Exercise L9 Voting 2021

Question 1

a) What outcome is winner if voting is plurality?

$$\bar{\omega}_1 = \{\omega_1, \omega_2, \omega_3, \omega_4\}$$
$$\bar{\omega}_2 = \{\omega_2, \omega_3, \omega_4, \omega_1\}$$
$$\bar{\omega}_3 = \{\omega_3, \omega_4, \omega_1, \omega_2\}$$

No winner in plurality since different first place for all voters

b) Do we have a Condorcet's paradox here?

If ω_1 is winner $\bar{\omega}_2$ prefer $\omega_2, \omega_3, \omega_4$
Yes $\bar{\omega}_3$ prefer ω_3, ω_4

if w_2 is winner $\bar{w}_1$ prefer w_1
yes $\bar{w}_3$ prefer w_3, w_4, w_1

if w_3 is winner $\bar{w}_1$ prefer w_1, w_2
yes $\bar{w}_2$ prefer w_2

$\Rightarrow$ We do have a Condorcet's paradox! No matter what candidate we pick the majority would prefer another candidate.

c) What is tactical voting? Is it possible for voters to manipulate this election? How can we safeguard against strategic manipulation?

Tactical voting is when an agent i unilaterally can influence the ordering of outcomes in the social preference list by missrepresenting their individual preference order

$$f(\bar{\omega}_1, \dots, \bar{\omega}_i', \dots, \bar{\omega}_n) \succ_i f(\bar{\omega}_1, \dots, \bar{\omega}_i, \dots, \bar{\omega}_n)$$

* $\bar{\omega}_1$ would rather have ω_2 than ω_3 or ω_4

  $\bar{\omega}_2$ would rather have ω_3 than ω_1 or ω_4

  $\Rightarrow$ Both $\bar{\omega}_1$ and $\bar{\omega}_2$ could change 1 and 2 place to secure ω_2 or ω_3 being chosen.

* $\bar{\omega}_3$ would rather have ω_4 than ω_1 or ω_2 (or ω_1 over ω_2)

  $\Rightarrow$ not possible for $\bar{\omega}_3$ to manipulate election on its own

(unless 3 rank is considered)

* By making it NP-hard to manipulate elections, e.g. Second-order Copeland.

d) What is the Borda count for each outcome? Apply the Borda rule. Is this outcome Pareto efficient (as in Arrow's theorem)? Explain.

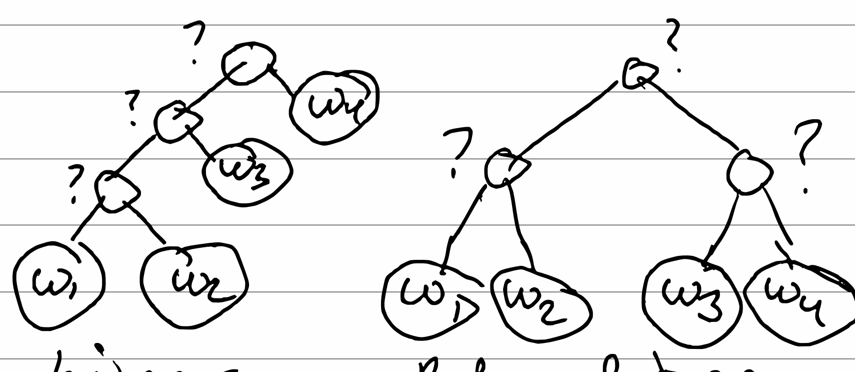
Calculate Borda count

$$BC(w_j) = \sum_{i=1}^{N} k - \text{rank}(\bar{\omega}_i(\omega_j))$$

	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	BC
ω_1	3	0	1	4
w_2	2	3	0	5
w_3	1	2	3	6
w_4	0	1	2	3

⇒ w_3 is optimal outcome corresponding with the Borda rule.
It is not possible to change outcome that makes someone better off without making someone else less off.
This is in accordence with Arrows theorem, BC is PO.

e) We are now going to look at this voting in terms of sequential majority voting. What types of sequential majority voting is there? Sketch the different types.



linear   Balanced tree

$\Rightarrow$ Sequence matters!

f) Write up all winners in pairwise elections and draw a majority graph representing the results. What is the preferred social ordering based on this graph?

