Exercises L12 Bargaining 2021

- Question 1

a) What would be a suitable protocol for bargaining in this case? Specify the needed requirements.

We have to reallocate resources among agents in order to increase mutual benefit.

Protocol for resource allocation
1) Z^0 is defined as current allocation
2) Any agent can propose a new allocation
$\langle Z^0, Z, \bar{p} \rangle$
where Z^0 is current allocation
Z is proposed allocation
$\bar{p}$ is sidepayments

3) If this deal is
a) Accepted by all agents and
b) Termination criteria is met
then z is implemented with
$\bar{p}$ side payments
4) If this deal is
a) Accepted by all agents
b) But termination criteria is not met
then z is implemented with side payment and negotiation continue with next agent performing step 2 using $z^{\circ} = z$
5) If this deal is
a) not accepted by all agents
then $z^{\circ} = z^{\circ}$ and negotiations continue with next agent performing step 2
6) If all rational proposals have been rejected current z° is implemented

Requirements:

1) Individual rationality

I $v_i(Z) - p_i > v_i(Z^0)$ buy

II $v_i(Z) + p_j > v_i(Z^0)$ sell

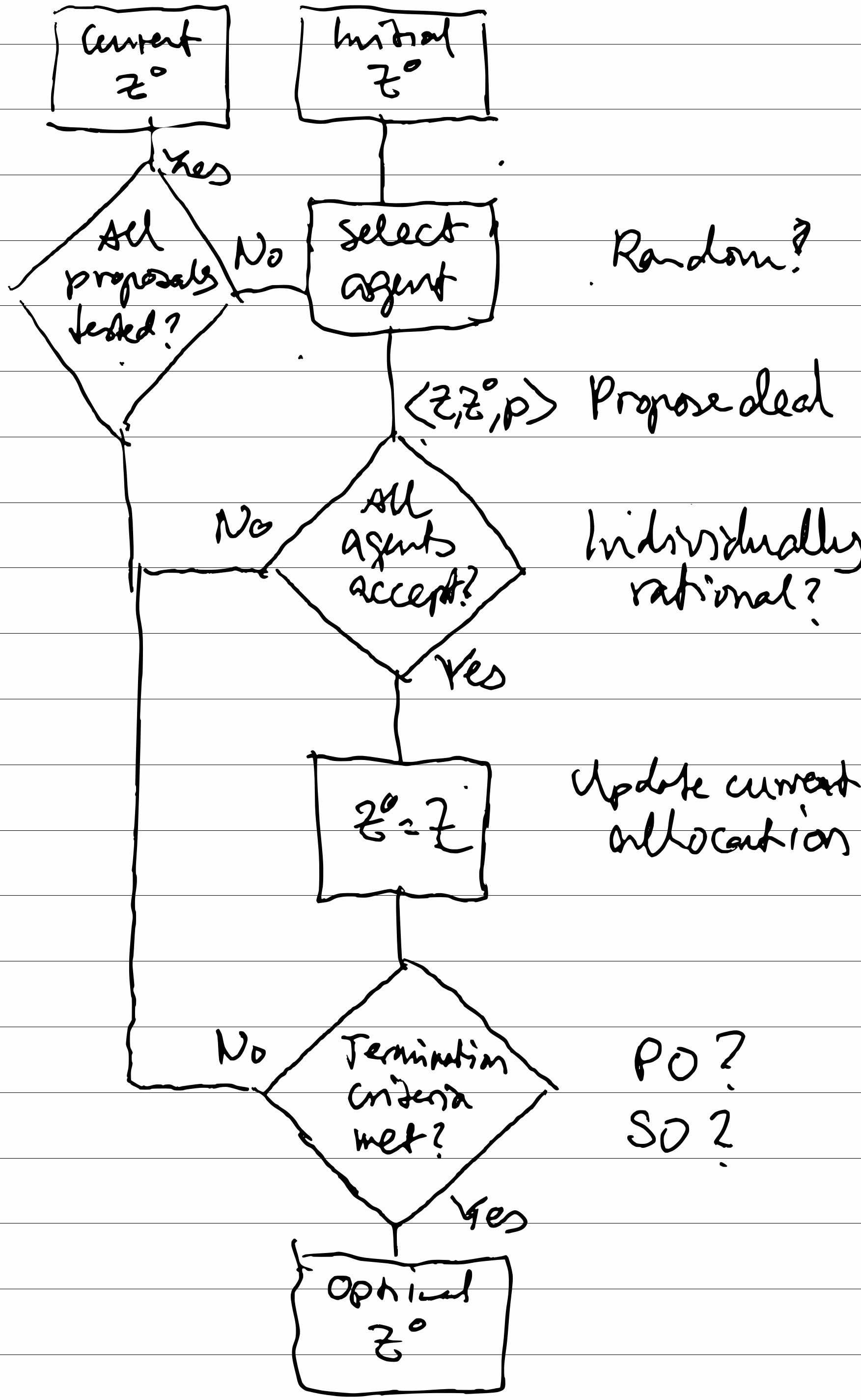
$$\sum_{i \in A_j}^{N} p_i = 0$$

2) Termination criteria

Pareto optimality or
Social optimum

PO is guaranteed if all deals
are individually rational
In one-contracts SO is reachable
but not guaranteed

PO is NP-hard to calculate


Current z^0
Initial z^0
Yes
All proposals tested?
No
Select agent
Random?
$\langle z, z^0, p \rangle$
Propose deal
No
All agents accept?
Individually rational?
Yes
$z^0 = z$
Update current allocation
No
Termination criteria met?
PO?
SO?
Yes
Optimal z^0

↳ What is the set of possible allocations? Could you calculate the social welfare of the different allocations? What allocations are Pareto optimal if no side payments are allowed?

