Lecture 4 – Message authentication, UF-CMA, CBC-MAC, CMAC

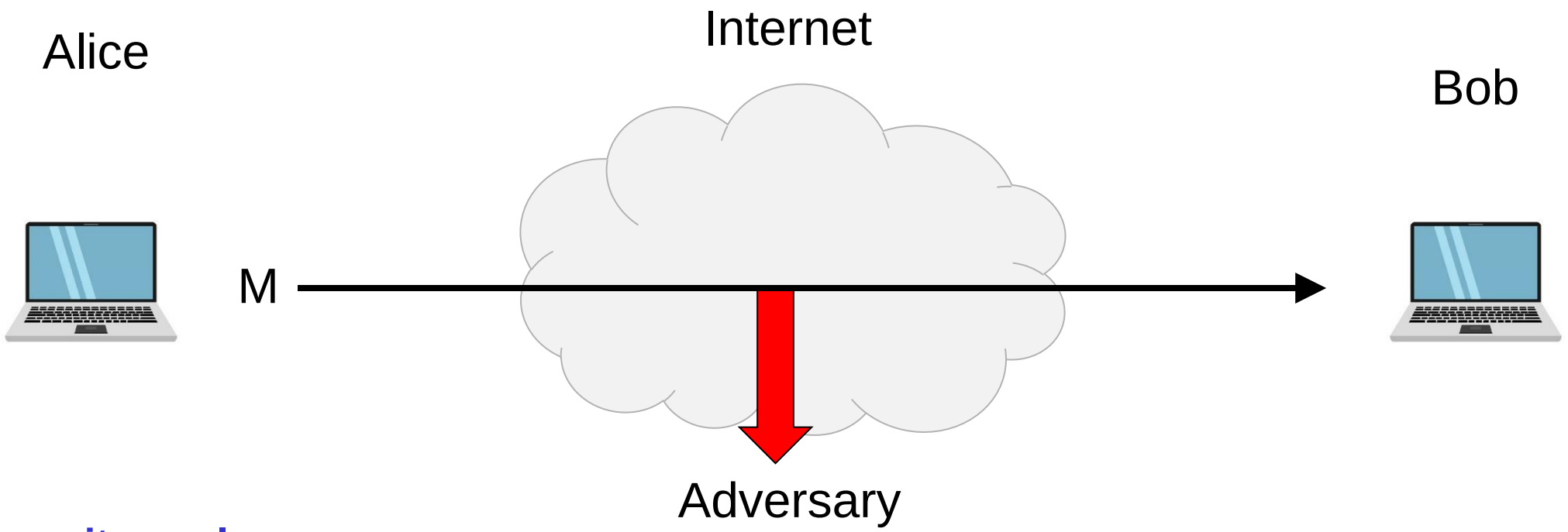
TEK4500

10.09.2025

Håkon Jacobsen

hakon.jacobsen@its.uio.no

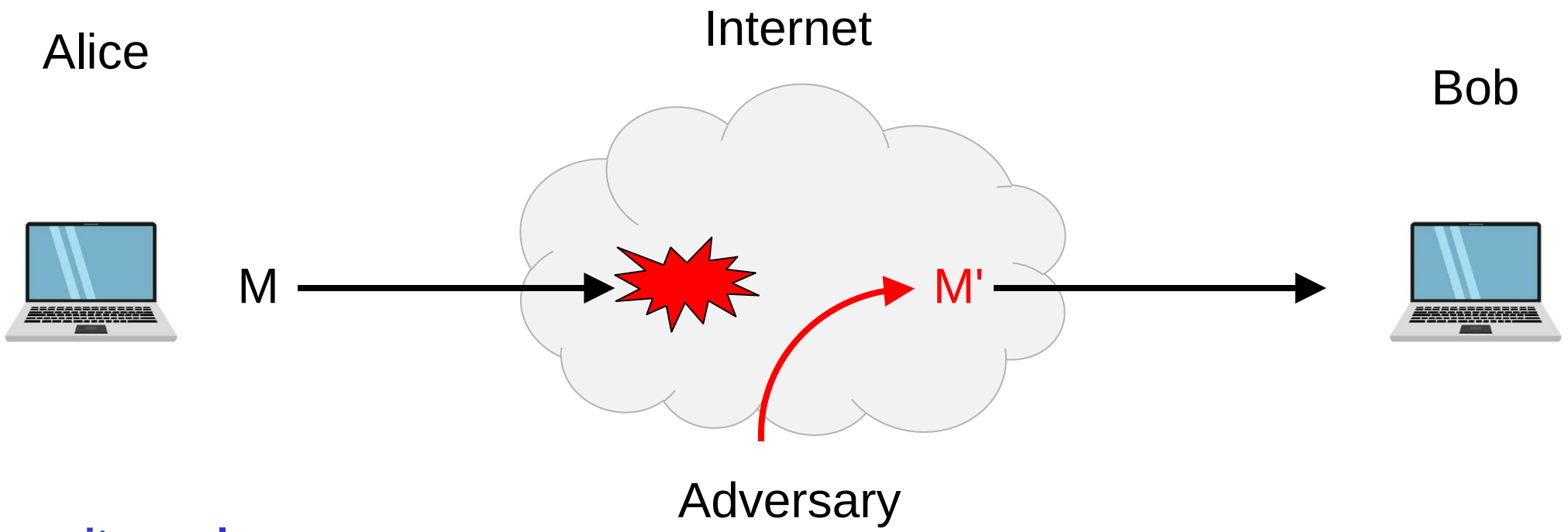
What is cryptography?



Security goals:

- **Data privacy:** adversary should not be able to read message M
- **Data integrity:** adversary should not be able to modify message M
- **Data authenticity:** message M really originated from Alice

What is cryptography?



Security goals:

- **Data privacy:** adversary should not be able to read message M
- **Data integrity:** adversary should not be able to modify message M
- **Data authenticity:** message M really originated from Alice

Basic goals of cryptography

	Message privacy	**Message integrity / authentication**
Symmetric keys	Symmetric encryption	Message authentication codes (MAC)
Asymmetric keys	Asymmetric encryption (a.k.a. public-key encryption)	Digital signatures

Motivation

- Goal: *integrity*, not privacy

- Examples:
  - Protecting OS system files against tampering

  - Browser cookies stored by web servers

  - Control signals in network management

Encryption ≠ integrity



CTR. Enc(K,·)

0110100 1110 100101100110

⊕

"Send Bob $10"

1001010 1010 000000001010

1111110 0100 **0110**01101100





CTR. Dec(K,·)

1111110 0100 **0110**01101100

⊕

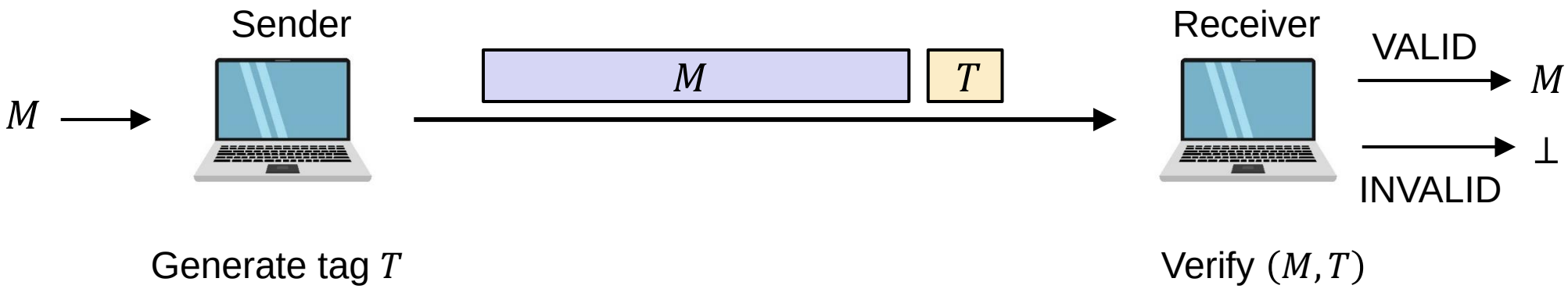
0110100 1110 100101100110

=

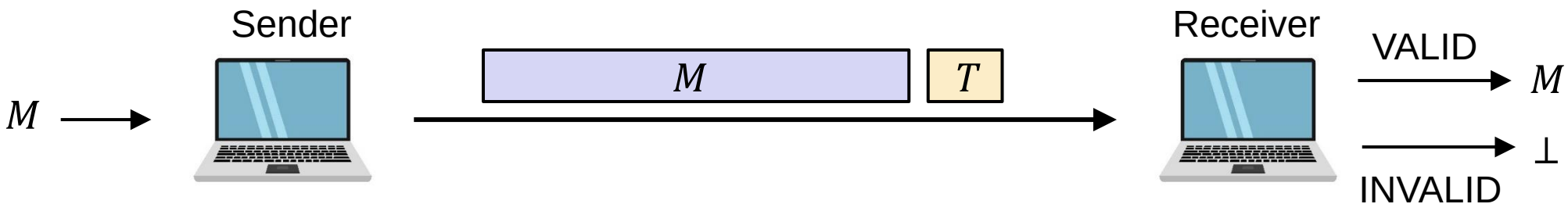
1001010 1010 **1111**00001010

"Send Bob **$3850**"

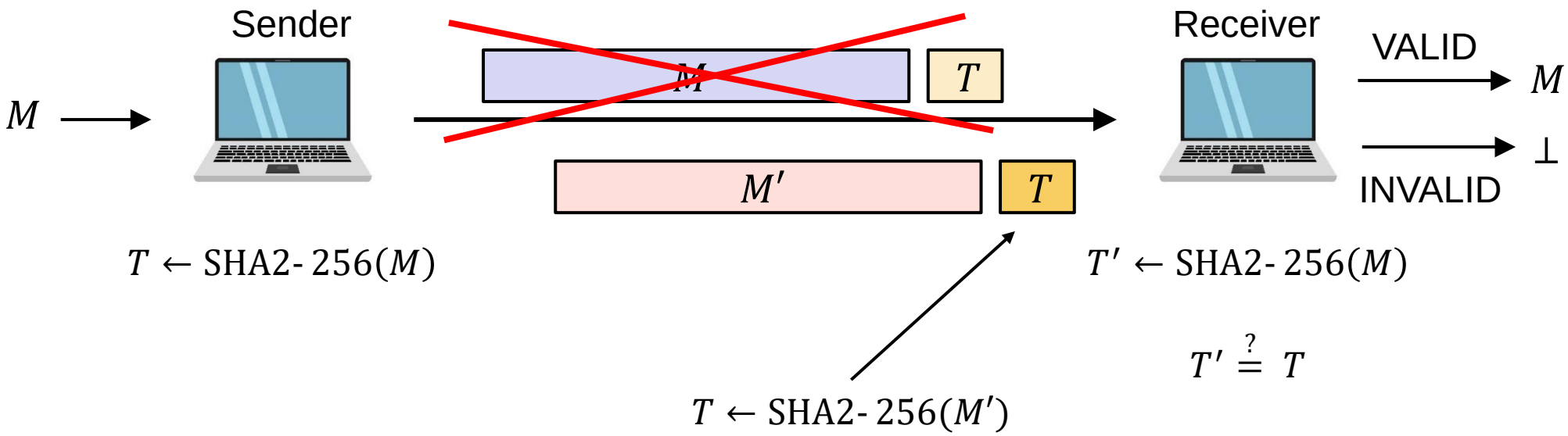
Message authentication – idea



Message authentication – idea



Message authentication – idea



Message authentication schemes – syntax

A **message authentication scheme** is a triple $\Sigma = (\mathrm{KeyGen}, \mathrm{Tag}, \mathrm{Vrfy})$ of algorithms:

