

# Exercise L4 Swarm robotics | 2021

## Question 1

a) In terms of using USL for modelling the performance of swarm systems, could you explain the variables in the model?

USL is from parallel processing systems

$$R(N) = C \frac{N}{1 + \alpha(N-1) + \beta N(N-1)}$$

where  $R$  is the performance measure

$N$  is the number of processes/agents

$C$  is a scalar

$\alpha$  is contention (limited resources)

$\beta$  is coherency (lack of communication)

b) Could you describe the performance of the swarm system when best fit of model is given by the parameters  $C=1$  and  $\alpha = \beta = 0$ ?  
Make a plot of the UB2 spanning at least  $N=200$  processes.

$C=1, \alpha = \beta = 0 \Rightarrow$  linear speedup

c) How would that change if best fit is given by  $C > 1$ ,  $\alpha = 0,0001$  and  $\beta = 0$ ?

$C=1, \alpha=0,0001, \beta=0 \Rightarrow$  Sublinear speedup

d) Or  $C=1, \alpha=0,0007, \beta=0,0003$ ?

$C=1, \alpha=0,0007, \beta=0,0003 \Rightarrow$  Decrease

e) Now, given the parameters  $C=0.25$ ,  $\alpha=-0.0335$ ,  $\beta=0.00032$ , how would you characterize the performance of this swarm system?

What is the optimal performance?

$$C=0.25, \alpha=-0.0335, \beta=0.00032$$

⇒ sub, super, optimal, infrence regions

⇒ optimal performance at  $N=57$

f) What would constitute an optimal operation level for this swarm system considering that a loss of processes would have to be replaced with a limited number of robots available?

\* optimal performance derived from graph

1) When  $N$  is unlimited

$$\frac{dR}{dN} = 0 \Rightarrow \text{optimal at } N \approx 57$$

2) When  $N$  is limited and expect a constant probability of loss of agents

$$\frac{d(R/N)}{dN} = 0 \Rightarrow \text{optimal at } N \approx 53$$

★ Analytic method for calculating optimality using the chain rule:

$$\left( \frac{f}{g} \right)' = \frac{f'g - g'f}{g^2}$$

1) The case of unlimited supply of robots

$$\frac{dR}{dN} = 0 \quad R = \frac{C \cdot N}{1 + \alpha(N-1) + \beta N(N-1)} = \frac{f}{g}$$

$$f = C \cdot N$$

$$g = 1 + \alpha(N-1) + \beta N(N-1) \\ = 1 + \alpha N - \alpha + \beta N^2 - \beta N$$

$$f' = C$$

$$g' = \alpha + 2\beta N - \beta$$

$$\frac{dR}{dN} = \frac{C \cdot (1 + \alpha N - \alpha + \beta N^2 - \beta N) - (\alpha + 2\beta N - \beta) \cdot CN}{[1 + \alpha(N-1) + \beta N(N-1)]^2} = 0$$

$$\Rightarrow C + \alpha N - (\alpha + (\beta N^2 - C\beta N - \alpha N - 2\epsilon\beta N^2) + C\beta N = 0$$

$$C - \alpha - C\beta N^2 = 0$$

$$C(1 - \alpha - \beta N^2) = 0$$

$$C = 0 \vee 1 - \alpha - \beta N^2 = 0$$

$$N^2 = \frac{1 - \alpha}{\beta}$$

$$N = \pm \sqrt{\frac{1 - \alpha}{\beta}}$$

$$= \pm \sqrt{\frac{1 - (-0.0335)}{0,00032}} = \pm 56,83$$

$$\underline{\underline{N \approx 57}}$$

2, The case of limited supply of robots

$$\frac{d(R/N)}{dN} = 0 \quad \frac{R}{N} = \frac{C \cdot N}{1 + \alpha(N-1) + \beta N(N-1)} \cdot \frac{1}{N}$$

$$= \frac{C}{1 + \alpha(N-1) + \beta N(N-1)} = \frac{f}{g}$$

$$f = C$$

$$g = 1 + \alpha(N-1) + \beta N(N-1)$$

$$f' = 0$$

$$g' = \alpha + 2\beta N - \beta \quad (\text{same as unlimited})$$

$$\frac{d(R/N)}{dN} = \frac{0 \cdot (1 + \alpha(N-1) + \beta N(N-1)) - (\alpha + 2\beta N - \beta) C}{[1 + \alpha(N-1) + \beta N(N-1)]^2} = 0$$

$$\frac{d(R/N)}{dN} = \frac{-(\alpha + 2\beta N - \beta) C}{[1 + \alpha(N-1) + \beta N(N-1)]^2} = 0$$

$$\Rightarrow -(\alpha + 2\beta N - \beta) C = 0$$

$$C(\alpha + 2\beta N - \beta) = 0$$

$$C = 0 \quad \vee \quad \alpha + 2\beta N - \beta = 0$$

$$2\beta N = \beta - \alpha$$

$$N = \frac{\beta - \alpha}{2\beta}$$

$$= \frac{0,00032 - (-0,0335)}{0,00032}$$

$$= 52,84$$

$$\underline{\underline{N \approx 53}}$$