COLOUR THEORY

Main ref.: Nakamura
**Response of the eye**

- The human eye has three kinds of cones S, M, and L which is sensitive to different wavelength ranges: Blue, green, red. Colour view.
- Rods has higher sensitivity but for one range only, slightly below the green range. Night vision without colour.

- ![Graph showing normalized absorbance over wavelength](image)

Ref.: [www.wikipedia.org](http://www.wikipedia.org)
Adaptive and Subtractive Colour Mixing

Figure 3. Additive colour mixtures of blue, green and red to produce cyan, magenta, yellow and white.

Figure 4. Subtractive colour mixtures of cyan, magenta and yellow to produce blue, green and red.

Ref.: http://webvision.med.utah.edu/KallColor.html
Colour Theory

Colour Matching Functions

How to quantify colours

- $X(\lambda)$, $Y(\lambda)$, and $Z(\lambda)$
- CIE 1931 standard
  (Commission Internationale de l'Eclairage)
  International Commission on Illumination
- The Colour space is associated with the colour sensitivity of the human eye.

Ref. Nakamura

Ref. Wikipedia
Colour Theory

Colour Matching Functions (cont.)

Tristimulus values: X, Y, Z is obtained by weighting the source intensity \( L(\lambda) \) and the illuminated object’s reflectance \( R(\lambda) \) with the colour matching functions \( x(\lambda), y(\lambda) \) and \( z(\lambda) \).

\[
X = \int L(\lambda)R(\lambda)x(\lambda)d\lambda \\
Y = \int L(\lambda)R(\lambda)y(\lambda)d\lambda \\
Z = \int L(\lambda)R(\lambda)z(\lambda)d\lambda
\]  
(7.1)

Luminance: \( Y \)

Chrominance: \( x, y \) values is represented in a 2 dimensional space.

\[
x = \frac{X}{X + Y + Z} \quad y = \frac{Y}{X + Y + Z}
\]  
(7.2)

Converting back to \( X, Z \)

\[
X = \frac{Y}{y}x \quad Z = \frac{Y}{y}(1 - x - y)
\]  
(7.3)

Ref.: efg’a Computer Lab

CIE 1931 Chromaticity Diagram

egf’a Computer Lab

www.efg2.com/llab
Improved geometric uniformity is achieved with the standard CIE 1976: L*ab* and L*u*v*.
Developed for gray background (L* = 50)

Common L*:

\[
L^* = \begin{cases} 
116 \left( \frac{Y}{Y_0} \right)^{1/3} - 16 & \frac{Y}{Y_0} > 0.008856 \\
903.9 \left( \frac{Y}{Y_0} \right) & \frac{Y}{Y_0} \leq 0.008856 
\end{cases} \quad (7.4)
\]

\[
u^* = 13L^*(u' - u'_0) \quad v^* = 13L^*(v' - v'_0) \quad (7.5)
\]

where

\[
u' = \frac{4X}{X + 15Y + 3Z} \quad \nu' = \frac{9Y}{X + 15Y + 3Z} \quad (7.6)
\]

Index 0 represents tristimulus values for white

\[
a^* = 500(X_n - Y_n) \quad b^* = 200(Y_n - Z_n) \quad (7.7)
\]

where

\[
X_n = \begin{cases} 
\left( \frac{X}{X_0} \right)^{1/3} & \frac{X}{X_0} > 0.008856 \\
7.787 \left( \frac{X}{X_0} \right) + \frac{16}{116} & \frac{X}{X_0} \leq 0.008856 
\end{cases} \quad (7.8)
\]

\[
Y_n = \begin{cases} 
\left( \frac{Y}{Y_0} \right)^{1/3} & \frac{Y}{Y_0} > 0.008856 \\
7.787 \left( \frac{Y}{Y_0} \right) + \frac{16}{116} & \frac{Y}{Y_0} \leq 0.008856 
\end{cases} \quad (7.9)
\]

\[
Z_n = \begin{cases} 
\left( \frac{Z}{Z_0} \right)^{1/3} & \frac{Z}{Z_0} > 0.008856 \\
7.787 \left( \frac{Z}{Z_0} \right) + \frac{16}{116} & \frac{Z}{Z_0} \leq 0.008856 
\end{cases} \quad (7.10)
\]
**Colour Theory**

L*a*b* (CIE1976)

---

Ref.: efq’a Computer Lab
L* C* H* (CIECAM02)

L* as in (7.4).

\[ C_{ab}^* = \sqrt{(a^*2 + b^*2)} \quad h_{ab} = \frac{180^\circ}{\pi} \arctan \left( \frac{b^*}{a^*} \right) \]

(7.11)

Ref.: Wikipedia
Colour temperature of light sources

[Graph showing the colour temperature of light sources with various light sources and their corresponding colour temperatures.]
Colour theory

Colour temperature of light sources (cont.)