$\mathrm{Tag} : \mathcal{K} \times \mathcal{M} \to \mathcal{T}$

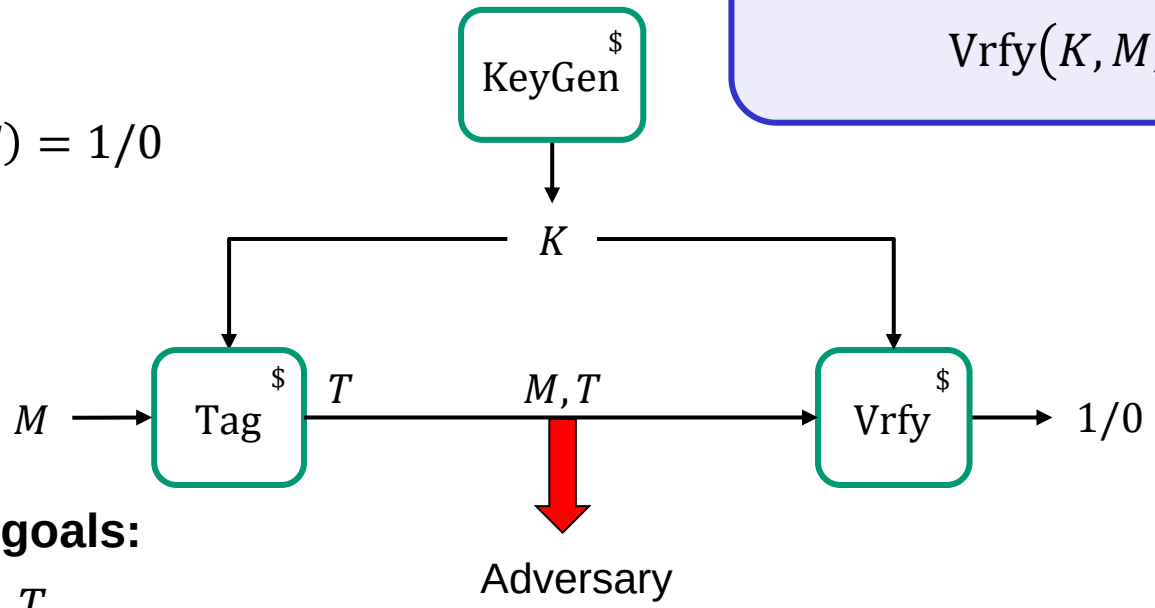
$\mathrm{Tag}(K, M) = \mathrm{Tag}_K(M) = T$

$\mathrm{Vrfy} : \mathcal{K} \times \mathcal{M} \times \mathcal{T} \to \{0,1\}$

$\mathrm{Vrfy}(K, M, T) = \mathrm{Vrfy}_K(M, T) = 1/0$

Correctness: $\forall K \xleftarrow{\$} \mathrm{KeyGen}$ and $\forall M \in \mathcal{M}$:

$$\mathrm{Vrfy}\big(K, M, \mathrm{Tag}(K, M)\big) = 1$$



Possible integrity security goals:

- Hard to recover K from M, T
- Hard to **forge** T for a specific message M
- Hard to forge T for *any* M
- Hard to forge T for any chosen M after seeing valid tags on (chosen) M_1, M_2, ... first

UF-CMA – Unforgability against chosen-message attacks

$\mathbf{Exp}^{\text{uf-cma}}_{\Sigma}(A)$

1. $K \xleftarrow{\$} \Sigma.\text{KeyGen}$
2. $\mathcal{S} \leftarrow [\,]$
3. forge $\leftarrow 0$
4. $A^{\text{TAG}(\cdot),\,\text{VF}(\cdot,\cdot)}$
5. **return** forge

TAG(M)

1. $T \leftarrow \Sigma.\text{Tag}(K, M)$
2. $\mathcal{S}.\text{add}(M)$
3. **return** T

VF(M, T)

1. $d \leftarrow \Sigma.\text{Vrfy}(K, M, T)$
2. **if** $d = 1$ **and** $M \notin \mathcal{S}$ **then:**
3. forge $\leftarrow 1$
4. **return** d



Challenger

$\text{Tag}(K,\cdot)$

$M_1,\ M_2,\ \ldots$

$T_1,\ T_2,\ \ldots$

$\text{Vrfy}(K,\cdot,\cdot)$

$(M'_1, T'_1),\ (M'_2, T'_2),\ \ldots$

$1/0,\quad 1/0,\ \ldots$

Adversary *wins* if:

- one (M'_i, T'_i) is *valid*, i.e., $\text{Vrfy}(K, M'_i, T'_i) = 1$
- and M'_i is *fresh*, i.e., $M'_i \notin \{M_1, M_2, \ldots,\}$

Definition: The **UF-CMA advantage** of an adversary A is

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma}(A) = \Pr\left[\mathbf{Exp}^{\text{uf-cma}}_{\Sigma}(A) \Rightarrow 1\right]$$

Properties of UF-CMA definition

- UF-CMA security implies:
  - Hard to recover K
  - Hard forge a *specific* message chosen by the adversary
  - Tag lengths must be *long enough*!
    - Suppose $|T| = 10$; then

$$\mathbf{Adv}_{\Sigma}^{\text{uf-cma}}(\text{"Guess } T\text{"}) = \frac{1}{2^{10}} \approx \frac{1}{1000}$$

- Does *not* give protection against **replay attacks**
  - Message counters
  - Nonces
  - Timestamps

$\mathbf{Exp}_{\Sigma}^{\text{uf-cma}}(A)$

1. $K \xleftarrow{\$} \Sigma.\text{KeyGen}$
2. $\mathcal{S} \leftarrow [\,]$
3. forge $\leftarrow 0$
4. $A^{\text{TAG}(\cdot),\,\text{VF}(\cdot,\cdot)}$
5. **return** forge

TAG(M)

1. $T \leftarrow \Sigma.\text{Tag}(K, M)$
2. $\mathcal{S}.\text{add}(M)$
3. **return** T

VF(M, T)

1. $d \leftarrow \Sigma.\text{Vrfy}(K, M, T)$
2. **if** $d = 1$ **and** $M \notin \mathcal{S}$ **then:**
3. forge $\leftarrow 1$
4. **return** d

$$\mathbf{Adv}_{\Sigma}^{\text{uf-cma}}(A) = \Pr\left[\mathbf{Exp}_{\Sigma}^{\text{uf-cma}}(A) \Rightarrow 1\right]$$

Message authentication *codes* (MACs)

deterministic *function*

$$\mathrm{Tag}_K(M) = T$$

$$\mathrm{Vrfy}_K(M, T) = \begin{cases} 1, & \text{if } \mathrm{Tag}_K(M) = T \\ 0, & \text{otherwhise} \end{cases}$$

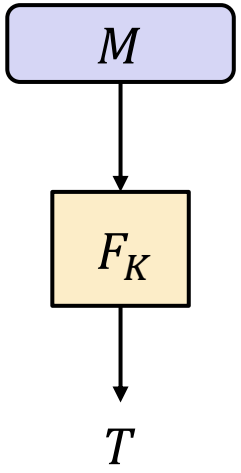
"recompute and check"

- MAC = special class of message authentication *schemes*
  …but actually most common in practice

- MAC ⟹ unique tags

PRFs are good MACs

$F : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^\tau$



$\Sigma_F.\mathrm{Tag}(K, M)$

1. **return** $F_K(M)$

$\Sigma_F.\mathrm{Vrfy}(K, M, T)$

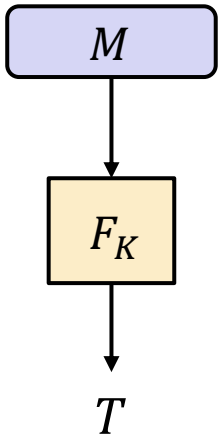
1. **return** $F_K(M) \stackrel{?}{=} T$

Theorem: If F is a secure PRF then Σ_F is UF-CMA secure for *fixed-length* messages

PRFs are good MACs – proof sketch

Theorem: For *any* UF-CMA adversy A, asking v VF queries, there is a PRF-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma_F}(A) \leq \mathbf{Adv}^{\text{prf}}_{F}(B) + \frac{v}{2^{\tau}}$$

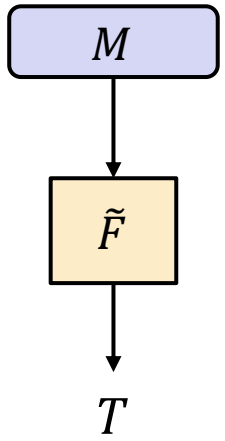


$$\Pr[F_K(M) = T] = \ ?$$

PRFs are good MACs – proof sketch

Theorem: For *any* UF-CMA adversy A, asking v VF queries, there is a PRF-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma_F}(A) \leq \mathbf{Adv}^{\text{prf}}_{F}(B) + \frac{v}{2^{\tau}}$$

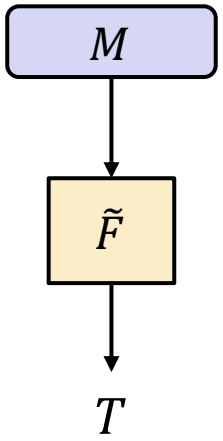


$\tilde{F} \xleftarrow{\$} \text{Func}[n, \tau]$

PRFs are good MACs – proof sketch

Theorem: For *any* UF-CMA adversy A, asking v VF queries, there is a PRF-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma_F}(A) \leq \mathbf{Adv}^{\text{prf}}_{F}(B) + \frac{v}{2^{\tau}}$$



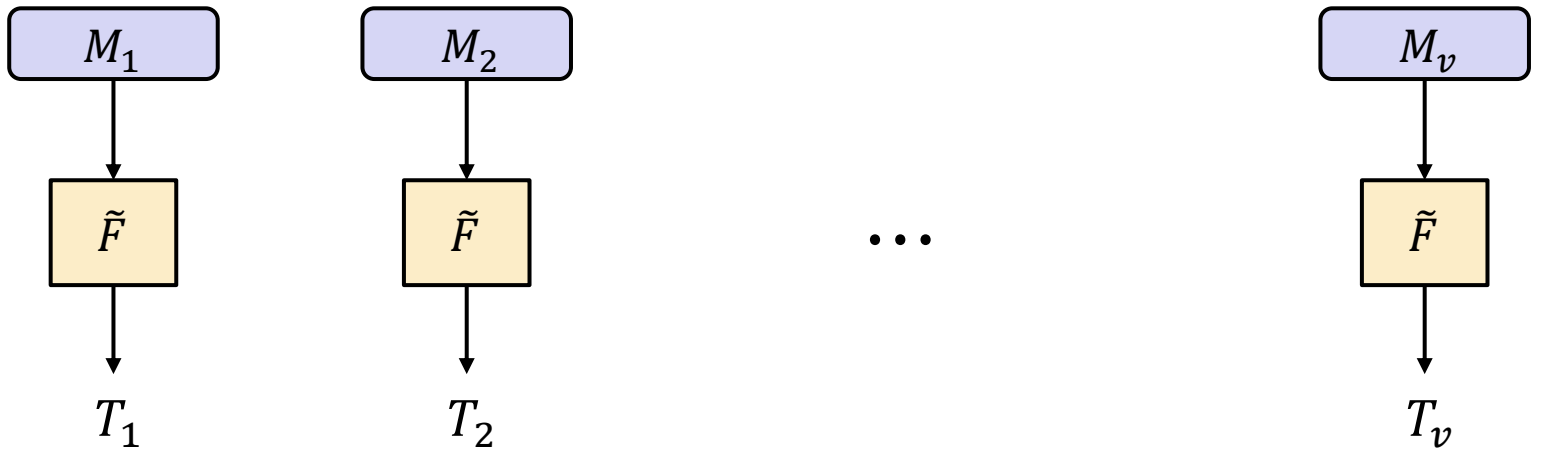
$\tilde{F} \xleftarrow{\$} \text{Func}[n, \tau]$

$$\Pr[\tilde{F}(M) = T] = \frac{1}{2^{\tau}}$$

PRFs are good MACs – proof sketch

Theorem: For *any* UF-CMA adversy A, asking v VF queries, there is a PRF-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma_F}(A) \leq \mathbf{Adv}^{\text{prf}}_{F}(B) + \frac{v}{2^{\tau}}$$



$$\Pr[\tilde{F}(M_1) = T_1 \ \vee \ \cdots \vee \ \tilde{F}(M_v) = T_v] \ \leq \ \sum_{i=1}^{v} \Pr[\tilde{F}(M_i) = T_i] \ = \ \frac{v}{2^{\tau}}$$

PRFs are good MACs – proof sketch

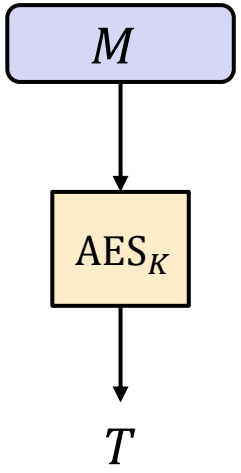
Theorem: For *any* UF-CMA adversy A, asking v VF queries, there is a PRF-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\Sigma_F}(A) \leq \mathbf{Adv}^{\text{prf}}_{F}(B) + \frac{v}{2^{\tau}}$$

