

# Lecture 8.4

## Pose from epipolar geometry

Thomas Opsahl



# Introduction

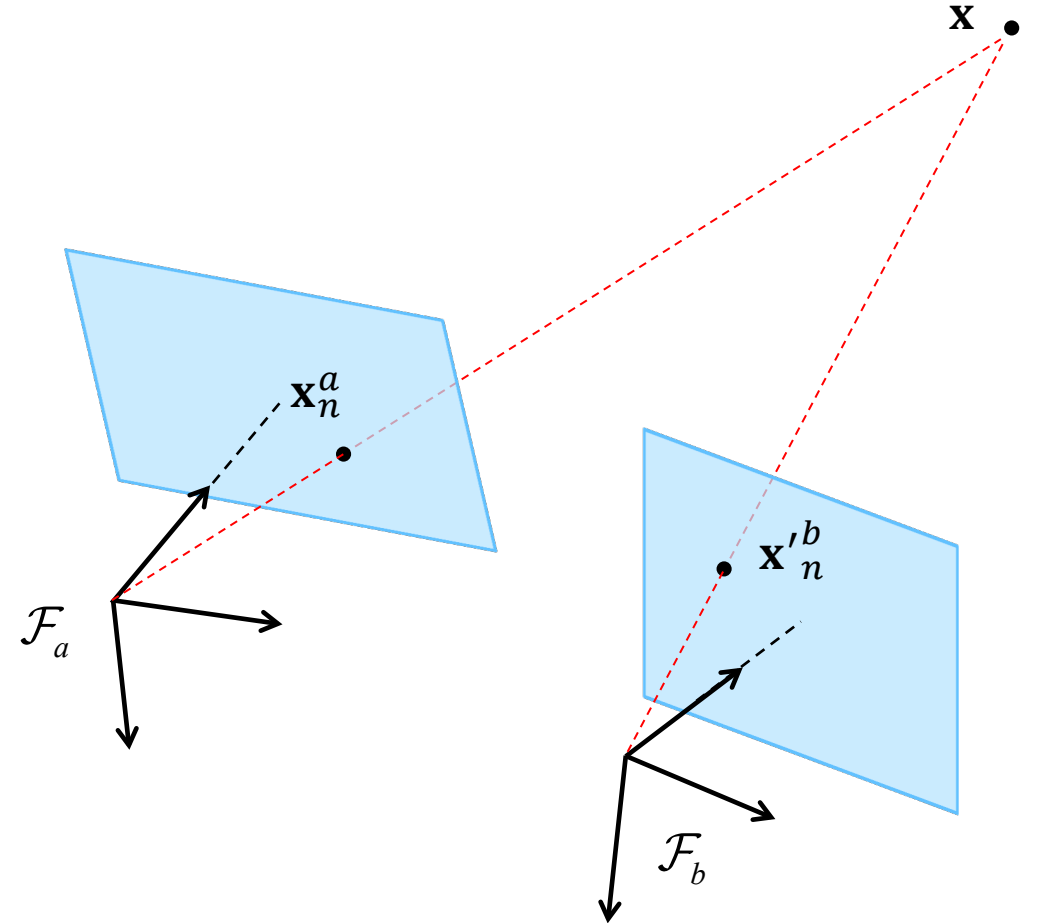
The essential matrix  $\mathbf{E}$  represents the epipolar constraint on normalized image points  $\mathbf{x}_n^a$  and  $\mathbf{x}_n^{b'}$  that correspond to the same 3D point  $\mathbf{x}$

$$\left(\tilde{\mathbf{x}}_n^{b'}\right)^T \mathbf{E}_{ba} \tilde{\mathbf{x}}_n^a = 0$$