Equal colour temperature is not the same as equal spectrum
Colour Rendering index (CRI)

A measure on a light source capability to reproduce colour.

- 100 is best, 0 is worst
- The light source is measured against a reference, a filament lamp (blackbody radiation) with CRI=100.
- A standardized set of colours are illuminated and compared.
- The reference is daylight for colour temperatures above 5000 °K.

Average colour rendering index $R_a$ is calculated according to formula

$$R_a = \frac{1}{8} \sum_{i=1}^{8} (100 - 4, 6\Delta E_i)$$

Where $\Delta E_i$ is the colour deviation in a given colour space, e.g. 1964 L*U*V*

$$\Delta E_i = \sqrt{(L_i - L_{i0})^2 + (U_i - U_{i0})^2 + (V_i - V_{i0})^2}$$

Index i gives the colour in the standard set, index 0 represents the reference source.
Colour Rendering index (forts.)

<table>
<thead>
<tr>
<th>Sample light source</th>
<th>F2</th>
<th>F7</th>
<th>F11</th>
<th>A</th>
<th>D65</th>
<th>D50</th>
<th>D55</th>
</tr>
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<tbody>
<tr>
<td>Chromaticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>0.3721</td>
<td>0.3129</td>
<td>0.3805</td>
<td>0.4476</td>
<td>0.3127</td>
<td>0.3457</td>
<td>0.3324</td>
</tr>
<tr>
<td>y</td>
<td>0.3751</td>
<td>0.3292</td>
<td>0.3769</td>
<td>0.4074</td>
<td>0.3290</td>
<td>0.3585</td>
<td>0.3474</td>
</tr>
<tr>
<td>Reference light source (P: black body; D: daylight)</td>
<td>P</td>
<td>D</td>
<td>P</td>
<td>P</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>Correlated color temperature</td>
<td>4200</td>
<td>6500</td>
<td>4000</td>
<td>2856</td>
<td>6500</td>
<td>5000</td>
<td>5500</td>
</tr>
<tr>
<td>Average color rendering index</td>
<td>Ra</td>
<td>64</td>
<td>90</td>
<td>83</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Special color rendering index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>7.5 R 6/4</td>
<td>56</td>
<td>89</td>
<td>98</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R2</td>
<td>5 Y 6/4</td>
<td>77</td>
<td>92</td>
<td>93</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R3</td>
<td>5 GY 6/8</td>
<td>90</td>
<td>91</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R4</td>
<td>2.5 G 6/6</td>
<td>57</td>
<td>91</td>
<td>88</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R5</td>
<td>10 BG 6/4</td>
<td>59</td>
<td>90</td>
<td>87</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R6</td>
<td>5 PB 6/8</td>
<td>67</td>
<td>89</td>
<td>77</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R7</td>
<td>2.5 P 6/8</td>
<td>74</td>
<td>93</td>
<td>89</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R8</td>
<td>10 P 6/8</td>
<td>33</td>
<td>87</td>
<td>79</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R9</td>
<td>4.5 R 4/13</td>
<td>-84</td>
<td>61</td>
<td>25</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>R10</td>
<td>5 Y 8/10</td>
<td>45</td>
<td>78</td>
<td>47</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
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<td>4.5 G 5/8</td>
<td>46</td>
<td>89</td>
<td>72</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R12</td>
<td>3 PB 3/11</td>
<td>54</td>
<td>87</td>
<td>53</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R13</td>
<td>5 YR 8/4</td>
<td>60</td>
<td>90</td>
<td>97</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R14</td>
<td>5 GY 4/4</td>
<td>94</td>
<td>94</td>
<td>67</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>R15</td>
<td>1 YR 6/4</td>
<td>47</td>
<td>88</td>
<td>96</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Camera Colour Properties

To generate tristimulus values unlikely, these have to be a linear combination of the camera’s sensitivity functions:

\[ X = c_{11}S_x + c_{12}S_y + c_{13}S_z \]
\[ Y = c_{21}S_x + c_{22}S_y + c_{23}S_z \]
\[ Z = c_{31}S_x + c_{32}S_y + c_{33}S_z \]

Called the Luther condition.