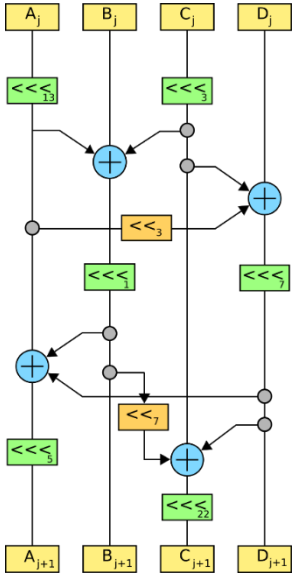


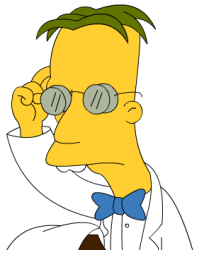
$$\Pr[\tilde{F}(M_1) = T_1 \ \vee \ \cdots \ \vee \ \tilde{F}(M_v) = T_v] \ \leq \ \sum_{i=1}^{v} \Pr[\tilde{F}(M_i) = T_i] \ = \ \frac{v}{2^{\tau}}$$

PRFs are good MACs



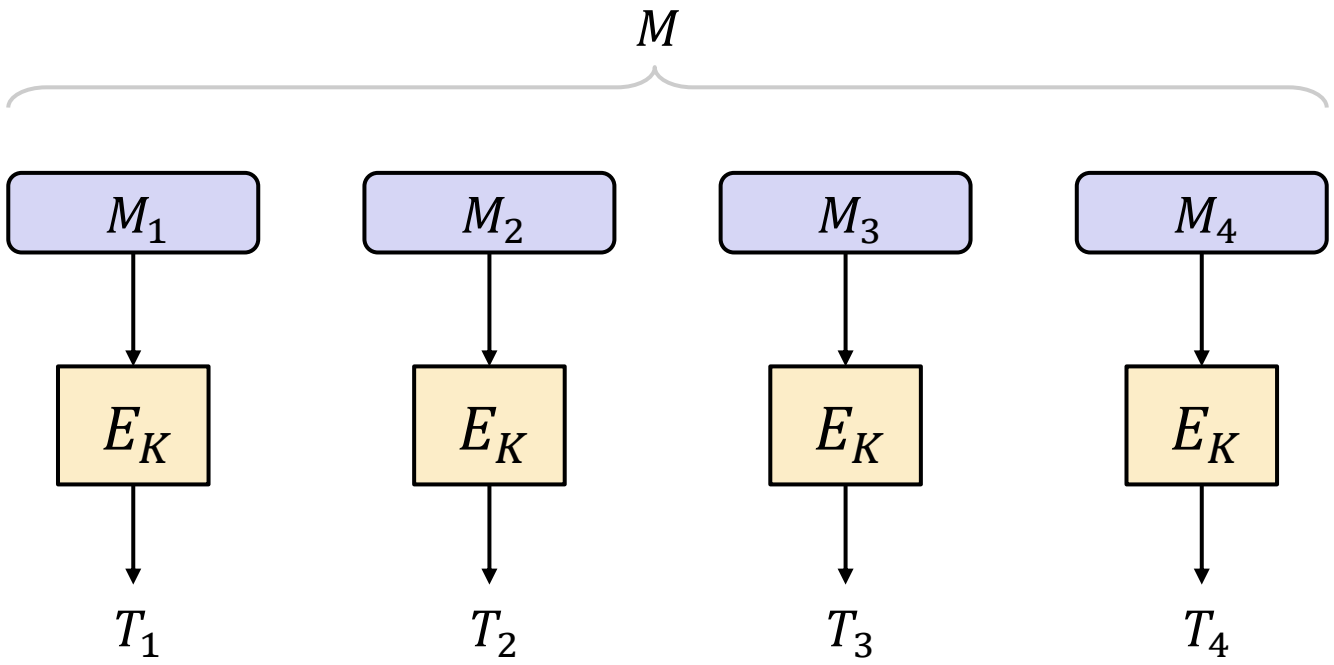
Conclusion: $\mathrm{AES\text{-}256} : \{0,1\}^{256} \times \{0,1\}^{128} \rightarrow \{0,1\}^{128}$ is a good MAC





