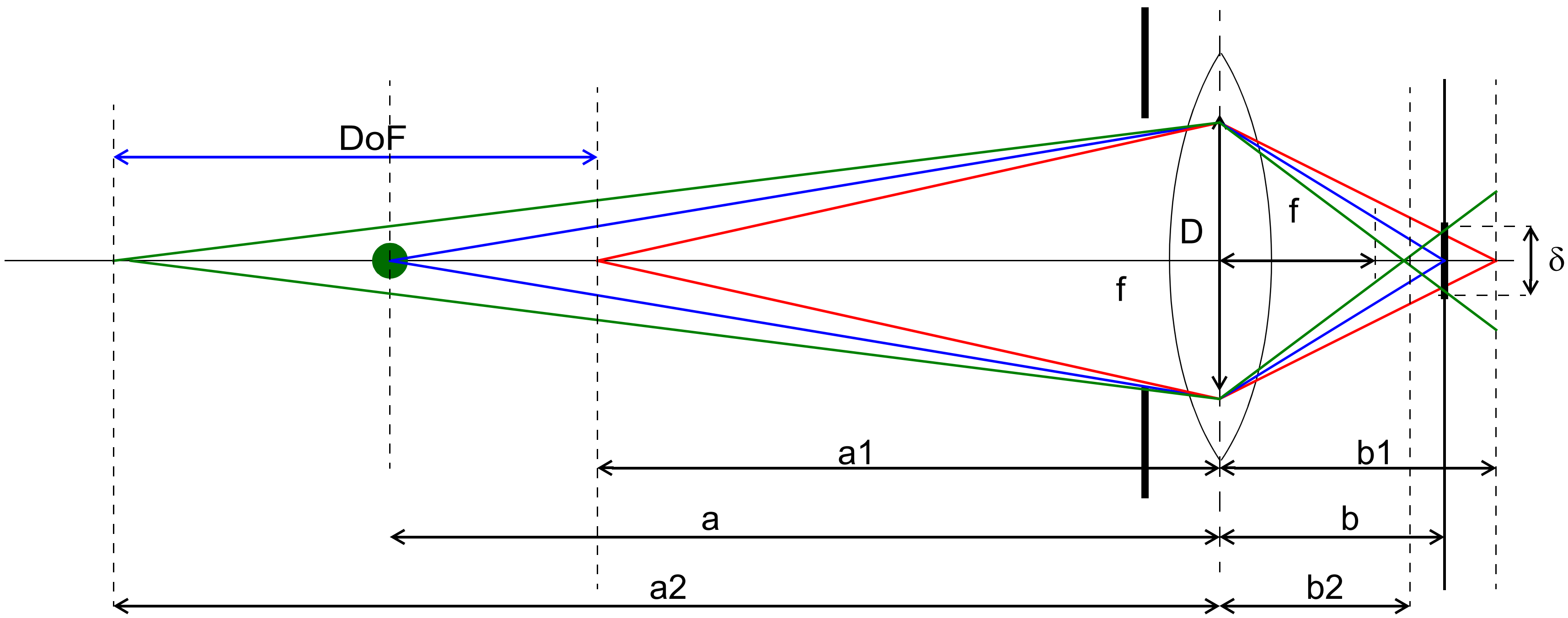
	f	No. pixels	Pixel pitch	Image circle	δ	Dh	Δ
35-mm film	**38 mm**			**ø 43.27 mm**	**35 µm**	**14.7 m**	**98 µm**
Type 1/1.8	7.8 mm	4 Mpixels	3.125 µm	ø 8.9 mm	6.3–7.5 µm	3.4–2.9 m	17–21 µm
Type 1/2.7	5.8 mm	3 Mpixels	2.575 µm	ø 6.6 mm	5.2–6.2 µm	2.3–1.9 m	14–17 µm



50mm lens,
f/16

hyperfocal
distance =
17 feet

Depth of Field



Using the same technique as above, it can be shown that Depth of Field can be written as:

$$a2 - a1 = \frac{\frac{2f^2}{\delta F}a(a-f)}{\left(\frac{2f^2}{\delta F}\right)^2 - (a-f)^2} \quad \textbf{(6.7)}$$

