Exercises LIO coop 2021

Question 1

a) Calculate the values of the following coalitions...

Marginal contribution net is given

$b \rightarrow 4$	rule 1
$c \rightarrow 6$	rule 2
$a \wedge b \rightarrow 7$	rule 3
$b \wedge c \rightarrow 3$	rule 4

Find the v's (the characteristic function)

$v(\emptyset) = 0$ (empty coalition)
$v(\{a\}) = 0$ (no rule apply)
$v(\{b\}) = 4$ (rule 1)
$v(\{c\}) = 6$ (rule 2)

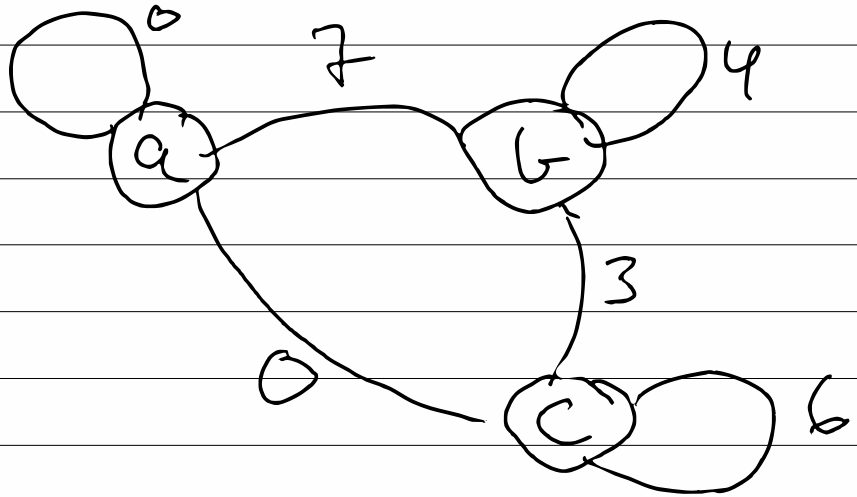
$v(\{a,b\}) = 4+7 = 11$ (Rule 1 & 3)

$v(\{a,c\}) = 6$ (Rule 2)

$v(\{b,c\}) = 4+6+3 = 13$ (Rule 1, 2 & 4)

$v(\{a,b,c\}) = 4+6+7+3 = 20$ (Rule 1, 2, 3, 4)

b) Draw the weighted graph representing this game



c) Is this game stable?