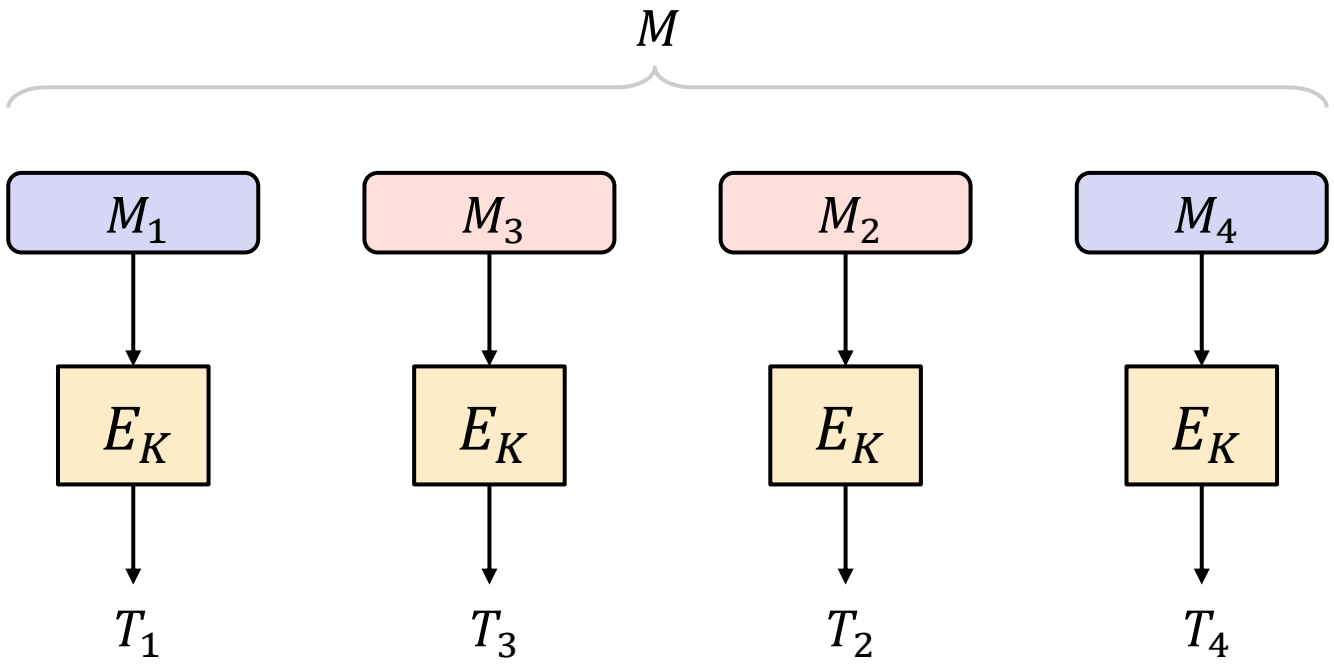
MACs for long messages

MACs from block ciphers – Attempt 1



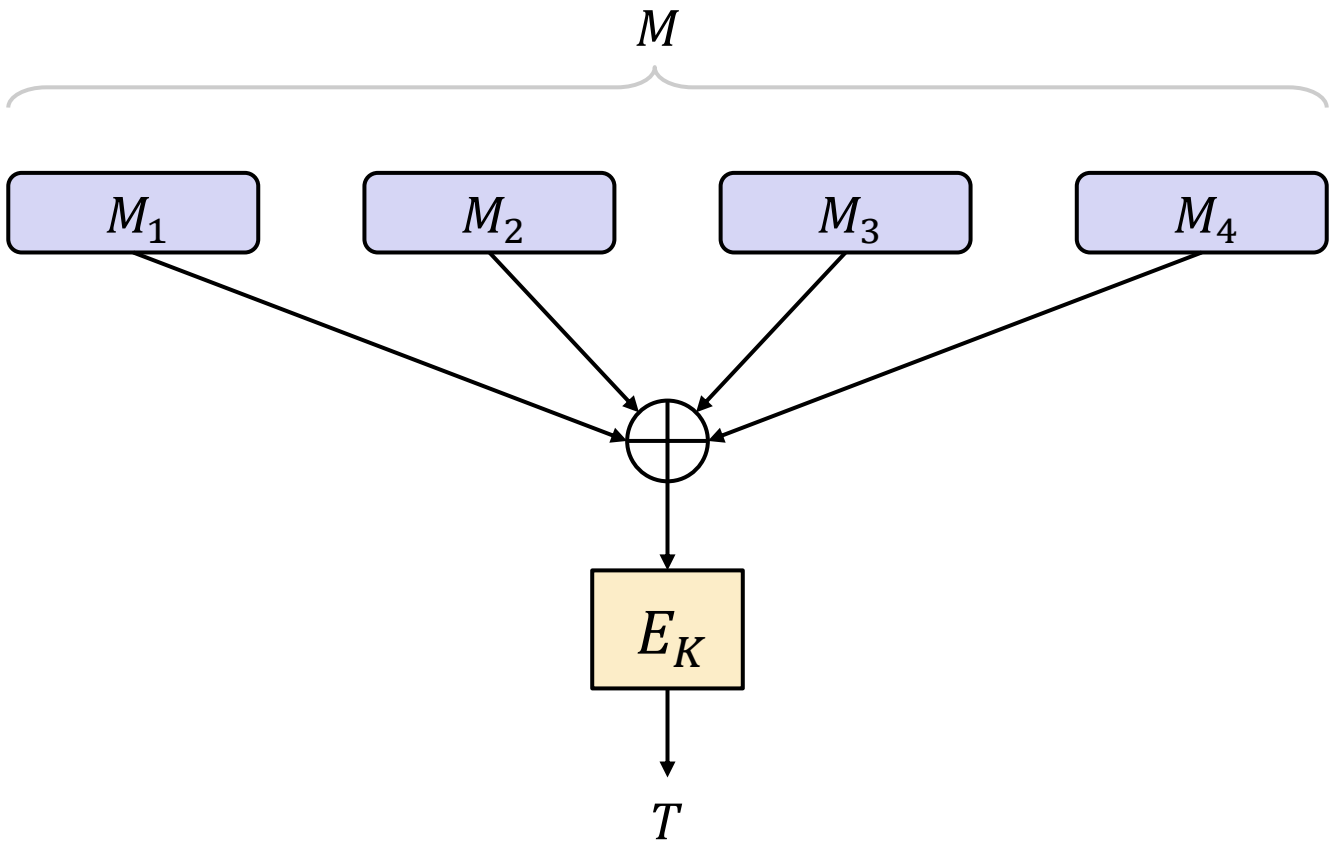
$$T = T_1||T_2||T_3||T_4$$

Attempt 1 – an attack

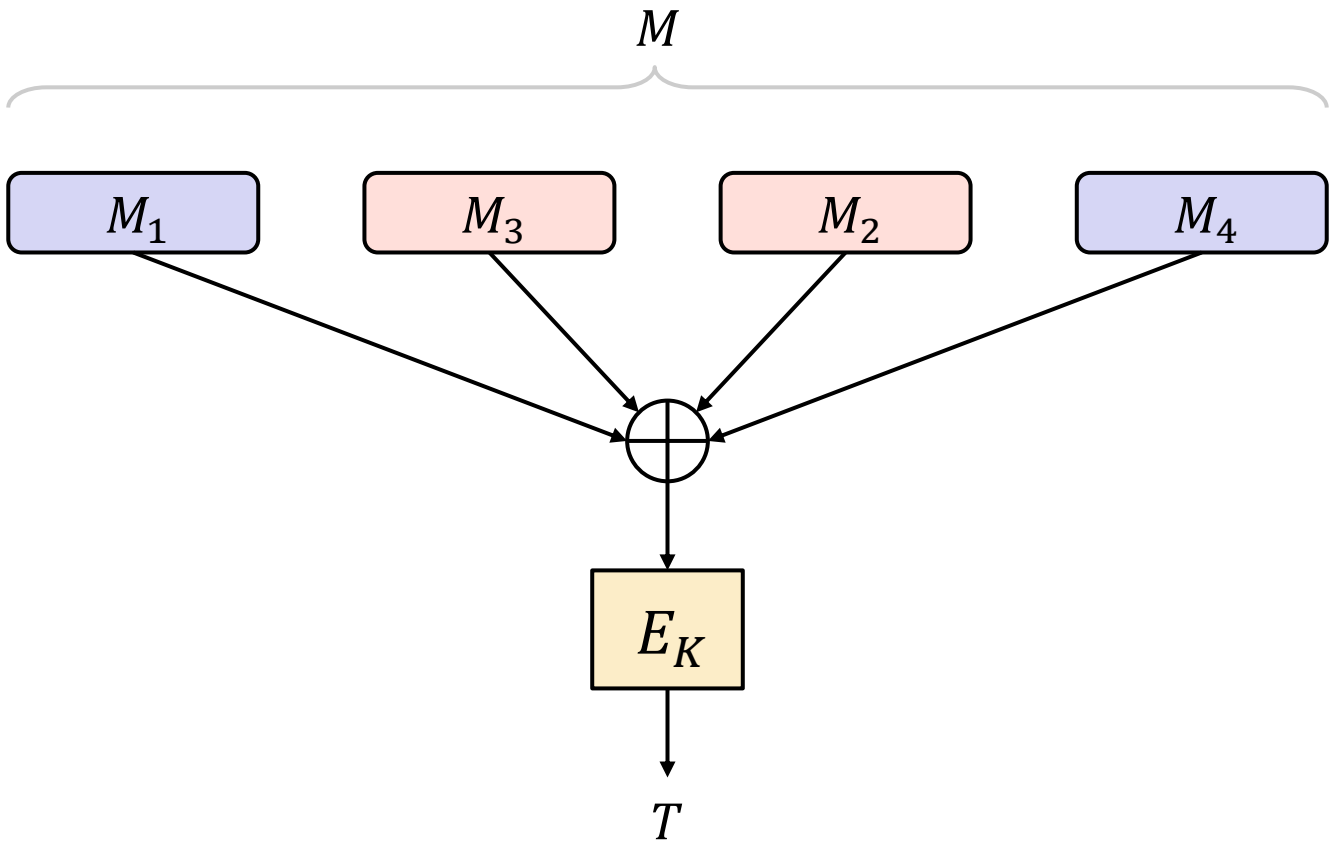


$$T = T_1 || T_3 || T_2 || T_4$$

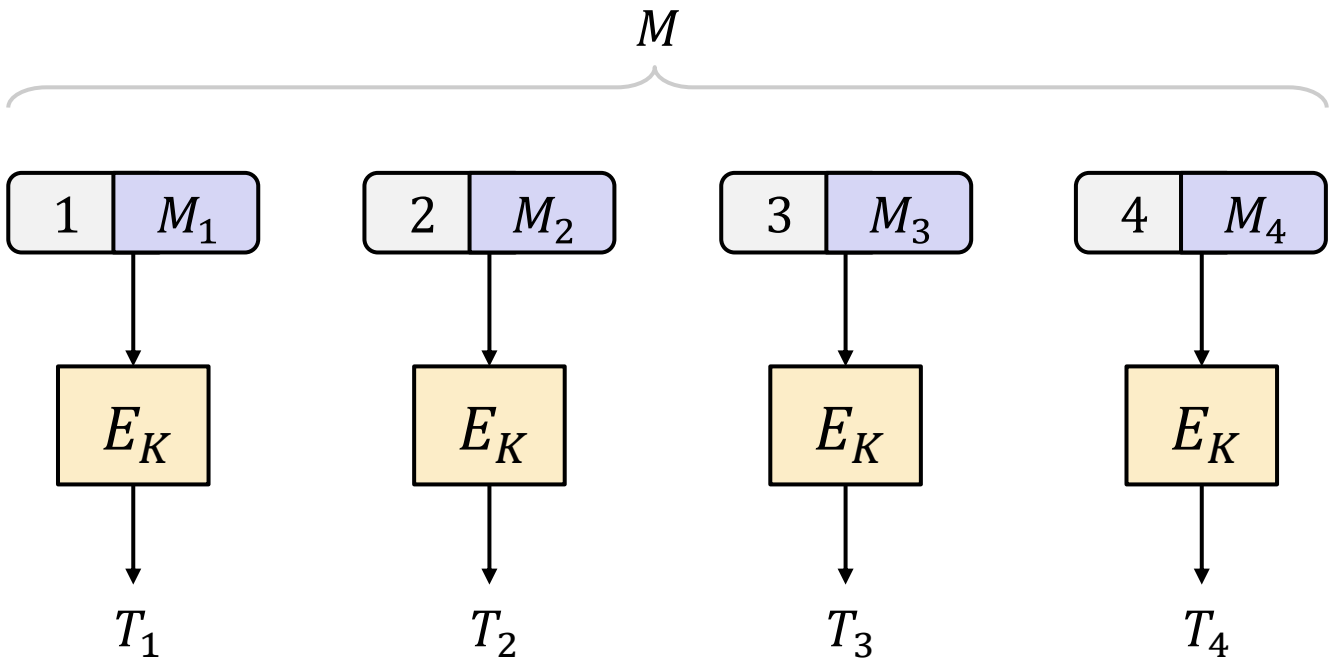
Attempt 2



Attempt 2 – an attack

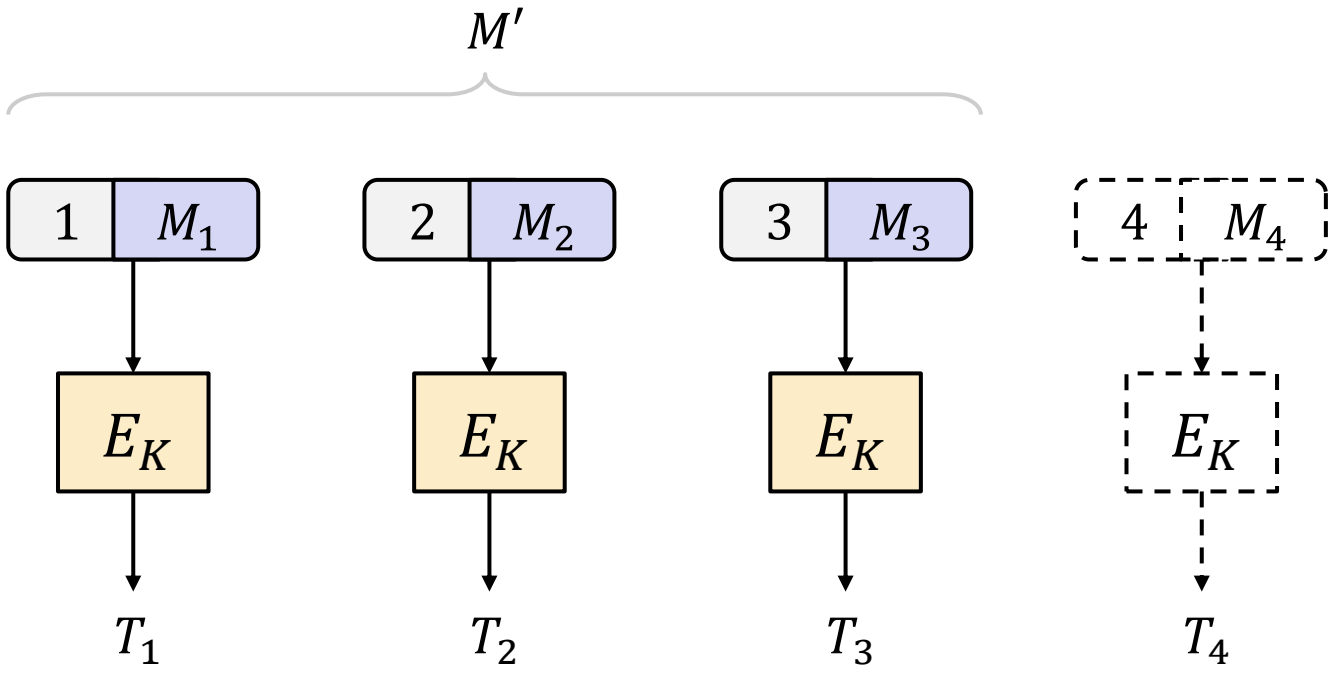


Attempt 3



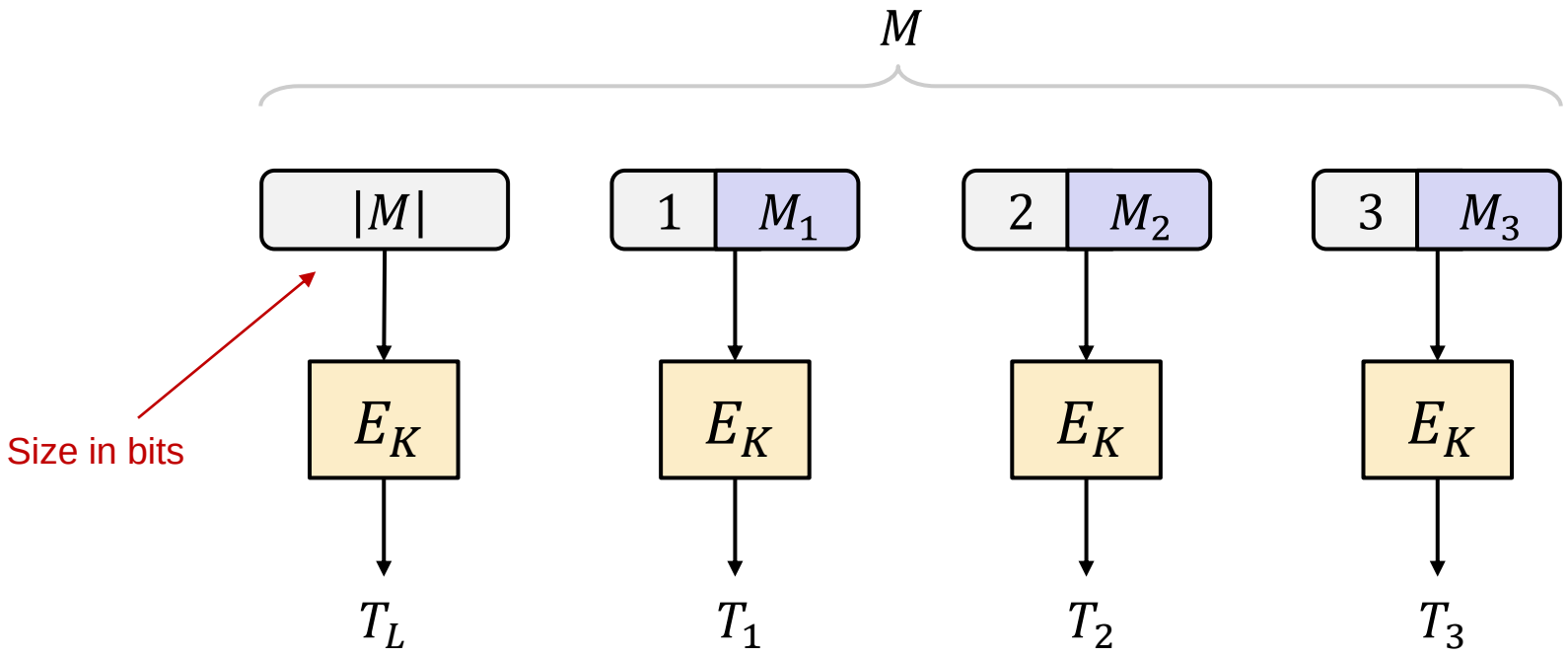
$$T = T_1||T_2||T_3||T_4$$

Attempt 3 – an attack



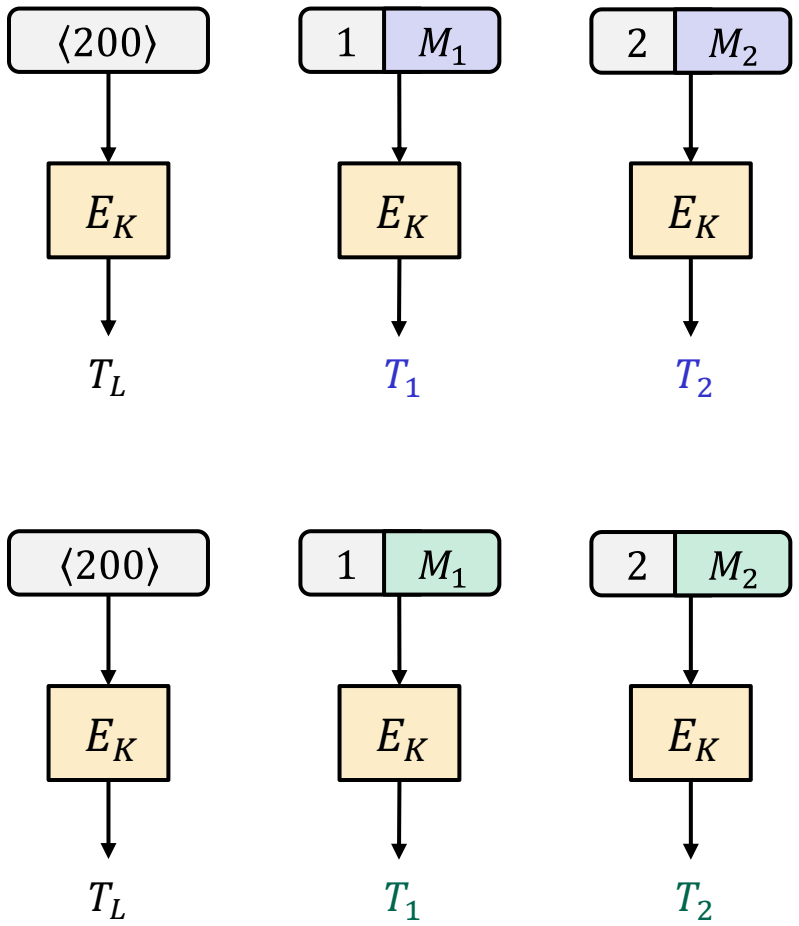
$$T = T_1 || T_2 || T_3 || T_4$$

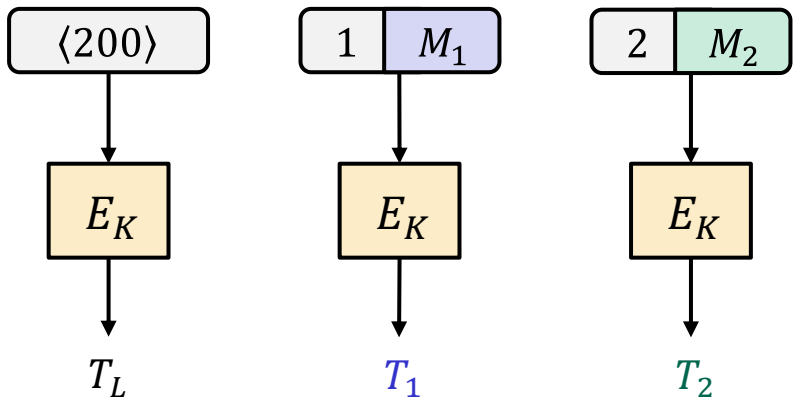
Attempt 4



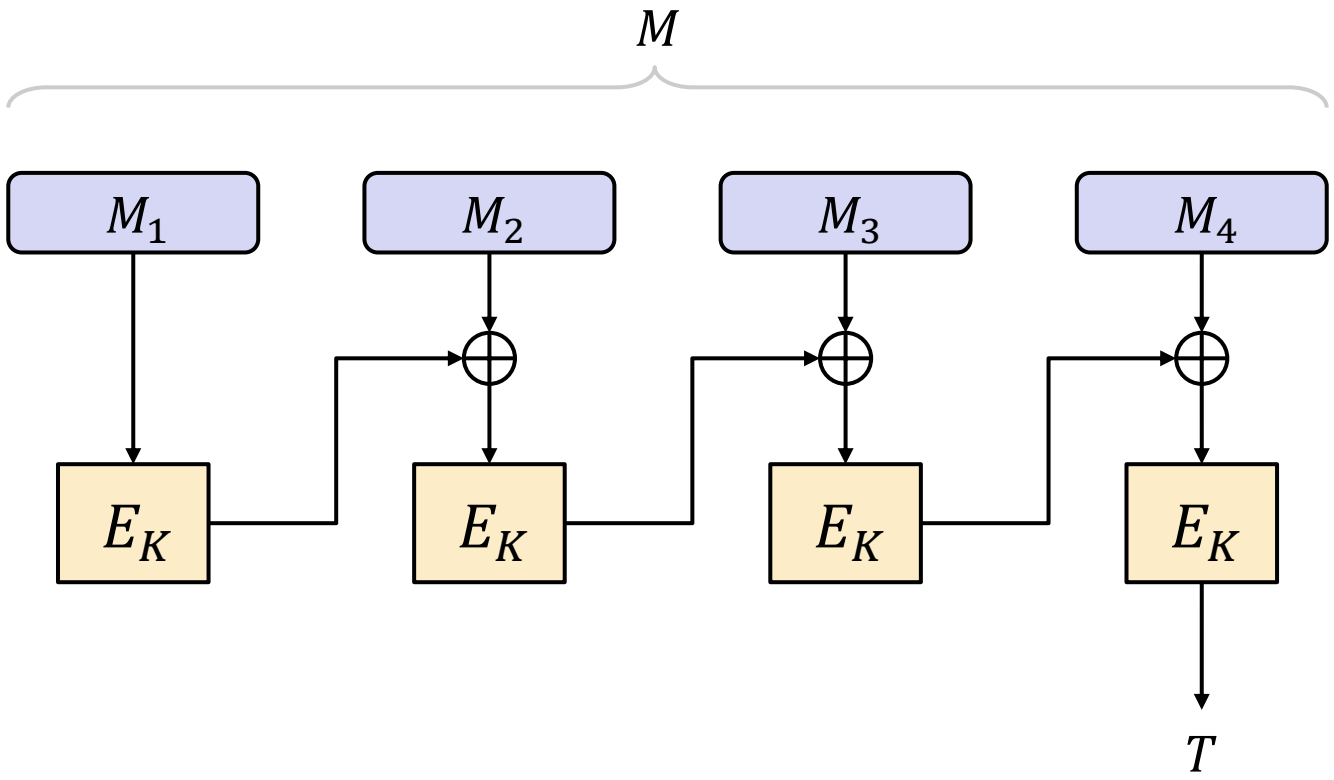
