Exercises from lecture 6 (Task allocation and self-assembly) TEK5010 Multiagent systems 2021

Question 1

a) Could you model and explain the stimuli response function T_{θ_i}?

b) Two types of workers in a swarm with different response thresholds θ_1 and θ_2 reacting to a stimulus s can be modelled by the coupled differential equations:

$$\frac{\partial x_1}{\partial t} = T_{\theta_1}(s)(1 - x_1) - px_1$$

$$\frac{\partial x_2}{\partial t} = T_{\theta_2}(s)(1 - x_2) - px_2$$

$$\frac{\partial s}{\partial t} = \delta - \frac{\alpha}{N}(N_1 + N_2)$$

Could you explain the variables used and describe the dynamics of the system?

c) An analytic solution to the above differential equation in terms of the probability of finding an active worker of type 1 is given by:

$$x_1 = \frac{\chi + \left(\chi^2 + 4f(p+1)(z-1)\left(\frac{\delta}{\alpha}\right)\right)^{1/2}}{2f(p+1)(z-1)}$$

where $\chi = (z-1)\left(f + (p+1)\left(\frac{\delta}{\alpha}\right)\right) - z$ is a shift variable, $z = \theta_1^2/\theta_2^2$ and $f = n_1/N$ is the fraction of type 1 worker in the population.

Could you model the average fraction of active workers x_1 as a function of the fraction f of worker of type 1 in the population using parameters $\theta_1 = 2$, $\theta_2 = 6$, $p = 0.2$, $\delta = 1$ and $\alpha = 3$.

d) What happens if $\alpha \approx \delta$? And what happens when $\alpha \gg \delta$? Explain.