Provided the reflectance has a continuous transitions in the spectrum such that the reflected spectrum can be decomposed into three basic components, an object’s tristimulus values can be calculated using any set of sensitivity functions.

Colour chart for test is made to simulate the reflectance of real objects.

The camera colour functions can be generated from the chart. Either an analytical or recursive method.

Gretag Macbeth colour checker
Camera Colour Properties (cont.)

The target:

\[
T = \begin{bmatrix}
X_1 & \cdots & X_i & \cdots & X_n \\
Y_1 & \cdots & Y_i & \cdots & Y_n \\
Z_1 & \cdots & Z_i & \cdots & Z_n
\end{bmatrix}
\]

Estimated value:

\[
\hat{T} = \begin{bmatrix}
\hat{X}_1 & \cdots & \hat{X}_i & \cdots & \hat{X}_n \\
\hat{Y}_1 & \cdots & \hat{Y}_i & \cdots & \hat{Y}_n \\
\hat{Z}_1 & \cdots & \hat{Z}_i & \cdots & \hat{Z}_n
\end{bmatrix}
\]

Compensation

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
r_1 & \cdots & r_i & \cdots & r_n \\
g_1 & \cdots & g_i & \cdots & g_n \\
b_1 & \cdots & b_i & \cdots & b_n
\end{bmatrix} = A \cdot S
\]

Solving for the compensation matrix:

\[
A = T \cdot S^T \cdot (S \cdot S^T)^{-1}
\]
Camera Colour Properties (cont.)

Recursive method to minimize colour differences:

$$J = \sum_{i=1}^{n} w_i \Delta E(X_i, Y_i, Z_i, \hat{X}_i, \hat{Y}_i, \hat{Z}_i)$$  \hfill (7.13)

Where $\Delta E$ is the direct colour difference (e.g. $L^*a^*b^*$)

$$\Delta E_{ab^*} = \sqrt{(L_1 - L_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2}$$  \hfill (7.14)

$J$: Visible colour difference (deviation from the object’s colour)

$w_i$: Coefficient associated to each colour on the palette.
Colour Theory

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\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix}
\]

Matrix to minimize $\Delta E$
http://neuroanthropology.net/2008/09/04/colour-is-it-in-the-brain/
White act as the reference for the colours, i.e. the colour of the light source. The eye adjusts the sensitivity of cones to adapt to the lightning conditions.

The question is: What is white in the image? There are several methods.

- Scene average.
  Assuming the “world is grey” and that the average colour is the colour of the light source.

- The brightest spot is white.
  Assuming the light that is reflected by the brightest elements represents the light source.

- Gamut of the scene (colour space).
  Assuming the total spectrum has a expected distribution, analyse the spectrum from all part of the scene and move the colours towards the highest correlation between expected and measured distribution.
White Balance - Chromatic adaption

von Kries model:

Images are viewed in a background illumination different from the recording background light.
Cone response at background illumination 1, is tristimulus values $L_1, M_1, S_1$ at the state white $= L_{w1}, M_{w1}, S_{w1}$
Cone response at background illumination 2, is tristimulus values $L_2, M_2, S_2$ at the state white $= L_{w2}, M_{w2}, S_{w2}$

\[
L_2 = \frac{L_{w2}}{L_{w1}} L_1 \quad M_2 = \frac{M_{w2}}{M_{w1}} M_1 \quad S_2 = \frac{S_{w2}}{S_{w1}} S_1
\]

New $r'$ $b'$ $g'$ values is calculated from $r$ $g$ $b$ by:

\[
\begin{bmatrix}
    r' \\
    g' \\
    b'
\end{bmatrix} = A^{-1}B^{-1}
\begin{bmatrix}
    L'_{w} \\
    L_{w} \\
    0 \\
    0 \\
    M'_{w} \\
    M_{w} \\
    0 \\
    0 \\
    S'_{w} \\
    S_{w}
\end{bmatrix}
\begin{bmatrix}
    r \\
    b \\
    g \\
    L \\
    M \\
    S
\end{bmatrix}
\]

(7.15)

A: Sensor RGB -> Tristimulus values XYZ
B: Tristimulus values XYZ -> cone response LMS.
Subjective properties of the eye ensures that an object’s colour stays nearly unchanged at varying light conditions (needed for recognition of objects). It is however required that the spectrum of the light has a certain width, that Ra (average colour rendering index) is high.