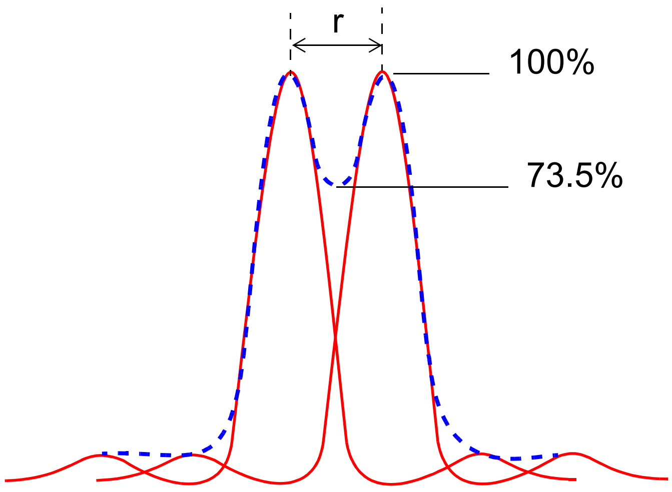
Diffraction

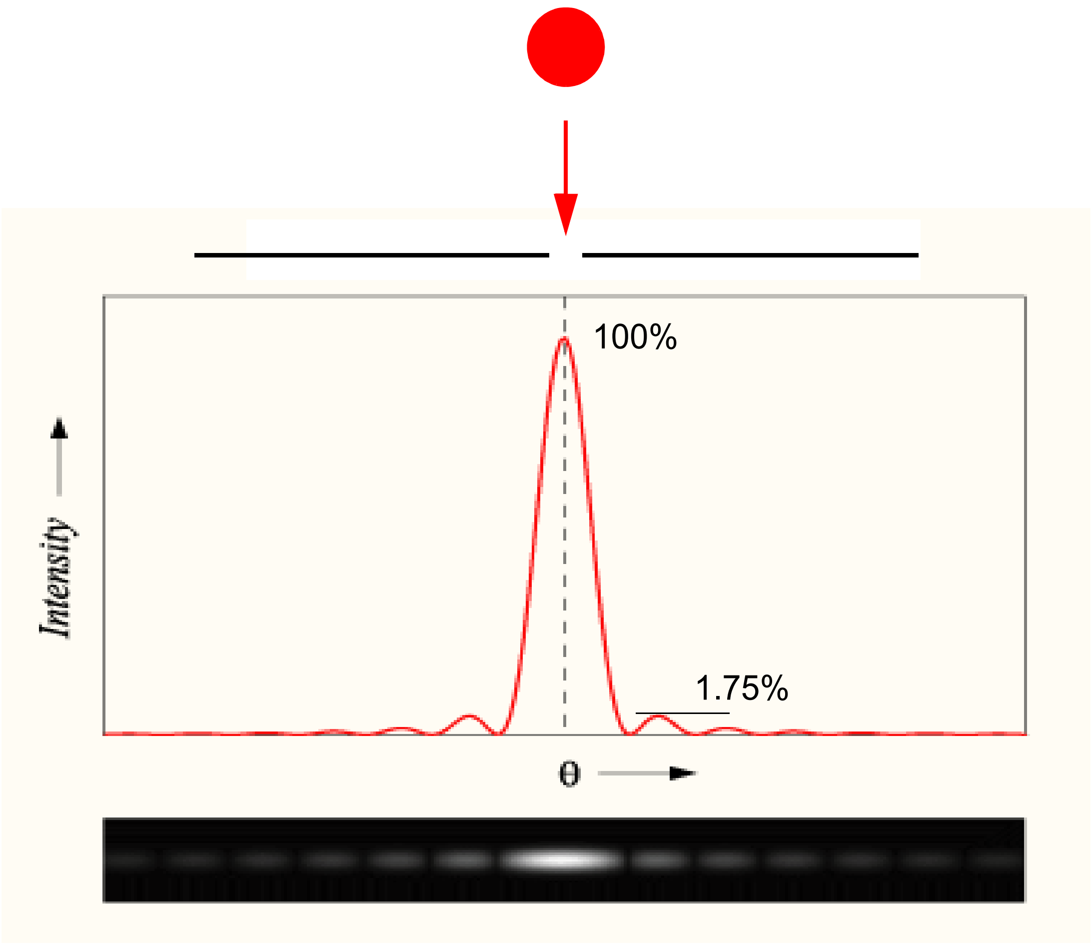
Even optics without aberrations and smear will be a source of diffraction due to the aperture.

Interference around small light points called airy discs.

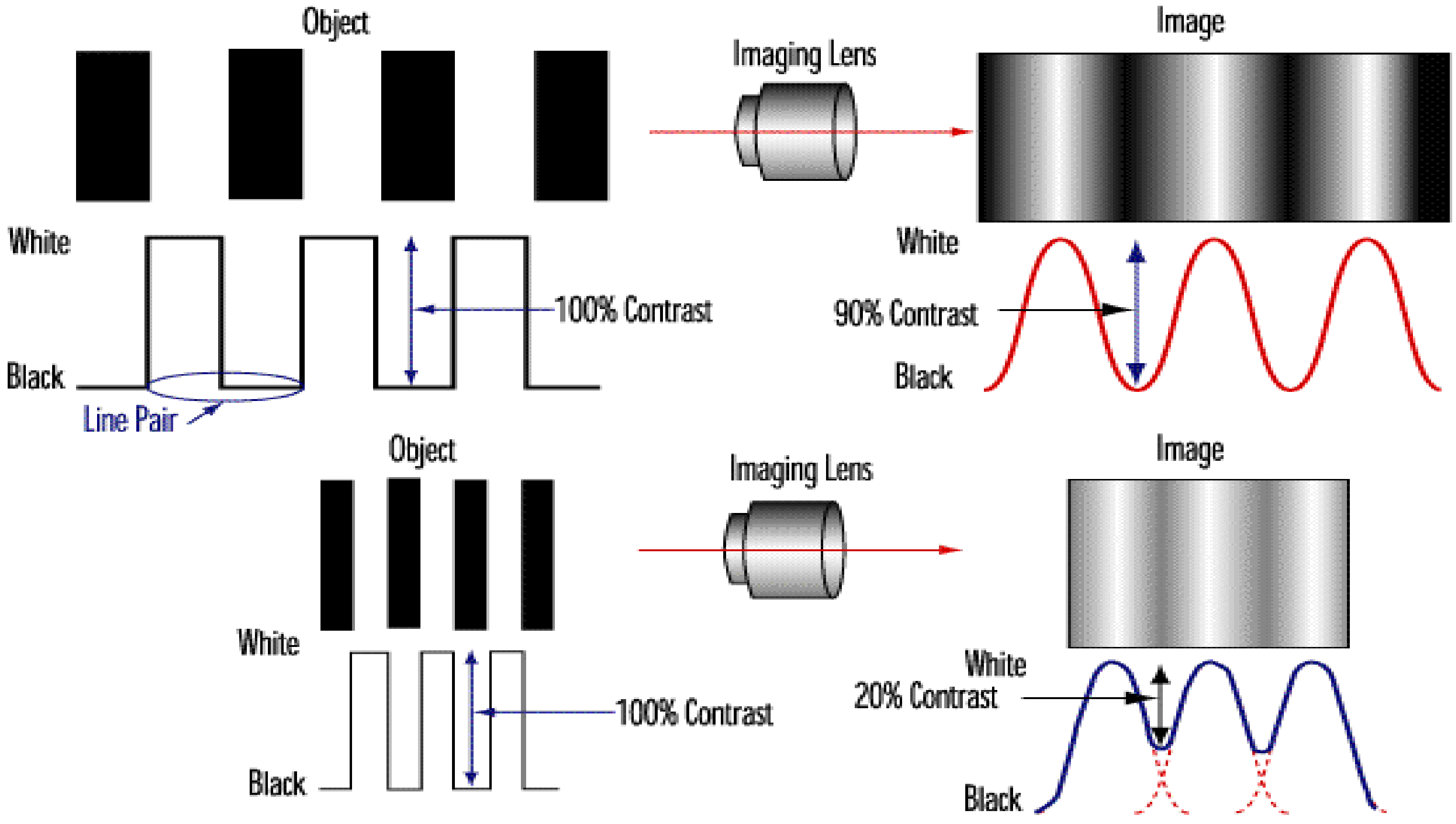
Radius of the first dark circle is the Rayleigh criterion for resolution, the separation of two points.