$v_1(\{z_1\}) = 4 \quad v_1(\{z_2\}) = 1$

$v_2(\{z_1\}) = 5 \quad v_2(\{z_2\}) = 7$

Ag1	Ag2	SW	PO
$\{z_1\}+\{z_2\} = 5$ $4+1$	$\{\varphi\} = 0$ 0	5	Yes
$\{z_1\} = 4$	$\{z_2\} = 7$	11	Yes
$z^0 \Rightarrow \{z_2\} = 1$	$\{z_1\} = 5$	6	No
$\{\varphi\} = 0$ 0	$\{z_1\}+\{z_2\} = 12$ $5+7$	12	Yes

c) What would be allocation if agent 1 is selected to give the first bargaining proposal and side payments are allowed?

- Agent 1 could either
  1, Sell z_2 to Ag_2
  2, Buy z_1 from Ag_2

* Ag_1 sell z_2 in round 1 $z^0(z_2, z_1)$

I $v_2(z_1 z_2) - p_2 > v_2(z_1)$ Rational Ag_2

$12 - p_2 > 5$

$p_2 < 7$

II $v_1(\varphi) + p_2 > v_1(z_2)$ Rat. Ag_1

$0 + p_2 > 1$

$p_2 > 1$

$\Rightarrow Ag_1$ will sell z_2 to Ag_2 if $p_1 > 1$,
Ag_2 will buy z_2 if $p_1 < 7$

* Ag_1 buy z_1 from Ag_2 in round 1

I $v_1(z_1 z_2) - p_1 > v_1(z_2)$ Rational Ag_1

$5 - p_1 > 1$

$p_1 < 4$

II $v_2(\varphi) + p_1 > v_2(z_1)$ Rational Ag_2

$0 + p_1 > 5$

$p_1 > 5$

$\Rightarrow$ Not possible for Ag_1 to buy z_1 from Ag_2 in round 1

Ag_1 will propose a deal to sell z_2 to Ag_2 for a side payment of $p_2 < 7$

$Ag_1 = \{\varphi\} + p_2 = 0 + 7 = 7$

$Ag_2 = \{z_1\} + \{z_2\} - p_2 = 12 - 7 = 5$

This is PO and so allocation so bargaining stops.

Lets check if true...

- Ag_2 is selected round 2
  1) sell z_1
  2) sell z_2

* Ag_2 sell z_1

I $v_1(z_1) - p_1 > v_1(\emptyset)$ Rational Ag_1

$4 - p_1 > 0$

$p_1 < 4$

II $v_2(z_2) + p_1 > v_2(z_1, z_2)$ Rational Ag_2

$7 + p_1 > 12$

$p_1 > 5$

$\Rightarrow$ Not possible to sell z_1

* Ag_2 sells z_2

I $v_1(z_2) - p_1 > v_1(\emptyset)$ Rational Ag_1
$1 - p_1 > 0$
$p_1 < 1$

II $v_2(z_1) + p_1 > v_2(z_1, z_2)$ Rational Ag_2
$5 + p_1 > 12$
$p_1 > 7$

$\Rightarrow$ Impossible to sell z_2

What if Ag_1 in round 1 does not know the valuation function of agent 2?

Ag_1 then proposes to sell z_2 to Ag_2 for $p_2 > 1$

Allocation is the same of goods but different net utility of agents.

$Ag_1 = \{\varphi\} + p_2 = 0 + 1 = 1$
$Ag_2 = \{z_1\} + \{z_2\} - p_2 = 12 - 1 = 11$

- Ag_2 is selected round 2
  1) sell z_1
  2) sell z_2

* Ag_2 sells z_1 to Ag_1

I $v_1(z_1) - p_1 > v_1(\varphi)$ rational Ag_1
II $v_2(z_2) + p_1 > v_2(z_1 z_2)$ rational Ag_2

Same as before!

* Ag_2 sells z_2 to Ag_1

I $v_1(z_2) - p_1 > v_1(\varphi)$ rational Ag_1
II $v_2(z_1) + p_1 > v_2(z_1 z_2)$ rational Ag_2

Same as before!

d) What would be allocation if agent 2 is selected for first proposal instead?

- Ag_2 could either
  1) sell z_1 to Ag_1
  2) buy z_2 from Ag_1

* Ag_2 sell z_1 to Ag_1 in round 1

I $v_1(z_1 z_2) - p_1 > v_1(z_2)$ Rational Ag_1

$$5 - p_1 > 1$$
$$p_1 < 4$$

II $v_2(\emptyset) + p_1 > v_2(z_1)$ Rational Ag_2

$$0 + p_1 > 5$$
$$p_1 > 5$$

$\Rightarrow$ Impossible to sell z_1 to Ag_1

* Ag_2 buy z_2 from Ag_1

I $v_1(\varphi) + p_2 > v_1(z_2)$ Rational Ag_1
$0 + p_2 > 1$
$p_2 > 1$

II $v_2(z_1 z_2) - p_2 > v_2(z_1)$ Rational Ag_2
$12 - p_2 > 5$
$p_2 < 7$

$\Rightarrow$ Ag_2 can buy z_2 from Ag_1
if price is between 1 and 7

Agent 2 will propose to buy z_2 from Ag_2 at a side payment of $p_2 > 1$, giving allocation

$Ag_1 = \{\varphi\} + p_2 = 0 + 1 = 1$
$Ag_2 = \{z_1\} + \{z_2\} - p_2 = 12 - 1 = 11$

This is PO and SO, bargaining stops!

e) This is one-contract bargaining with one resource and one side payment, will always end up in PO (but not guaranteed to be SO).

However, when PO is reached net utility among agents are dependent on bargaining history. Side payments are sensitive to information of other agents valuation function.