A simple method is to use (7.15) with the adaption of B to a standard light source.

http://en.wikipedia.org/wiki/colour_constancy
Perception of colour depend on the colour of the background

The pink sheet in top image looks more like white in the bottom image.

http://en.wikipedia.org/wiki/colour_constancy
A is at the same grey level as B?

http://en.wikipedia.org/wiki/colour_constancy
http://en.wikipedia.org/wiki/colour_constancy
Colour Interpolation

**Sampling of Bayer pattern CFA (Colour Filter Array)**

Red and blue distance = 2. Green distance = $\sqrt{2}$. Nyquist frequency is 1.4x higher for green, in the directions 45° and 135°
**Bilinear Interpolation**

Interpolation: Insert a new point with values based on averaging of the neighbouring points, i.e. spatial low pass filtering of the data.

Position 1:
\[
G(x_1,y_1)|_R = \frac{R(x_1,y_1+1) + R(x_1,y_1-1)}{2} \\
G(x_1,y_1)|_B = \frac{B(x_1+1,y_1) + B(x_1-1,y_1)}{2}
\]

Position 2:
\[
B(x_2,y_2)|_G = \frac{G(x_2,y_2+1) + G(x_2,y_2-1) + G(x_2+1,y_2) + G(x_2-1,y_2)}{4} \\
B(x_2,y_2)|_R = \frac{G(x_2+1,y_2+1) + G(x_2+1,y_2-1) + G(x_2-1,y_2+1) + G(x_2-1,y_2-1)}{4}
\]

Corresponding for red pixel.


Colour Theory

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Colour correction

\[
\begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix} = \begin{bmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_3
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]  
(7.16)

The coefficients at the diagonal adjust the relation between R, G and B, white balance, directly. The matrix elements outside the diagonal can be used to compensate for colour crosstalk.

As the eye has lower spatial response for chrominance than for luminance, separation of these is advantageous. This can be done by converting RGB to $YCbCr$. This is especially useful in data compression.

The ITU standard:

\[
\begin{bmatrix}
Y \\
C_b \\
C_r
\end{bmatrix} = \begin{bmatrix}
0.2988 & 0.5869 & 0.1143 \\
-0.1689 & -0.3311 & 0.5000 \\
0.5000 & -0.4189 & -0.0811
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix},
\begin{bmatrix}
Y \\
C_b \\
C_r
\end{bmatrix} = \begin{bmatrix}
1.0000 & 0.00000 & 1.40200 \\
1.0000 & -0.34410 & -0.71410 \\
1.0000 & 1.77200 & -0.00015
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
(7.17)

Fixed point representation:

\[
\begin{bmatrix}
Y \\
C_b \\
C_r
\end{bmatrix} = \begin{bmatrix}
306 & 601 & 117 \\
-173 & -339 & 512 \\
512 & -429 & -83
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
(7.18)
Colour Reproduction

Method: Recording of Macbeth colour checker with a well defined reflectance for each colour, converting to Chrominance (and luminance using tristimulus functions. Quantifying deviation from expected Chrominance values for each colour.

Our perception of colour depends on the illuminance, background light, and some properties of our vision system that are not caught by the tristimulus functions.

Therefore the latest adjustment is done by observing the chart plus some typical objects.

COLOR FIGURE 10.8 Test chart for measuring color reproduction: (a) color bar chart; (b) Macbeth’s color chart.
Colour Theory

Gamma Correction and Hue Correction

Must take into account the non-linear response, the form of $y = x^\gamma$. For example, the Cathode Ray Tube (CRT) has a typical gamma of 0.45. The transform from one response to another is the gamma correction. Can be done in both the RGB space or in the YC_bC_r space.
References:

Nakamura

Image Sensors and Signal Processing for Digital Cameras
Junichi Nakamura (editor)
CRC - Taylor & Francis

Wikipedia

www.wikipedia.org
http://en.wikipedia.org/wiki/CIE_1931_color_space
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http://en.wikipedia.org/wiki/colour_constancy

efg’s Computer Lab

www.efg2.com/Lab