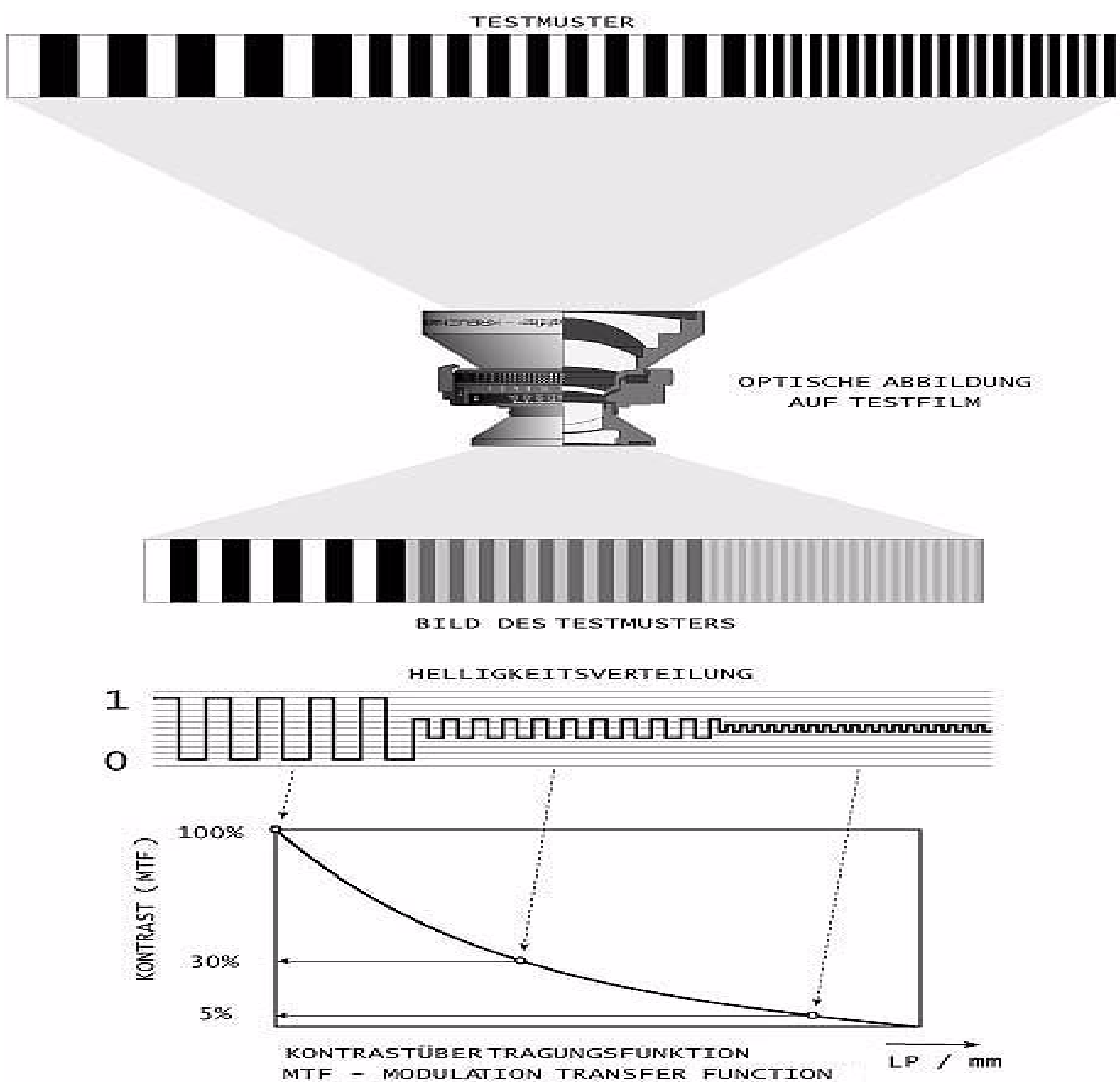
$$r = 1,22\lambda f/\# \quad \textbf{(6.8)}$$