$$T = T_L||T_1||T_2||T_3$$

Attempt 4 – an attack





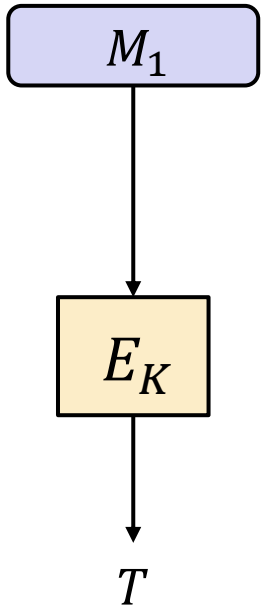
CBC-MAC

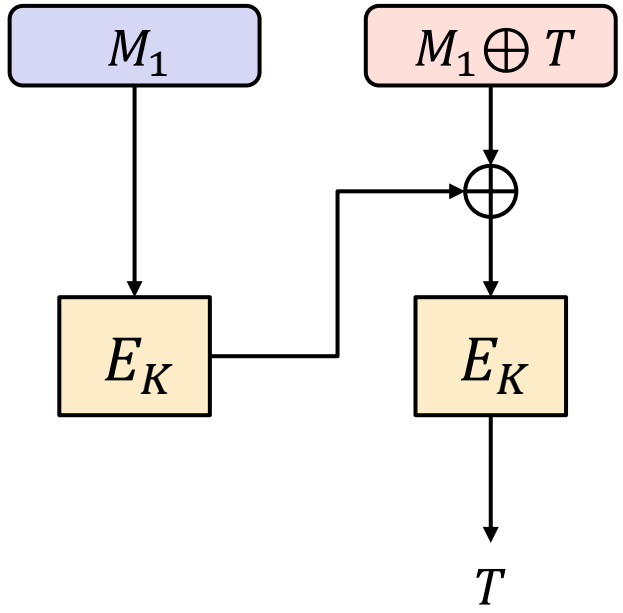


Theorem: secure if all messages have the *same* length

Warning: not secure if messages have *different* lengths!

CBC-MAC

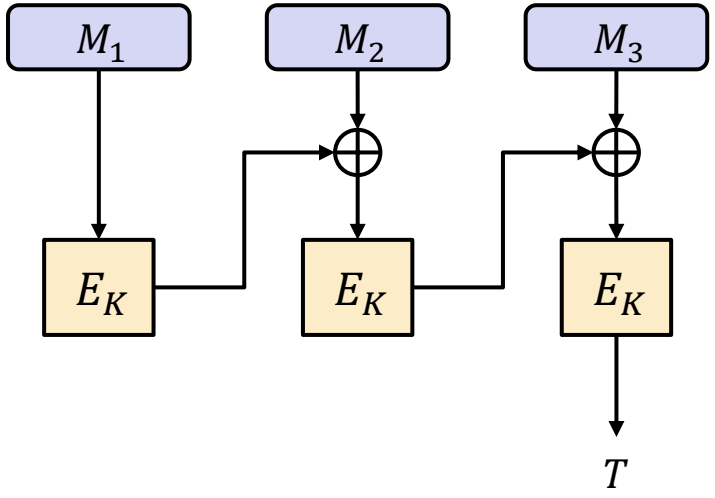


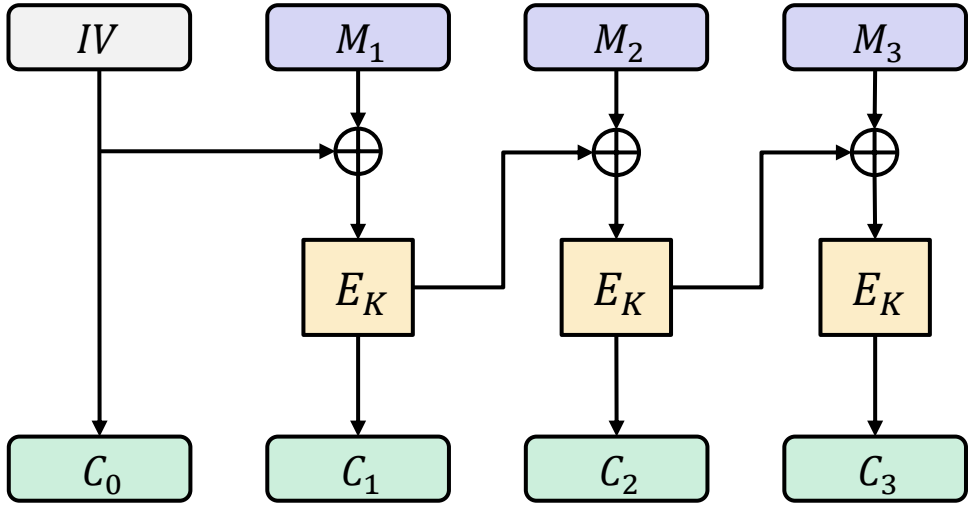


$$\mathbf{Adv}^{\text{uf–cma}}_{\text{CBC–MAC}}(A) = 1$$

Warning: not secure if messages have *different* lengths!

CBC-MAC vs. CBC$-ENC





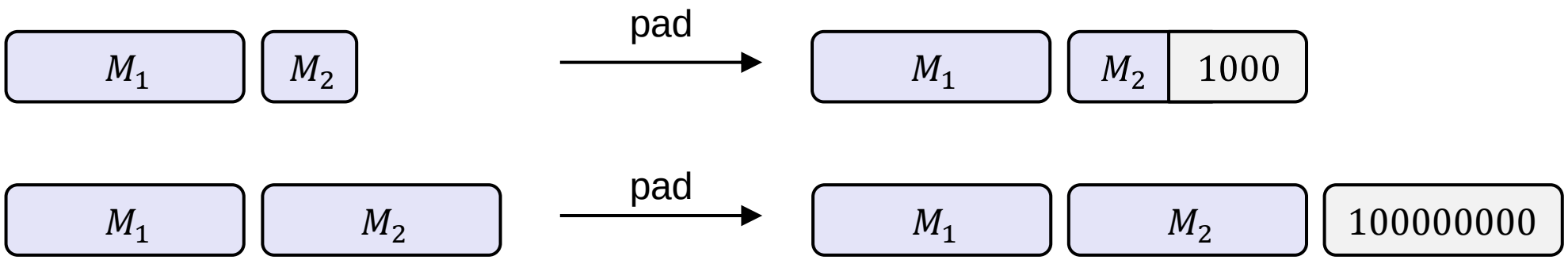
Warning 1: CBC-MAC is insecure *with* an IV!

Warning 2: CBC$-ENC is insecure *without* an IV!