This game is stable if we have a non-empty core

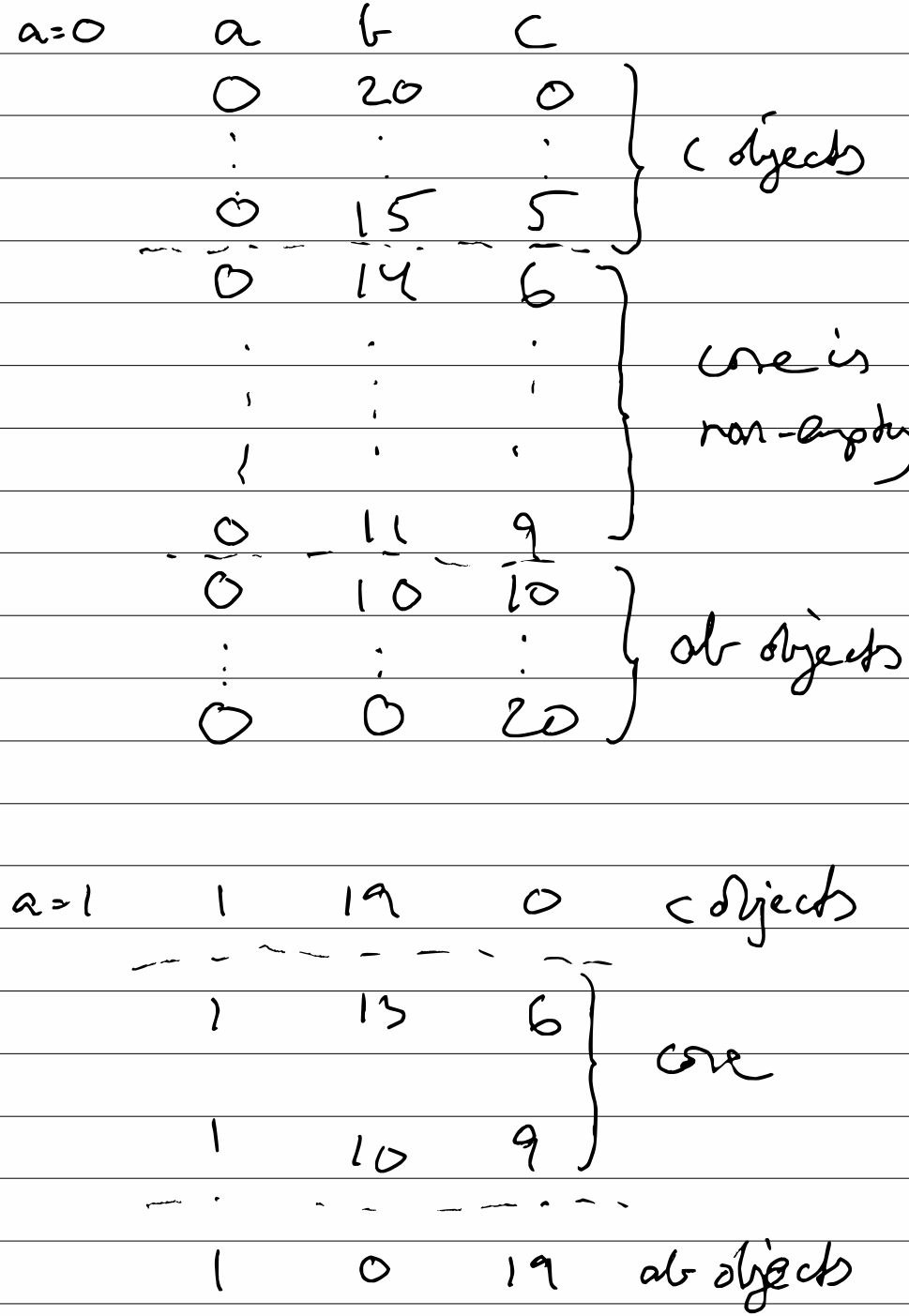
$a \quad b \quad c$

level 1 $\quad 0 \quad 4 \quad 6$ Singleton C

level 2 $\quad 11 \quad 13$

level 3 $\quad 20$ Grand C

lets look at the grand coalition first


a=0
a
b
c
0 20 0
0 15 5
0 14 6
1
0 11 9
0 10 10
0 0 20
objects
core is non-empty
ab objects
a=1
1 19 0
objects
1 13 6
1 10 9
core
1 0 19
ab objects

There exist some non-empty core, i.e. the game is stable

d) Calculate the Shapley value for each player in this game.

$$Shi = \frac{1}{|Ag|!} \sum_{O \in \Pi(Ag)} Mi(Gi(O))$$

where $\Pi(Ag)$ is the set of all possible orderings of coalition C

O is an ordering of a coalition

$Mi(C) = v(C \cup \{i\}) - v(C)$

given that $C \subseteq Ag \setminus \{i\}$,

i.e. the marginal contribution of agent i to C

Two ways of calculating Shapley

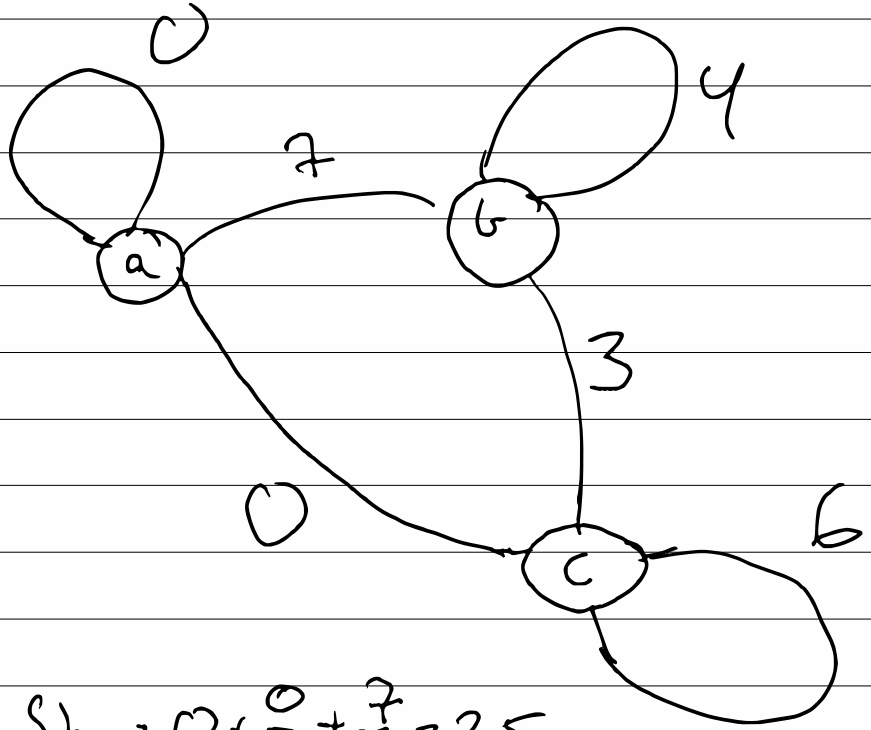
1) Enumeration
2) from the induced subgraph

* Enumeration

Different ordering (permutations)	Marginal contribution of players		
	a	b	c
abc	0	11	9
acb	0	14	6
bac	7	4	9
bca	7	4	9
cab	0	14	6
cba	7	7	6

$Sh_i \Rightarrow$ $21/6 = 3,5$ $54/6 = 9$ $45/6 = 7,5$

* from the induced subgraph



$Sh_a = 0 + \frac{0}{2} + \frac{7}{2} = 3,5$

$Sh_b = 4 + \frac{7}{2} + \frac{3}{2} = 9$

$Sh_c = 6 + \frac{0}{2} + \frac{3}{2} = 7,5$