



TEK4500 – Introduction to Cryptography

Problem Set 7

Fall 2025

Problem 1.

Read Sections 9.1, 9.3, 9.5 and Sections 10.1–10.2 in [BR], and Sections 14.1–14.2 in [Ros].

Problem 2.

Let $G_3 = \{e, a, b\}$ and $G_4 = \{e, a, b, c\}$. Decide whether $(G_3, \diamond)$, $(G_3, \triangleright)$, $(G_4, \odot)$, $(G_4, \oslash)$, $(G_4, \square)$ are groups, where $\diamond, \triangleright, \odot, \oslash, \square$ are the binary operations defined by the following multiplication tables.

$\diamond$	e	a	b
e	e	a	b
a	b	e	e
b	a	b	a

$\triangleright$	e	a	b
e	b	e	a
a	e	a	b
b	a	b	e

$\odot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	c	b	a	e
c	b	c	e	a

$\oslash$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

$\square$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

Problem 3.

- List all elements of $\mathbf{Z}_{17}$.
- List all elements of $\mathbf{Z}_{17}^*$.

Exp_{G,g}^{dlog}(A):

- 1: $x \xleftarrow{\$} \{0, 1, \dots, |\mathbf{G}| - 1\}$
- 2: $X \leftarrow g^x$
- 3: $x' \leftarrow A(X)$
- 4: **return** $x' \stackrel{?}{=} x$

Adv_{G,g}^{dlog}(A) = Pr[Exp_{G,g}^{dlog}(A) $\Rightarrow$ true]

Adv_{G,g}^{dh}(A) = Pr[Exp_{G,g}^{dh}(A) $\Rightarrow$ true]

Exp_{G,g}^{dh}(A):

- 1: $x, y \xleftarrow{\$} \{0, 1, \dots, |\mathbf{G}| - 1\}$
- 2: $X \leftarrow g^x$
- 3: $Y \leftarrow g^y$
- 4: $z \leftarrow A(X, Y)$
- 5: **return** $g^z \stackrel{?}{=} g^{xy}$

Figure 1: Formal security experiments for the discrete logarithm (DLOG) problem and the Diffie-Hellman problem in a cyclic group $\mathbf{G} = \langle g \rangle$.

- c) Find a generator for $(\mathbf{Z}_{17}, +)$ different from 1.
- d) Find a generator for $(\mathbf{Z}_{17}^*, \cdot)$.

Problem 4.

In this exercise we'll see why $(\mathbf{Z}_n, +)$ —the additive group of integers modulo n —is not a good choice for Diffie-Hellman. Specifically, we'll see that the discrete logarithm problem (DLOG) is very easy to solve in $(\mathbf{Z}_n, +)$ (thus, as we saw in the lecture, the Diffie-Hellman problem is also easy to solve).

- a) Suppose $n = 12490834970001235812402453450348235053453953450092421$ and you're given $X = 7120379023127712316719401209481487111824712987129479 \in \mathbf{Z}_n$. What's the discrete logarithm of X in $(\mathbf{Z}_n, +)$ assuming $g = 1$?
- b) What's the discrete logarithm of X if $g = 7$?

Hint: X is not a prime number (which you can verify [here](#)).

Problem 5.

Given the prime $p = 541$, let $\mathbf{G} = (\mathbf{Z}_p^*, \cdot)$.

- a) What are the possible orders that the subgroups of $\mathbf{G}$ can have?
- b) Is $\mathbf{G}$ a good choice to use for the Diffie-Hellman key exchange protocol?

Problem 6.

Given the prime $p = 402947$. Alice and Bob use the group $(\mathbf{Z}_p^*, \cdot)$ for the Diffie-Hellman key exchange protocol. They use 2 as the generator for $(\mathbf{Z}_p^*, \cdot)$.

- a) Verify that 2 is a generator for $(\mathbf{Z}_p^*, \cdot)$. You can either write a program that verifies this, or you can use the theory in Section 9.3¹ of [BR] to prove it yourself (hint: p is a safe prime).
- b) Suppose $A = 2^a = 97592$ is the value sent from Alice to Bob, and that $B = 2^b = 112792$ is the value sent from Bob to Alice. Implement the [baby-step, giant-step algorithm](#) in a programming language of your choice, and use it to find the key $Z = 2^{ab} \pmod{p}$ shared by Alice and Bob.

Hint: The inverse of 2 in $(\mathbf{Z}_p^*, \cdot)$ is $2^{-1} = 201474 \pmod{p}$

Problem 7.

Formally prove that DH security (according to the right-hand side of Fig. 1) implies DLOG security (according to the left-hand side of Fig. 1).

Hint: Prove the equivalent contrapositive statement: DLOG insecurity implies DH insecurity. That is, suppose you had an algorithm $\mathcal{A}$ that could solve the DLOG problem. Show how you could use $\mathcal{A}$ to create an algorithm $\mathcal{B}$ that solves the DH problem. What's the DH-advantage of $\mathcal{B}$, i.e., what's $\text{Adv}_{G,g}^{\text{dh}}(\mathcal{B})$?

Problem 8. [Problem 9.3 in [KL07]]

Consider the key-exchange protocol shown in Fig. 2. Show that Alice and Bob output the same key. Analyze the security of the scheme (i.e., either argue why an adversary can't obtain the shared key or show a concrete attack).

References

- [BR] Mihir Bellare and Phillip Rogaway. *Introduction to Modern Cryptography*. <https://web.cs.ucdavis.edu/~rogaway/classes/227/spring05/book/main.pdf>.
- [KL07] Jonathan Katz and Yehuda Lindell. *Introduction to Modern Cryptography*. Chapman and Hall/CRC Press, 2007.
- [Ros] Mike Rosulek. *The Joy of Cryptography*, (draft Jan 3, 2021). <https://web.engr.oregonstate.edu/~rosulekm/crypto/crypto.pdf>.

¹In particular, look at Proposition 9.13. Example 9.15 is also useful.

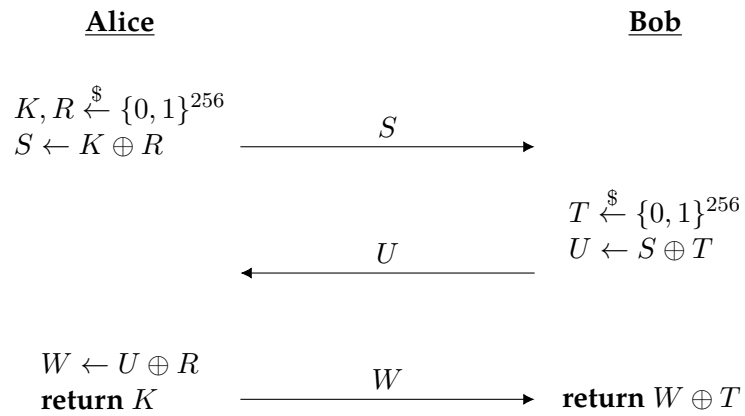


Figure 2: A key-exchange proposal.