MTF
MODULATED TRANSFER FUNCTION

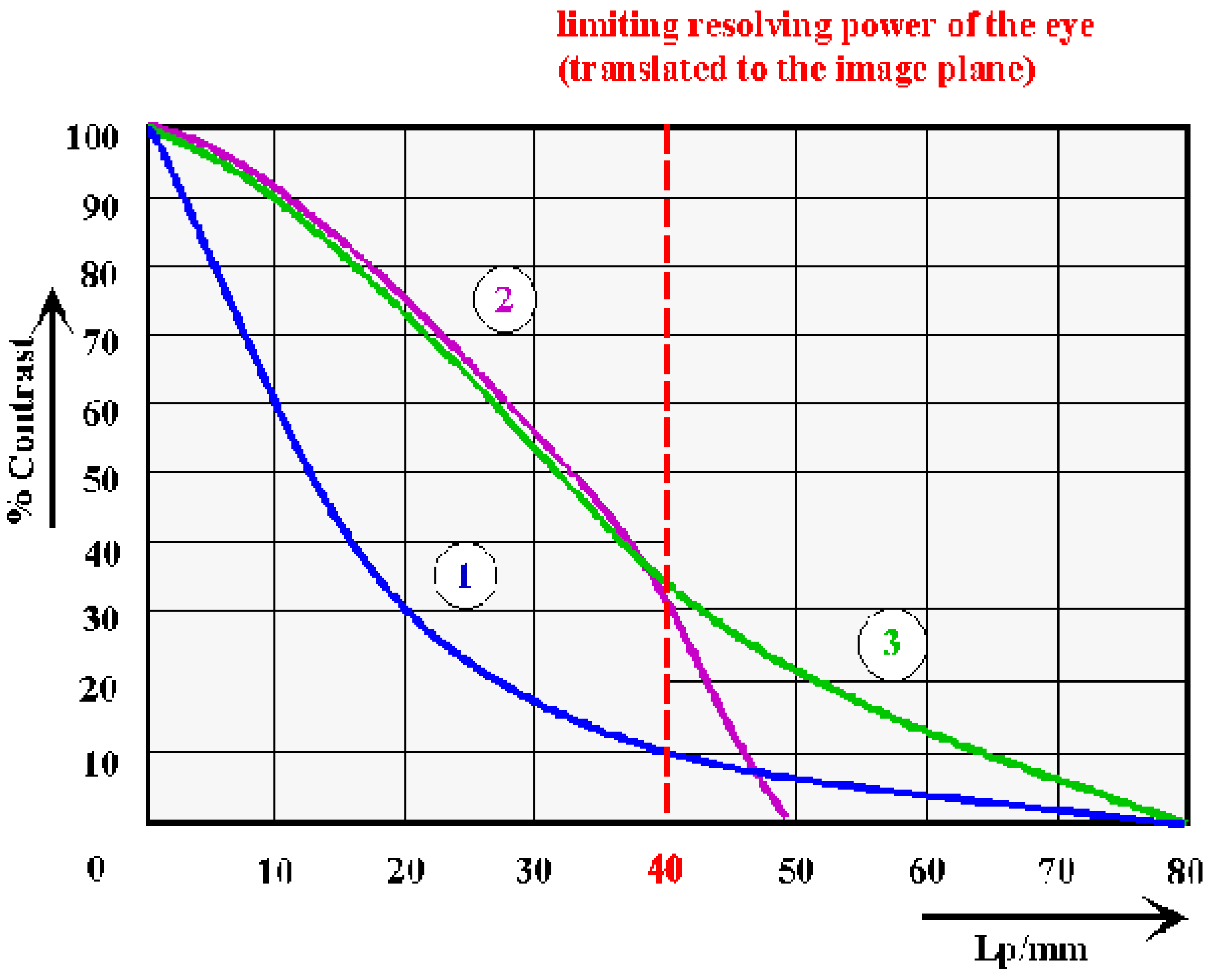

Object
Imaging Lens
Image
White
100% Contrast
Black
Line Pair
White
90% Contrast
Black
Object
Imaging Lens
Image
White
100% Contrast
Black
White
20% Contrast
Black


TESTMUSTER
OPTISCHE ABBILDUNG
AUF TESTFILM
BILD DES TESTMUSTERS
HELLIGKEITSVERTEILUNG
1
0
100%
30%
5%
KONTRAST (MTF)
KONTRASTÜBERTRAGUNGSFUNKTION
MTF - MODULATION TRANSFER FUNCTION
LP / mm

MTF of the optics can have low contrast at intermediate spatial frequencies, but still high resolution (1),

or high contrast at intermediate spatial frequencies, but low resolution (2),

or both high contrast and high resolution (3).