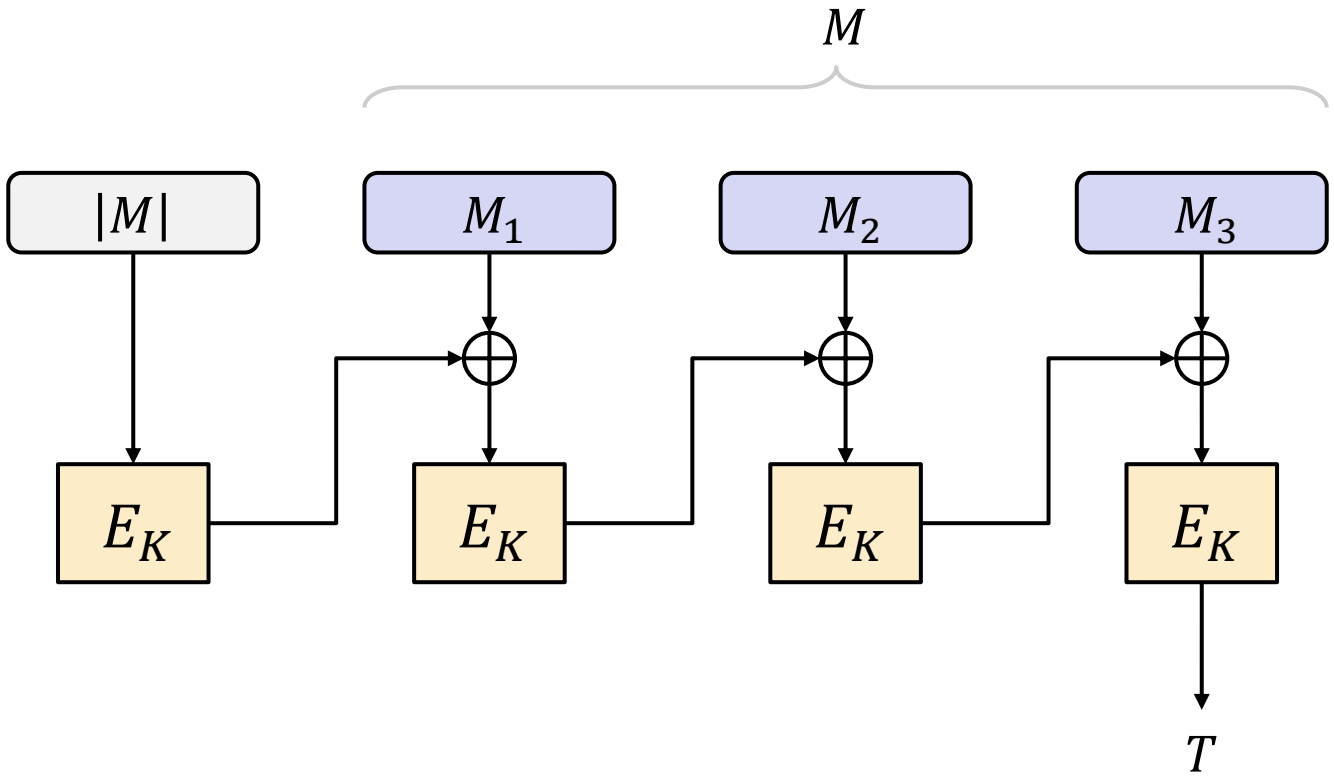
Allowing variable-length messages

- Want to support different-length messages
  - Could use different keys for each length ⟹ not practical

- Want to support message not a multiple of the block length
  - **Padding** needed



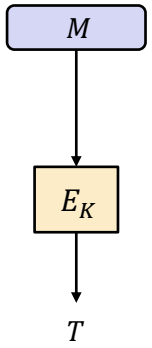
Length-prepended CBC-MAC

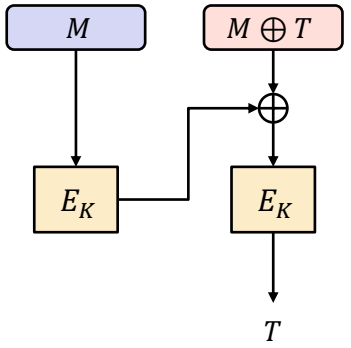


Theorem: Length-prepended CBC-MAC is UF-CMA secure if E is a secure PRP

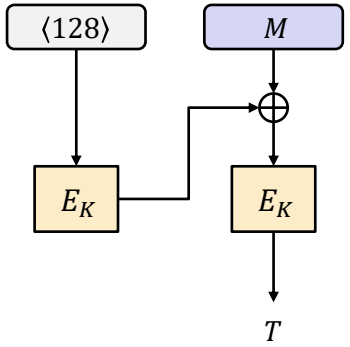
Question: What if you instead add the length to the end?

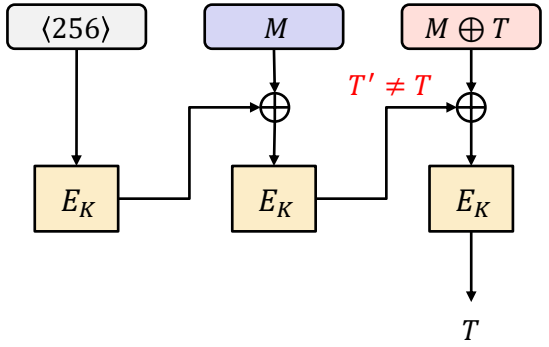
Length-prepended CBC-MAC





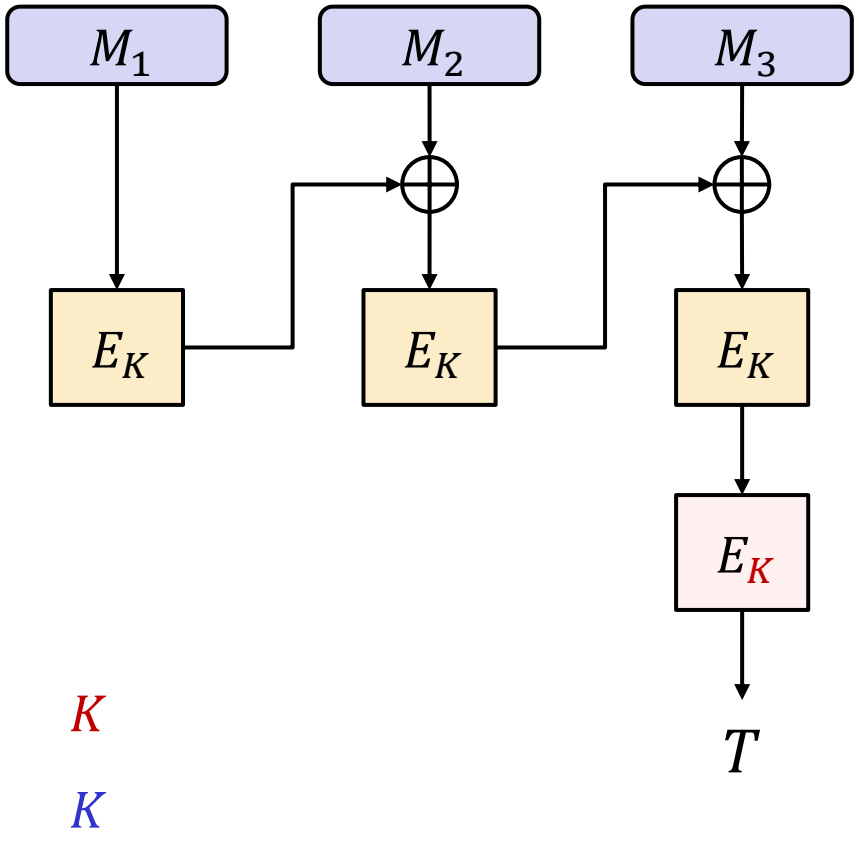
Forgery!

