

Exercise L11 Auction 2021

Question 1

a) What is the valuation function for bidders in terms of the following bundles of good $Z = \{a, b, c\}$

$$\begin{aligned} &v(\{a\}), v(\{b\}), v(\{c\}) \\ &v(\{ab\}), v(\{ac\}), v(\{bc\}) \\ &v(\{abc\}) \end{aligned}$$

given the bids of $A_j = \{1, 2, 3\}$

$$B_1 = (\{a\}, 3) \text{ XOR } (\{c\}, 1) \text{ XOR } (\{a, b\}, 5) \text{ XOR } (\{a, b, c\}, 7)$$

$$B_2 = (\{c\}, 5) \text{ XOR } (\{ab\}, 6) \text{ XOR } (\{a, b, c\}, 14)$$

$$B_3 = (\{b\}, 3) \text{ XOR } (\{c\}, 4) \text{ XOR } (\{a, b\}, 11) \text{ XOR } (\{b, c\}, 15)$$

ps Only one bundle per agent

	1	2	3
$v(\{a\})$	3	0	0
$v(\{b\})$	0	0	3
$v(\{c\})$	1	5	4
$v(\{a,b\})$	5	6	11
$v(\{a,c\})$	3	5	4
$v(\{b,c\})$	1	5	15
$v(\{a,b,c\})$	7	14	15

4) Determine the winner in this auction assuming auctioneer is maximizing social welfare. (Hint, enumerate all possible bundles)

	1	2	3	T
$\{a\}, \{b\}, \{c\}$	$\{a\} = 3$	$\{c\} = 5$	$\{b\} = 3$	11
$\{ab\}, \{c\}$	0	$\{c\} = 5$	$\{ab\} = 11$	16
$\{ac\}, \{b\}$	0	$\{ac\} = 5$	$\{b\} = 3$	8
$\{bc\}, \{a\}$	$\{a\} = 3$	0	$\{bc\} = 15$	18
$\{abc\}$	0	0	$\{abc\} = 15$	15

\Rightarrow We would give $\{a\}$ to $A_{S_1} = 3$,
 $\{c\}$ to $A_{S_2} = 5$ and $\{b\}$ to $A_{S_3} = 15$
 if max SD

c) What is the price each agent
 must pay if we use the
 VCG mechanism instead?

$\overline{A_{j_1}}$	1	2	3	T
$\{a\}\{b\}\{c\}$	x	$\{c\}=5$	$\{b\}=3$	8
$\{ab\}\{c\}$	x	$\{c\}=5$	$\{ab\}=11$	16
$\{ac\}\{b\}$	x	$\{ac\}=5$	$\{b\}=3$	8
$\{bc\}\{a\}$	x	0	$\{bc\}=15$	15
$\{abc\}$	x	0	$\{abc\}=15$	15

$$p_i = \sum_{j \in A_{j_1}/\{i\}} (A_{j, \overline{A_{j_1}}} - A_{j, A_{j_1}})$$

$$p_i = (5 - 0) + (11 - 15) = 5 - 4 = \underline{\underline{1}}$$

A_2	1	2	3	T
{a} {b} {c}	{a}=3	x	{c}=4	7
{a} {b} {c}	{c}=1	x	{a}=11	12
{a} {b} {c}	{a}=3	x	{b}=3	6
{b} {c} {a}	{a}=3	x	{b}=15	<u>18</u>
{a} {b} {c}	0	x	{b}=15	15

$$n_2 = (3-3) + (5-15) = \underline{\underline{0}}$$

Dummy player

$\overline{A_3}$	1	2	3	T
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$\{a\} \{b\} \{c\}$	$\{a\} = 3$	$\{c\} = 5$	\times	8
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$\{ab\} \{c\}$	$\{ab\} = 5$	$\{c\} = 5$	\times	10
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$\{ac\} \{b\}$	0	$\{ac\} = 5$	\times	5
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$\{bc\} \{a\}$	$\{a\} = 3$	$\{bc\} = 5$	\times	8
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$\{abc\}$	0	$\{abc\} = 14$	\times	<u>14</u>
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$$p_3 = (0 - 3) + (14 - 0) = -3 + 14 = \underline{\underline{11}}$$