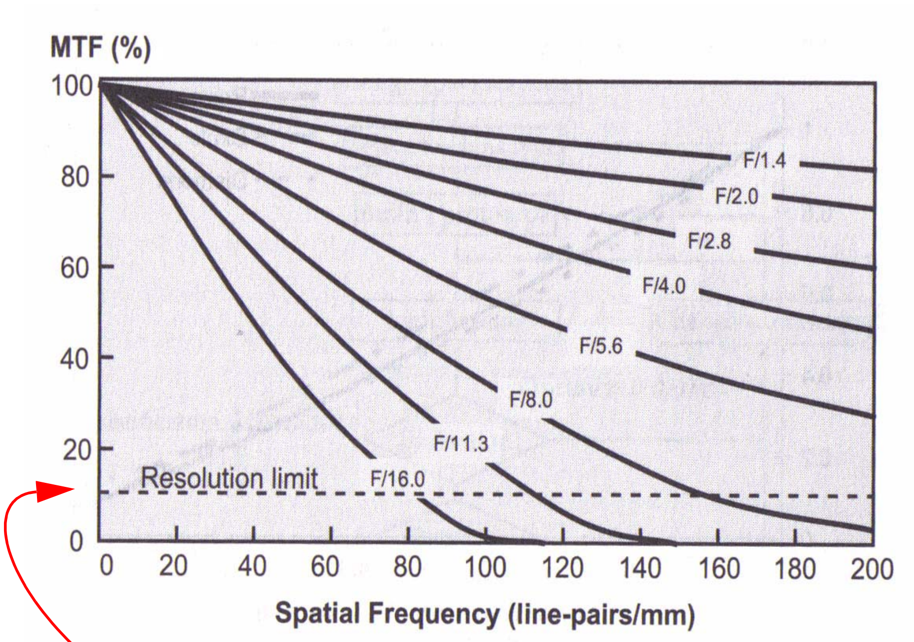


$$\mathrm{MTF}(\nu) = \frac{2}{\pi}\left(\mathbf{acos}(\lambda F\nu) - \lambda F\nu\sqrt{1-(\lambda F\nu)^2}\right) \qquad \mathbf{(6.9)}$$

Using (6.8), and $\nu = \frac{1}{r} = \frac{1}{1,22\lambda F}$,

Rayleigh limit can be expressed by MTF:

$$\mathrm{MTF}\left(\frac{1}{r}\right) = 0,0894$$



MTF (1/r) = 9%

Ref.: Nakamura

PIXEL PITCH ARRAY SIZE AND RESOLUTION

Optical format

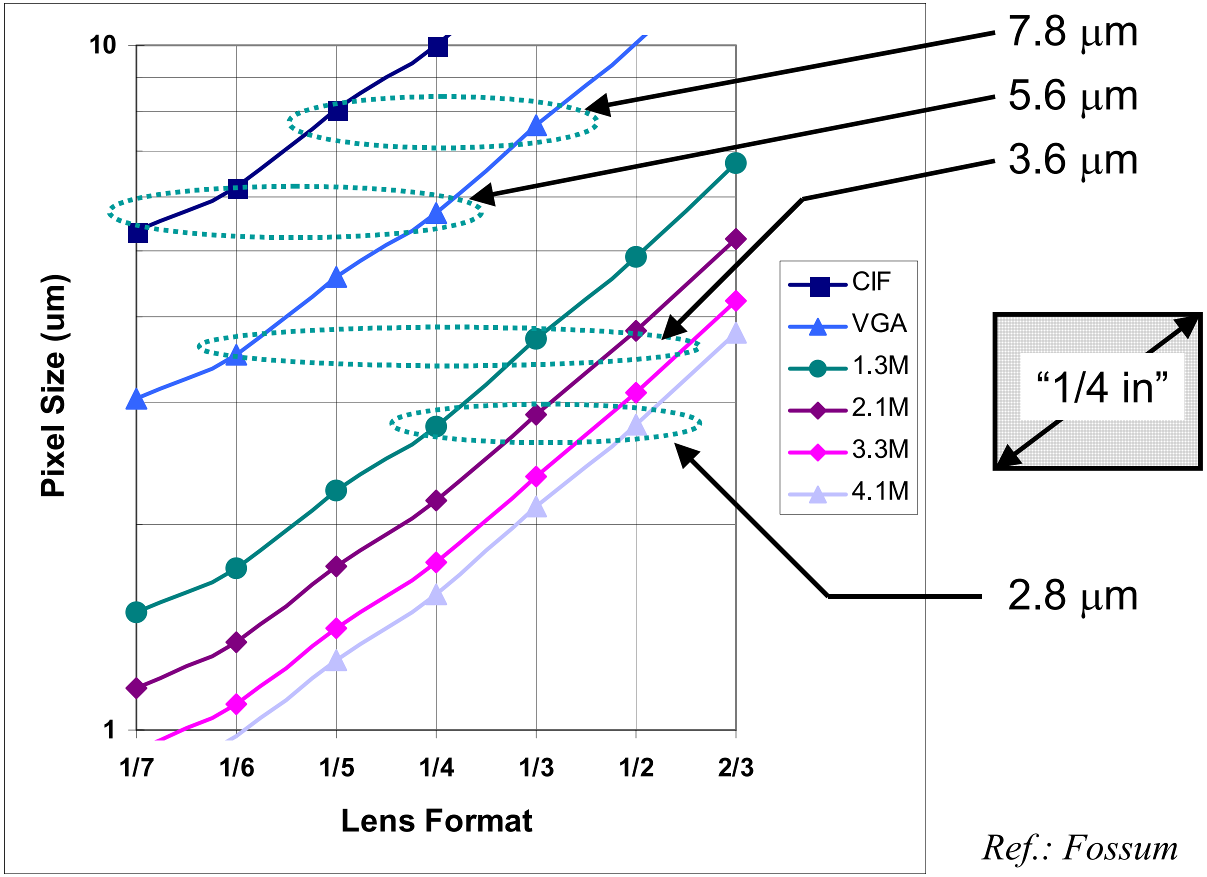
- Standard lens sizes are measured in inches, i.g. 2/3”, 1/2”, 1/3”, 1/4” (kind of diameter)
- The size has influence on the light sensitivity (responsivity), and cost.
- Sensor diagonal is less than the lens diameter
- Example: The format 1/3” (8.5 mm) need a sensor diagonal of 6.1 mm

Optical Format	Sensor diagonal
1/7 inch (=3.63 mm)	**~2.3 mm**
1/6 inch (=4.23 mm)	**~2.7 mm**
1/5 inch (=5.08 mm)	**~3.2 mm**
1/4 inch (=6.35 mm)	**~4 mm**
1/3 inch (=8.47 mm)	**~6 mm**
1/2 inch (=12.7 mm)	**~8 mm**
2/3 inch (=16.9 mm)	**~11 mm**
1 inch (=25.4 mm)	**~20 mm**

Ref.: Nakamura

Optical format (cont.)