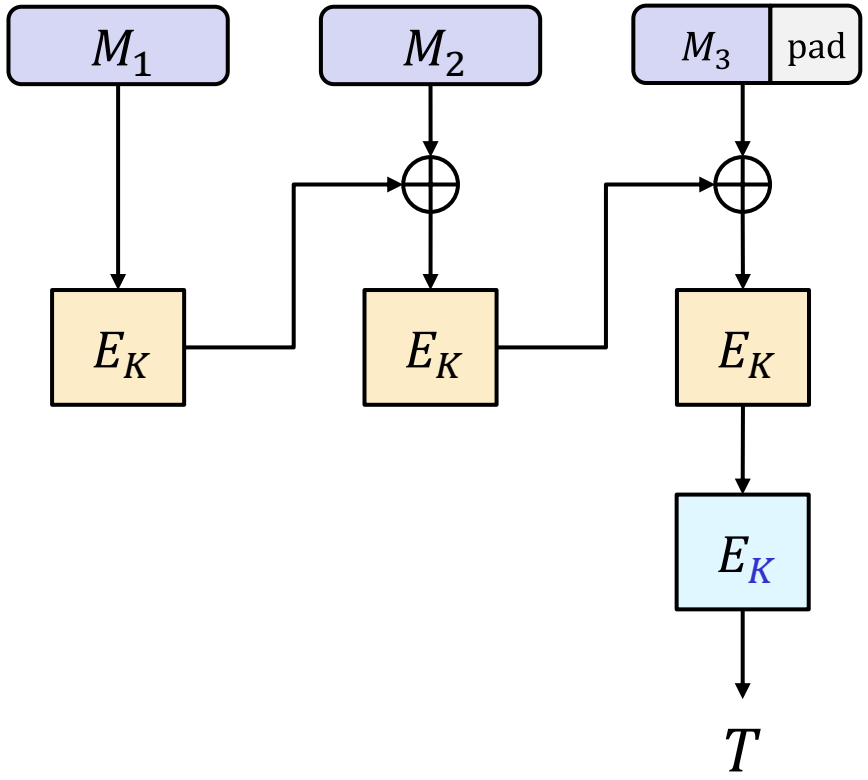




Not a forgery

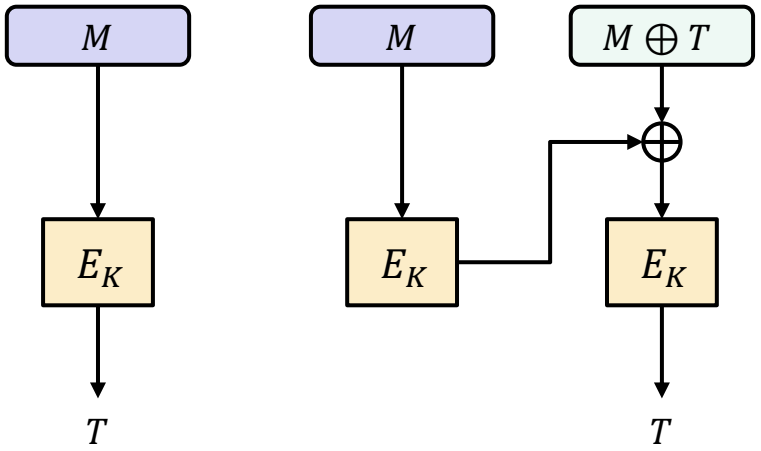
ECBC-MAC





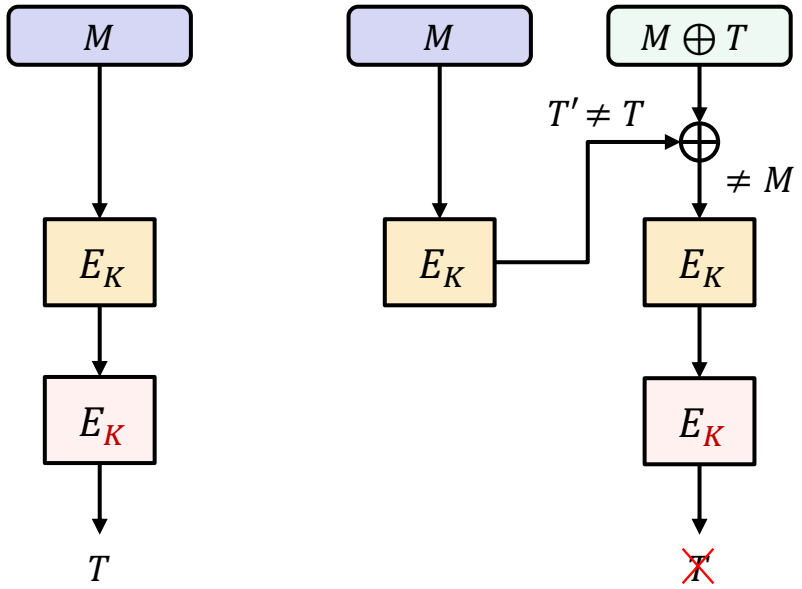
CBC-MAC vs ECBC-MAC

CBC-MAC



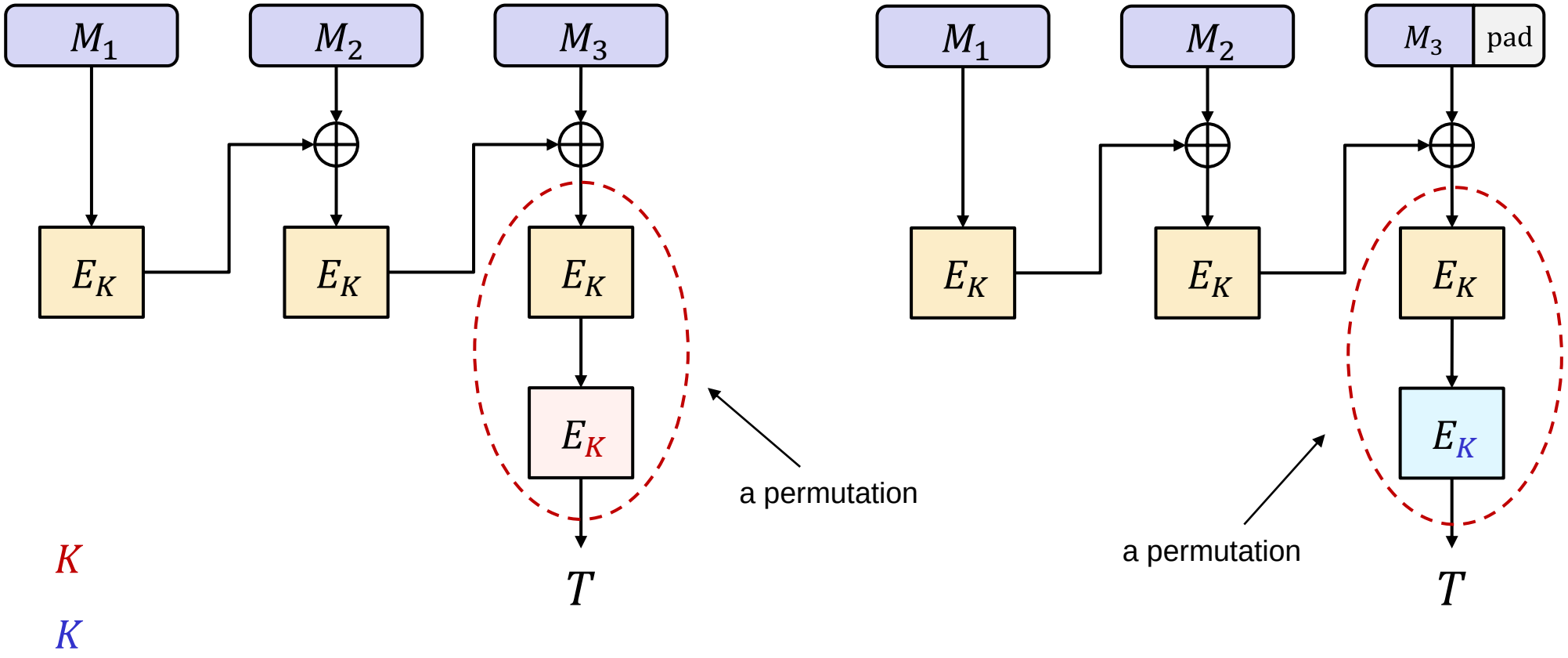
Forgery!

ECBC-MAC

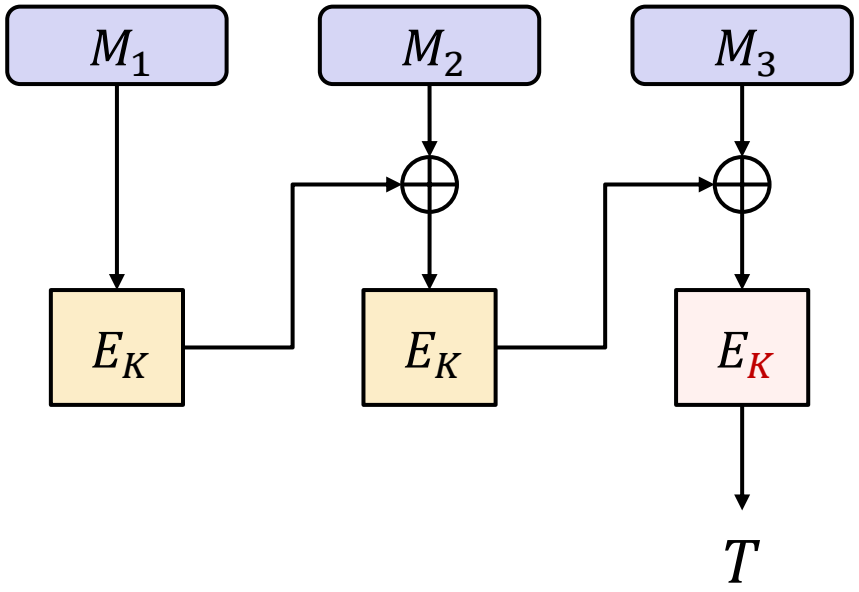


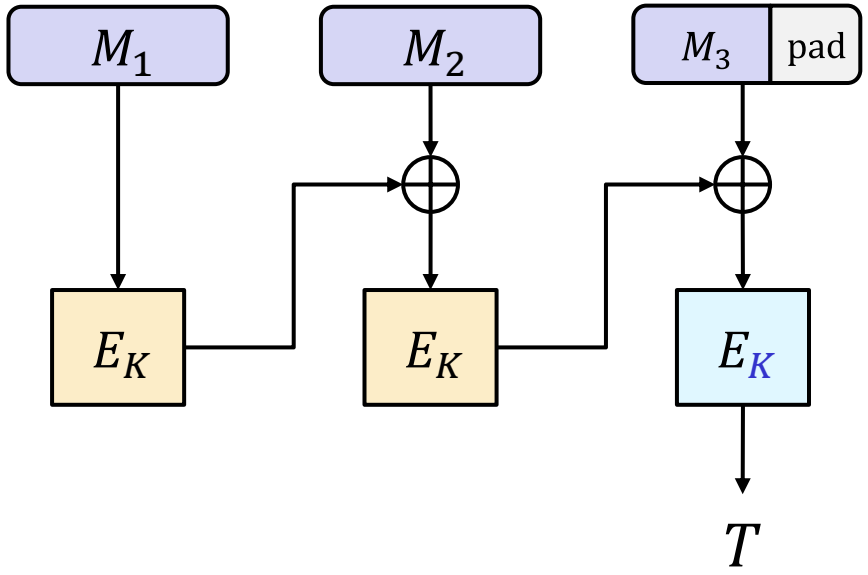
Not a forgery

ECBC-MAC


M_1
M_2
M_3
E_K
E_K
E_K
E_K
T
a permutation
K
K
M_1
M_2
M_3
pad
E_K
E_K
E_K
E_K
T
a permutation