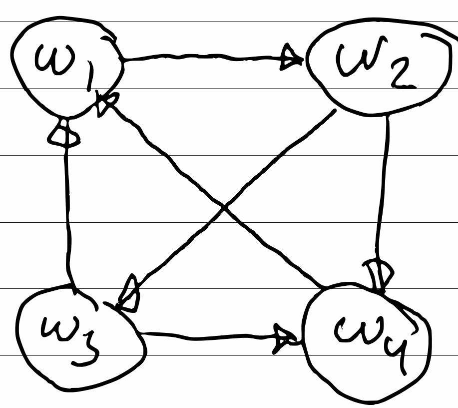
Pairwise elections

$w_1 \succ w_2$ $\quad w_2 \succ w_3$ $\quad w_3 \succ w_4$

$w_1 \succ w_3$ $\quad w_2 \succ w_4$

$w_1 \succ w_4$

All outcomes are possible winners!



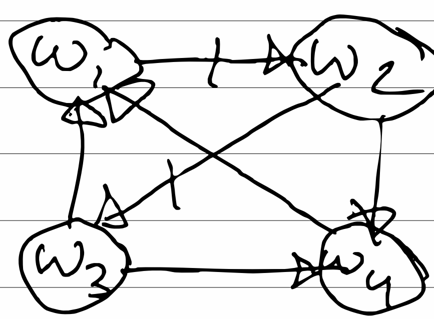
g) What edges in the majority graph must be flipped in order to have a social ordering with a Condorcet winner (assuming minimal number of edges to flip)? What is the best outcome in terms of the Slater rule? Why is the Slater rule problematic in the general case?

We could change

$\omega_3 \prec \omega_2$ to $\omega_3 \succ \omega_2$

$\omega_2 \prec \omega_1$ to $\omega_2 \succ \omega_1$

to have ω_2 or ω_3 as Condorcet winner with only one edge flip



Is this the least SC ?
(All other Condorcet's winners must have a SC > 1)

* Lets look at $w_2 \succ^*$

$w_2 \succ^* w_1 \succ^* w_3 \succ^* w_4$ 3
$w_2 \succ^* w_1 \succ^* w_4 \succ^* w_3$ 4
$w_2 \succ^* w_3 \succ^* w_1 \succ^* w_4$ 2
$w_2 \succ^* w_3 \succ^* w_4 \succ^* w_1$ 1
$w_2 \succ^* w_4 \succ^* w_1 \succ^* w_3$ 3
$w_2 \succ^* w_4 \succ^* w_3 \succ^* w_1$ 2

* Lets look at $w_3 \succ^*$

$w_3 \succ^* w_1 \succ^* w_4 \succ^* w_2$ 3
$w_3 \succ^* w_1 \succ^* w_2 \succ^* w_4$ 2
$w_3 \succ^* w_2 \succ^* w_1 \succ^* w_4$ 3
$w_3 \succ^* w_2 \succ^* w_4 \succ^* w_1$ 2
$w_3 \succ^* w_4 \succ^* w_1 \succ^* w_2$ 2
$w_3 \succ^* w_4 \succ^* w_2 \succ^* w_1$ 2

⇒ According to the Slater rule the social outcome

$$\omega_2 \succ^* \omega_3 \succ^* \omega_4 \succ^* \omega_1$$

is the most socially acceptable

⇒ SR/SC is generally NP-hard!

h, In terms of the analysis above, which task should the robots do first?

Summary of analysis:

Plurality	No ranking possible
Borda	$\omega_3 \succ \omega_2 \succ \omega_1 \succ \omega_4$
Slater	$\omega_2 \succ \omega_3 \succ \omega_4 \succ \omega_1$

For low $|\Omega|$ Slater
For high $|\Omega|$ Borda