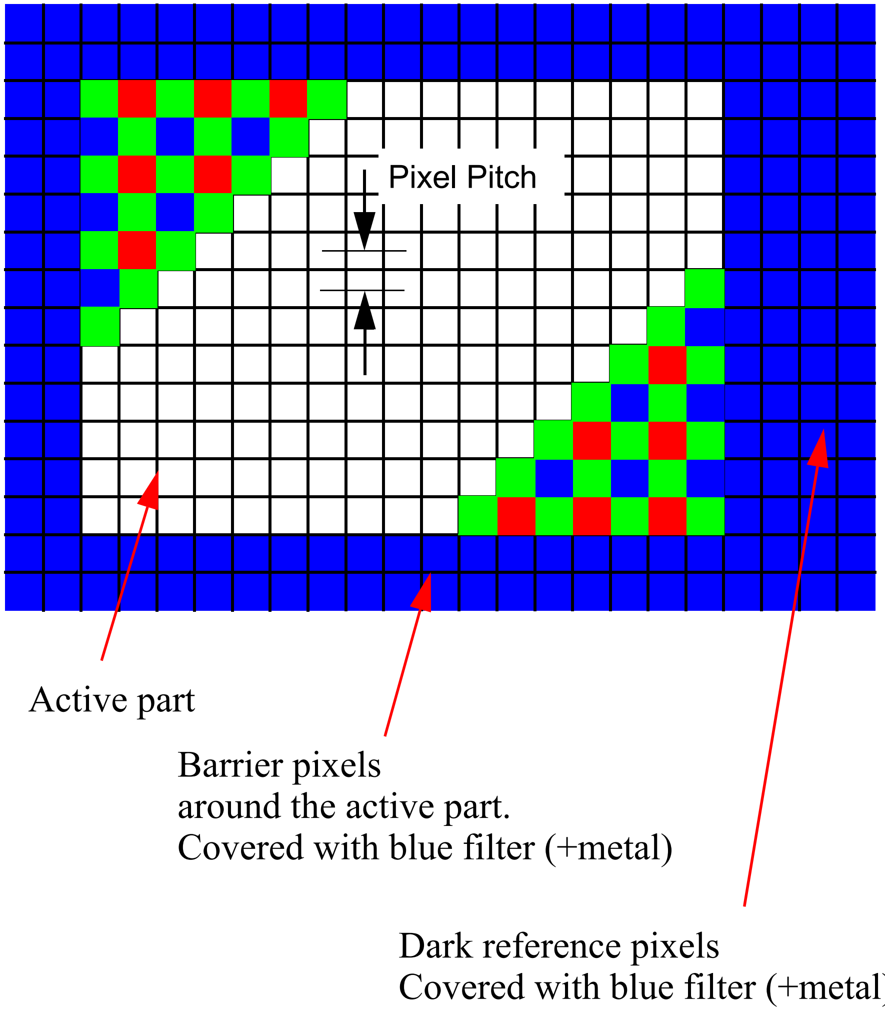
The figure shows the connection between pixel size, array format, and optical format.



Ref.: Fossum

Array formats

CIF (Common Intermediate Format):	352H x 288V
QCIF (Quarter CIF):	176H x 144V
VGA (Video Graphics Array):	640H x 489V
QVGA (Quarter VGA):	320H x 240V
QQVGA:	160H x 120V
SVGA (Super VGA):	800H x 600V
XGA (Extended Graphics Array):	1024H x 768V
SXGA:	1280H x 1024V
1.3M	1280H x 1024V
2.1M	1680H x 1248V
3.3M	2088H x 1550V
4.1M	2310H x 1732V
~12M	~¨4000H x 3000V