FCBC-MAC

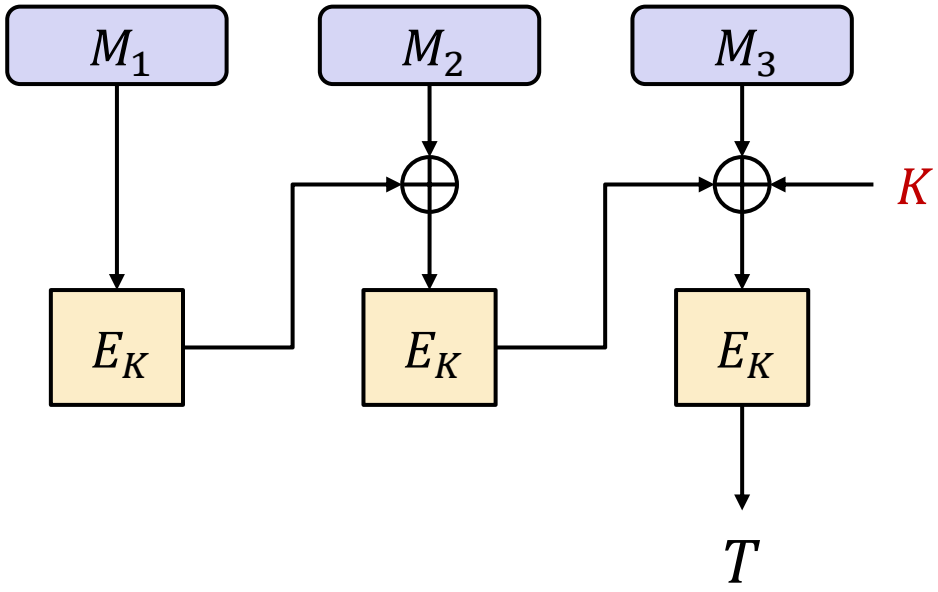


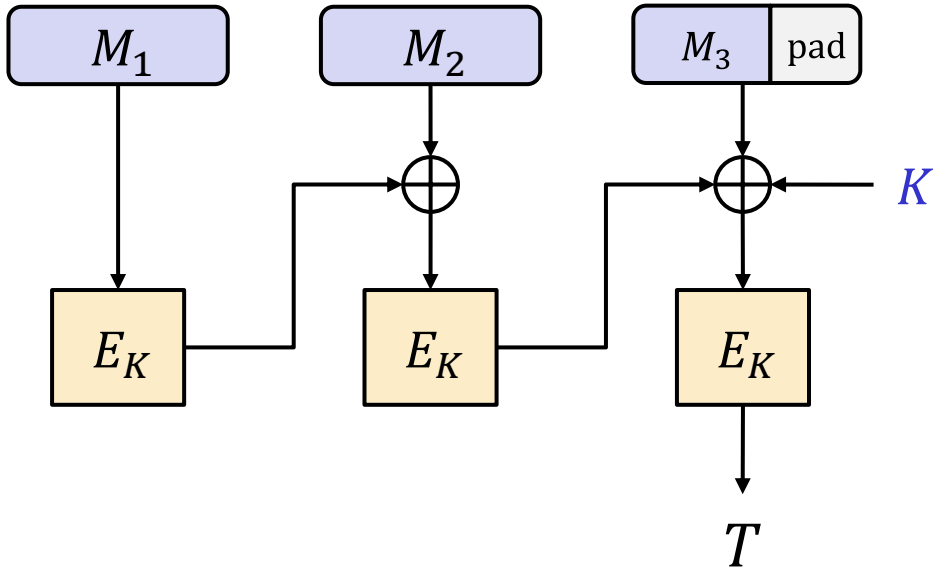


K

K

XCBC-MAC

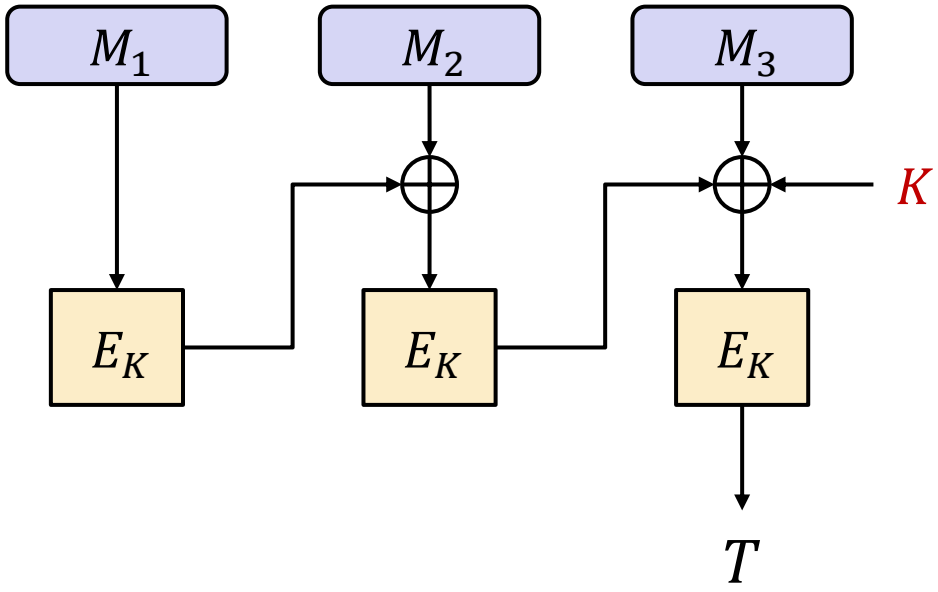


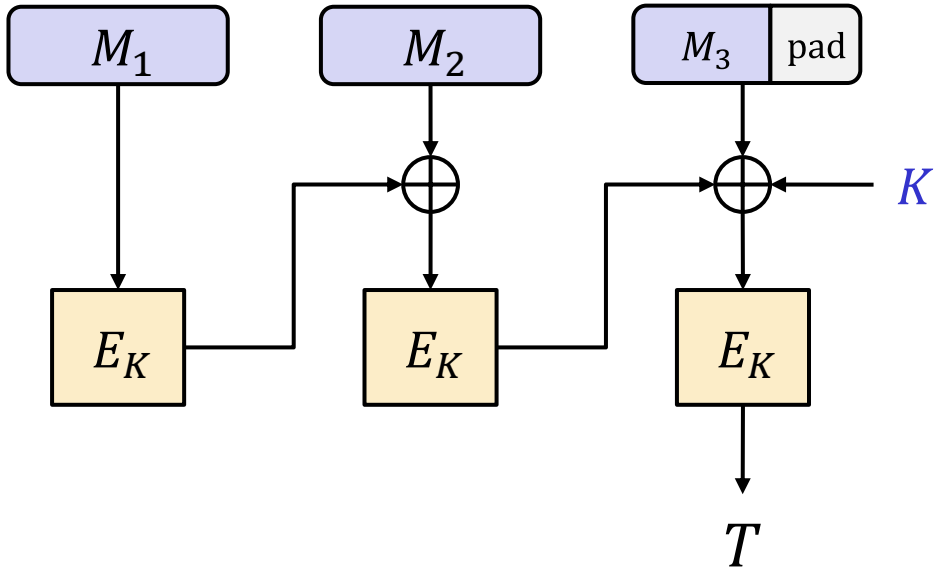


K

K

CMAC a.k.a One-key MAC





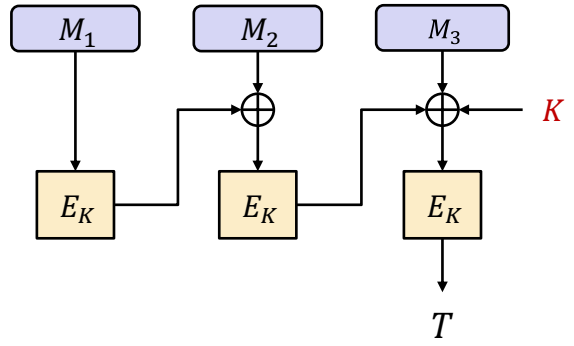
$K = 2 * E_K(0^n)$

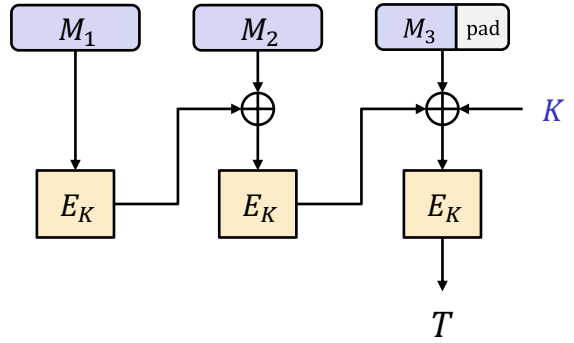
$K = 4 * E_K(0^n)$

Theorem: secure for messages of *all* lengths

CMAC

- **Theorem:** UF-CMA secure if E is a secure PRP
- 1 E-call per message block + 1 E-call to derive K_2, K_3
- Fully sequential
  - Cannot utilize parallel AES cores
- Only one key
- Standardized by NIST
  - Widely used





$K = 2 * F_K(0^n)$

$K = 4 * F_K(0^n)$

CMAC security

Theorem: For *any* UF-CMA adversary making q TAG-queries (each having ℓ blocks), and v VF-queries, there is a PRP-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\text{CMAC}}(A) \leq \mathbf{Adv}^{\text{prp}}_{E}(B) + \frac{5 \cdot \ell \cdot q^2}{2^n} + \frac{v}{2^n}$$

- **Example:**
  - $E = \text{AES} \quad (n = 128)$
  - Number of Tag-queries: $q = 2^{40} \approx 1$ trillion queries
  - Max blocks per message: $\ell = 2^{10} \approx 16$ kB
  - Number of Vrfy-queries: $v = 2^{20}$
  - $\mathbf{Adv}^{\text{prp}}_{E}(B) \approx 0$

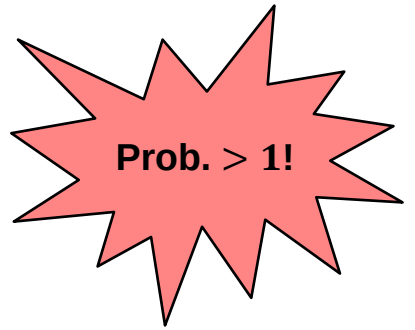
$$\mathbf{Adv}^{\text{uf-cma}}_{\text{AES-CMAC}}(A) \leq \frac{5 \cdot 2^{10} \cdot (2^{40})^2}{2^{128}} + \frac{2^{20}}{2^{128}} \approx \frac{2^{3+10+80}}{2^{128}} = \frac{1}{2^{35}}$$

CMAC security

Theorem: For *any* UF-CMA adversary making q TAG-queries (each having ℓ blocks), and v VF-queries, there is a PRP-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\text{CMAC}}(A) \leq \mathbf{Adv}^{\text{prp}}_{E}(B) + \frac{5 \cdot \ell \cdot q^2}{2^n} + \frac{v}{2^n}$$

- **Example:**
  - $E = \text{DES} \quad (n = 64)$
  - Number of Tag-queries: $q = 2^{40} \approx 1$ trillion queries
  - Max blocks per message: $\ell = 2^{10} \approx 16$ kB
  - Number of Vrfy-queries: $v = 2^{20}$
  - $\mathbf{Adv}^{\text{prp}}_{E}(B) \approx 0$



$$\mathbf{Adv}^{\text{uf-cma}}_{\text{DES-CMAC}}(A) \leq \frac{5 \cdot 2^{10} \cdot (2^{40})^2}{2^{64}} + \frac{2^{20}}{2^{64}} \approx \frac{2^{3+10+80}}{2^{64}} = 2^{29}$$

CMAC security

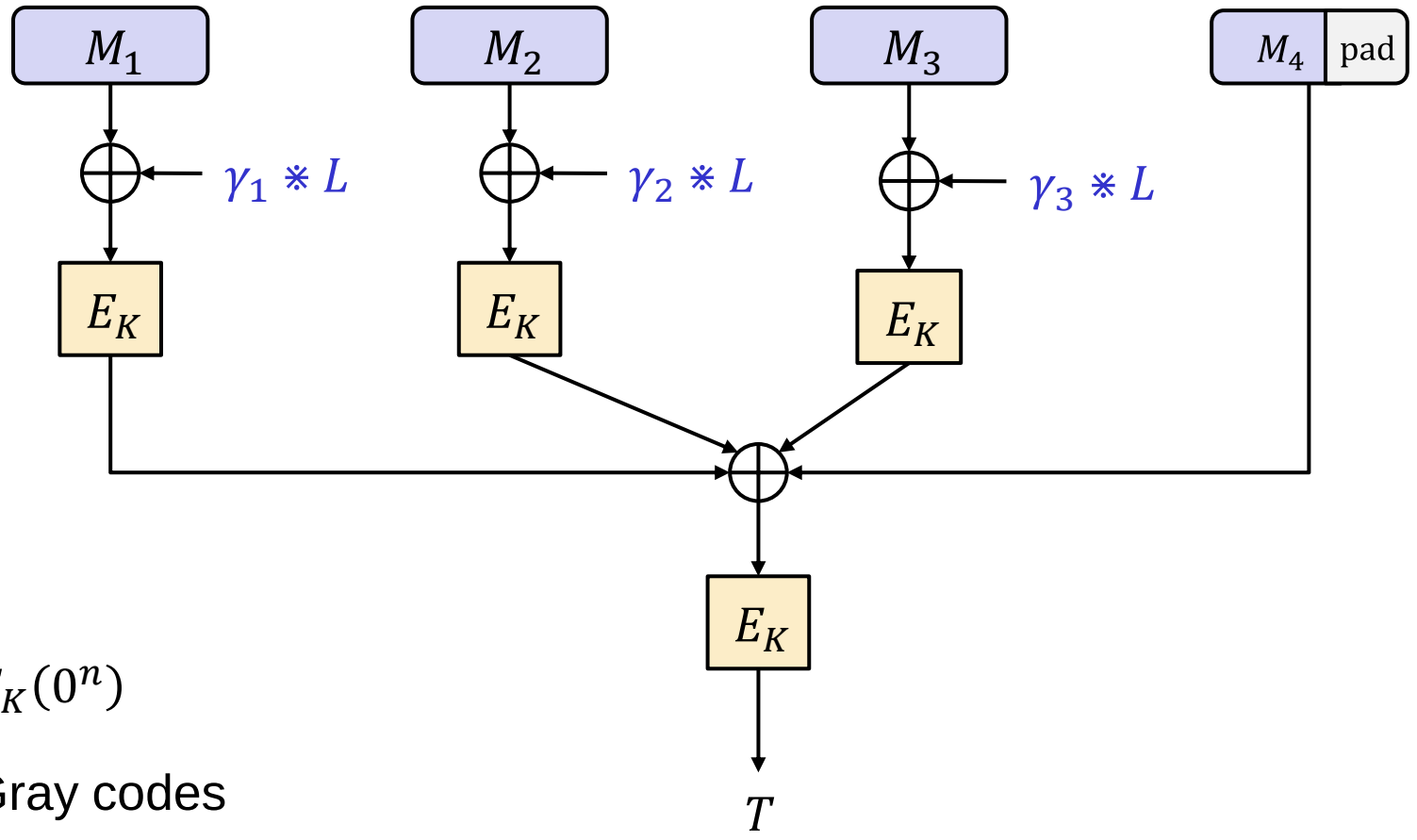
Theorem: For *any* UF-CMA adversary making q TAG-queries (each having ℓ blocks), and v VF-queries, there is a PRP-adversary B such that

$$\mathbf{Adv}^{\text{uf-cma}}_{\text{CMAC}}(A) \leq \mathbf{Adv}^{\text{prp}}_{E}(B) + \frac{5 \cdot \ell \cdot q^2}{2^n} + \frac{v}{2^n}$$

- **Example:**
  - $E = \text{DES} \quad (n = 64)$
  - Number of Tag-queries: $q = 2^{16} \approx 65\,000$ queries
  - Max blocks per message: $\ell = 2^{10} \approx 16$ kB
  - Number of Vrfy-queries: $v = 2^{20}$
  - $\mathbf{Adv}^{\text{prp}}_{E}(B) \approx 0$

$$\mathbf{Adv}^{\text{uf-cma}}_{\text{DES-CMAC}}(A) \leq \frac{1}{2^{19}}$$

PMAC

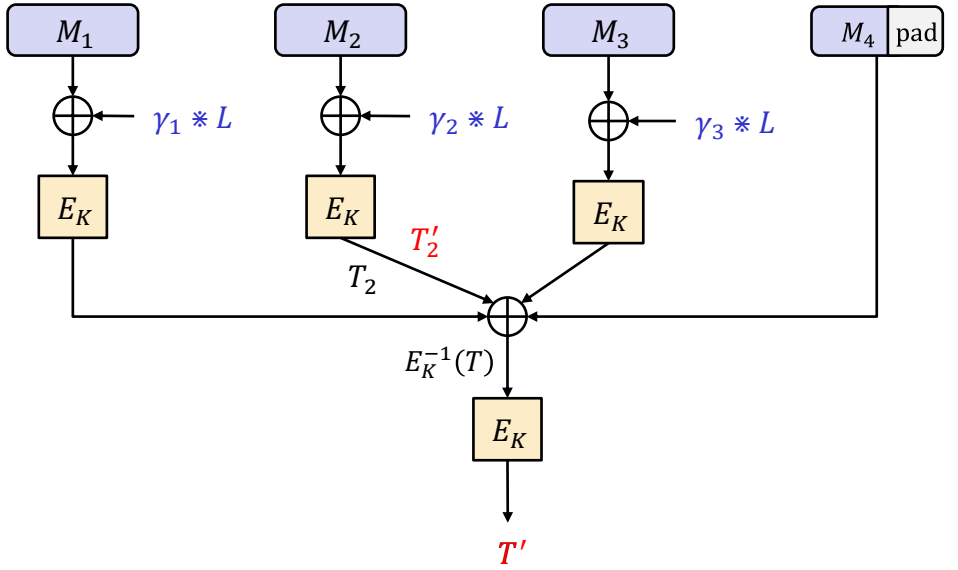


$L = E_K(0^n)$

γ_i = Gray codes

PMAC properties

- **Theorem:** UF-CMA secure if E is a secure PRP
- One key
- Fully parallelizable
- Incremental
- …not used in practice



$$\mathrm{PMAC}_K(M_1||M_2||\ldots||M_{1000}) = T$$

$$\mathrm{PMAC}_K(M_1||M_2'||\ldots||M_{1000}) = E_K(E_K^{-1}(T) \oplus T_2 \oplus T_2') = T'$$

$$T_2 = E_K(M_2 \oplus \gamma_2 * L)$$

$$T_2' = E_K(M_2' \oplus \gamma_2 * L)$$

Summary

- UF-CMA the right security notion for message integrity
  - Does not cover replay attacks
- PRFs are good MACs
  - But only for short (fixed) input length
- CBC-MAC good MAC for messages of a ***single*** *fixed* length
- CMAC upgrades CBC-MAC to variable-length messages