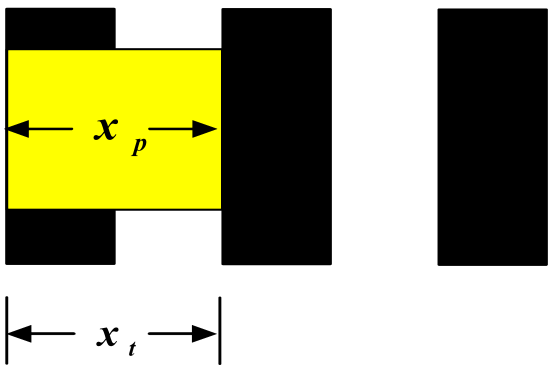


Ref.: Fossum

Regular pixel array. The image is spatially sampled

$$f_T = \frac{1}{x_T} = \text{spatial frequency of target}$$

$$f_s = \frac{1}{\Delta x_p} = \text{spatial frequency of sensor}$$



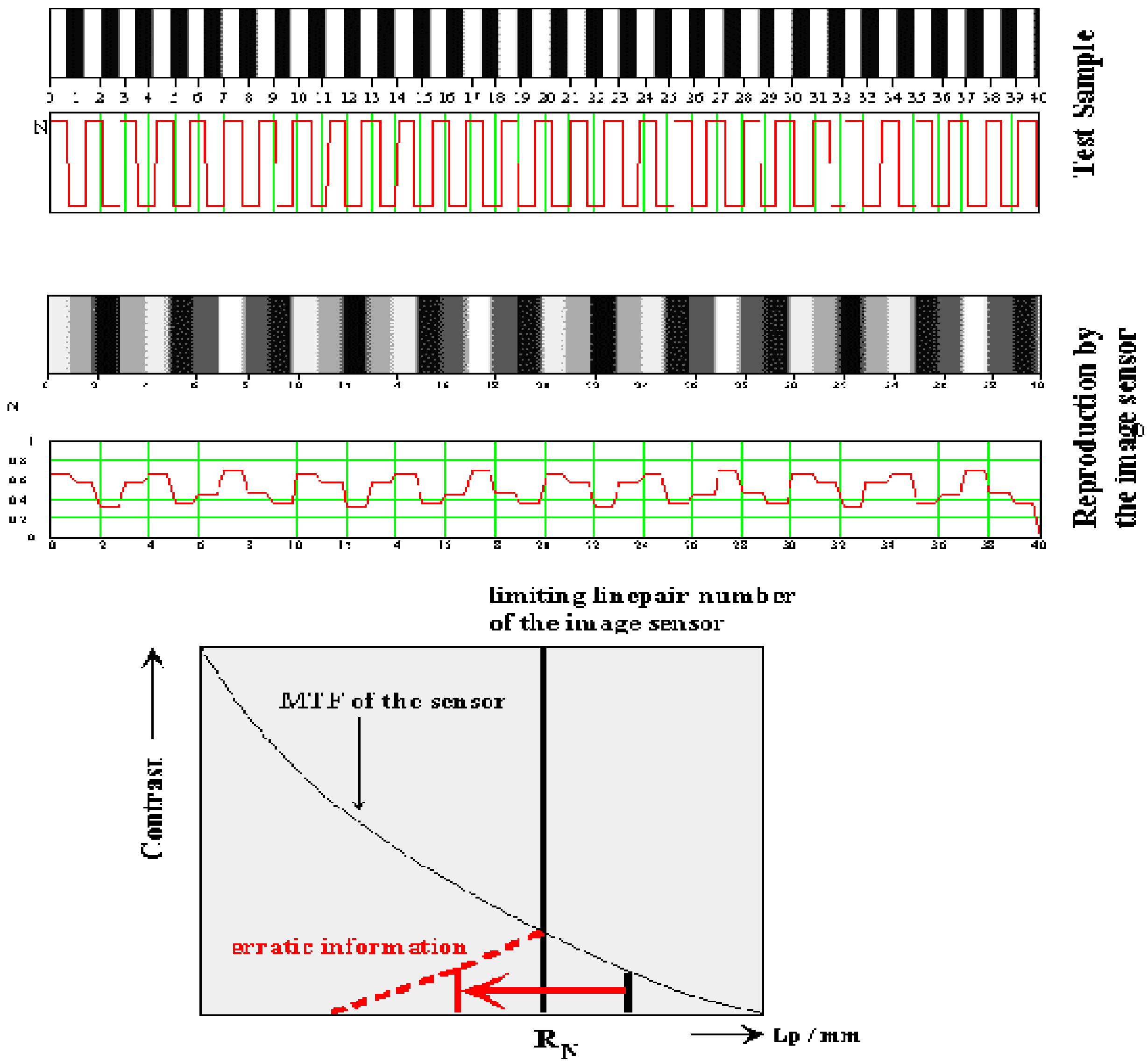
$$f_s = \frac{1}{2} f_N$$

(under sampled)

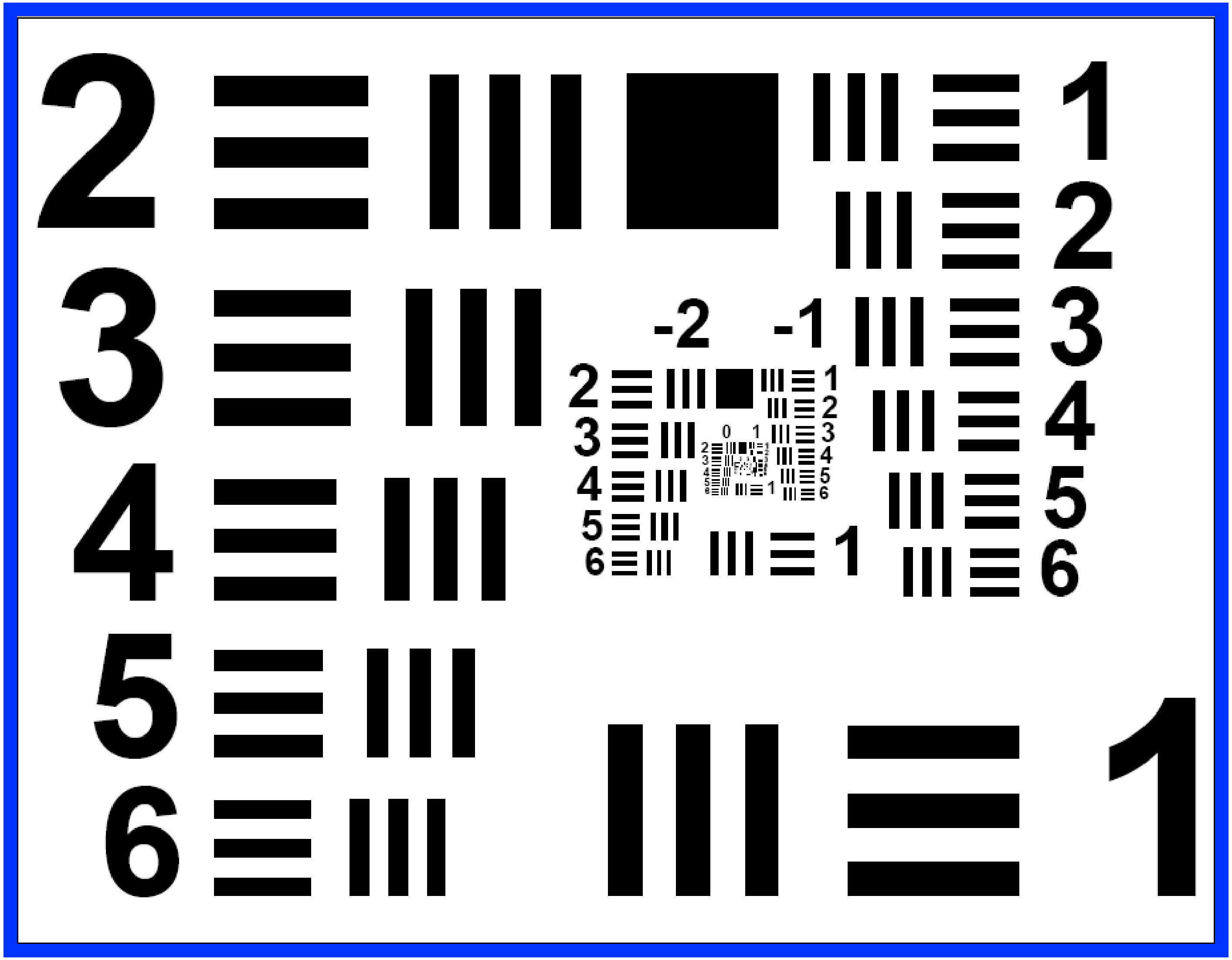
$$f_s = f_N$$

(correctly sampled)

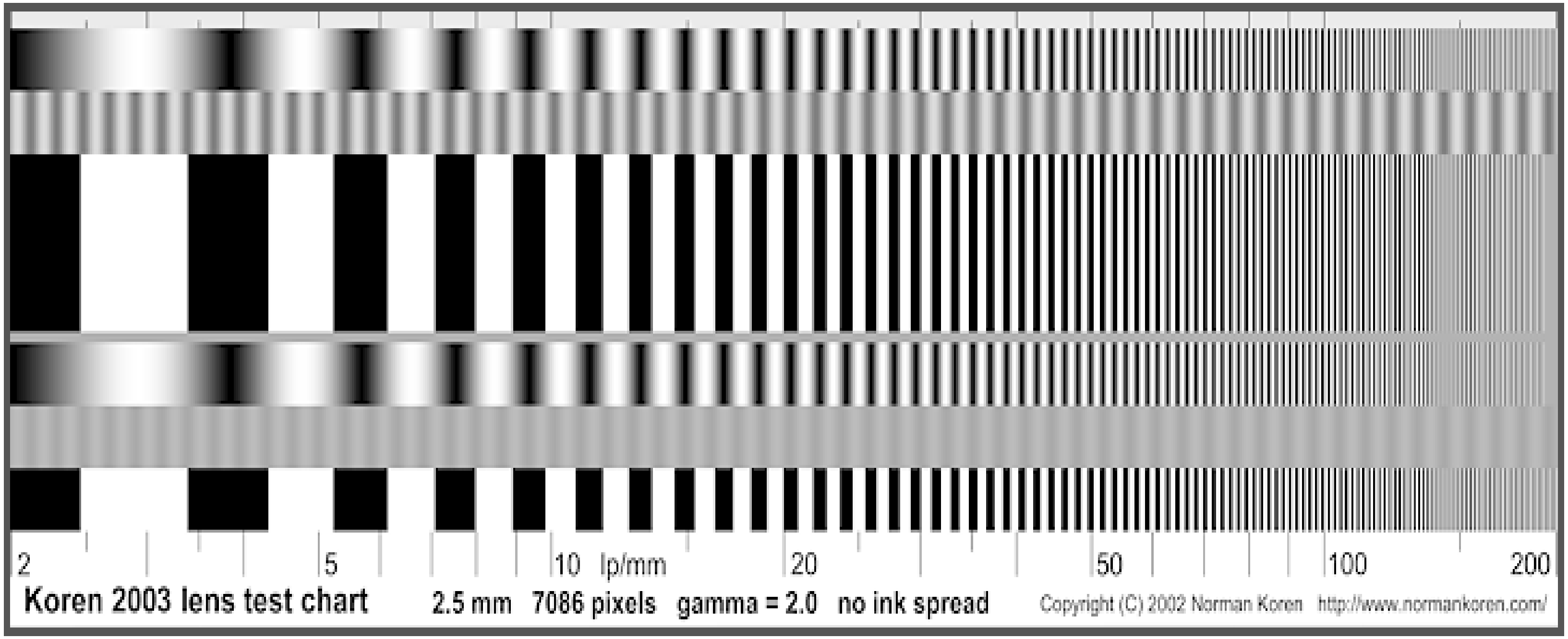
Aliasing


Test Sample
Reproduction by
the image sensor
limiting linepair number
of the image sensor
MTF of the sensor
Contrast
erratic information
R_N
Lp / mm

Test pattern



Test pattern



References

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Taylor & Francis

Fossum

See INF 5440 introductory lecture