The essential matrix is closely related to the relative pose between the cameras

$$\mathbf{E}_{ba} = \left(\mathbf{t}_{ba}^b\right)^\wedge \mathbf{R}_{ba}$$

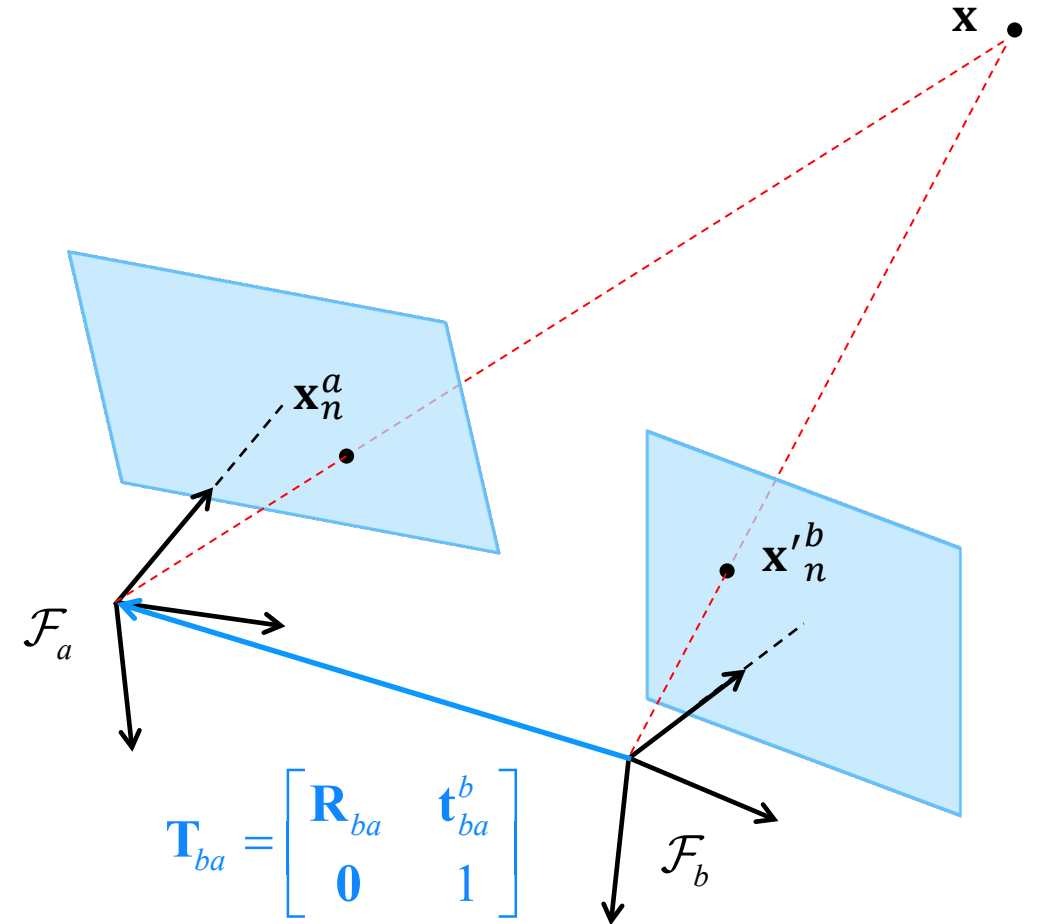
But  $\mathbf{E}$  is homogeneous by nature, so the equality is only up to scale



# Introduction

- The essential matrix can be estimated from five or more point correspondences  $\mathbf{x}_n^a \leftrightarrow \mathbf{x}'_n^b$
- One of the most important results in computer vision is that for a given essential matrix it is possible to determine the relative pose between the cameras up the scale of the translation vector

$$\mathbf{E}_{ba} \rightarrow \mathbf{T}_{ba} = \begin{bmatrix} \mathbf{R}_{ba} & \lambda \mathbf{t}_{ba}^b \\ \mathbf{0} & 1 \end{bmatrix}$$



# Pose from epipolar geometry

One of the properties of  $\mathbf{E}_{ba}$  is that it only has two nonzero singular values that always are equal

This means that we can force these singular values to be 1 by rescaling  $\mathbf{E}_{ba}$

Then its singular value decomposition has the form

$$\mathbf{E}_{ba} = \mathbf{USV}^T = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \mathbf{v}_3^T \end{bmatrix}$$

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There are four different poses that satisfy the equation  $\mathbf{E}_{ba} = (\mathbf{t}_{ba}^b)^\wedge \mathbf{R}_{ba}$

$$\mathbf{T}_{ba,1} = \begin{bmatrix} \mathbf{U}\mathbf{W}\mathbf{V}^T & \mathbf{u}_3 \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{ba,2} = \begin{bmatrix} \mathbf{U}\mathbf{W}\mathbf{V}^T & -\mathbf{u}_3 \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{ba,3} = \begin{bmatrix} \mathbf{U}\mathbf{W}^T\mathbf{V}^T & \mathbf{u}_3 \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{ba,4} = \begin{bmatrix} \mathbf{U}\mathbf{W}^T\mathbf{V}^T & -\mathbf{u}_3 \\ \mathbf{0} & 1 \end{bmatrix}$$

where

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Pose from epipolar geometry

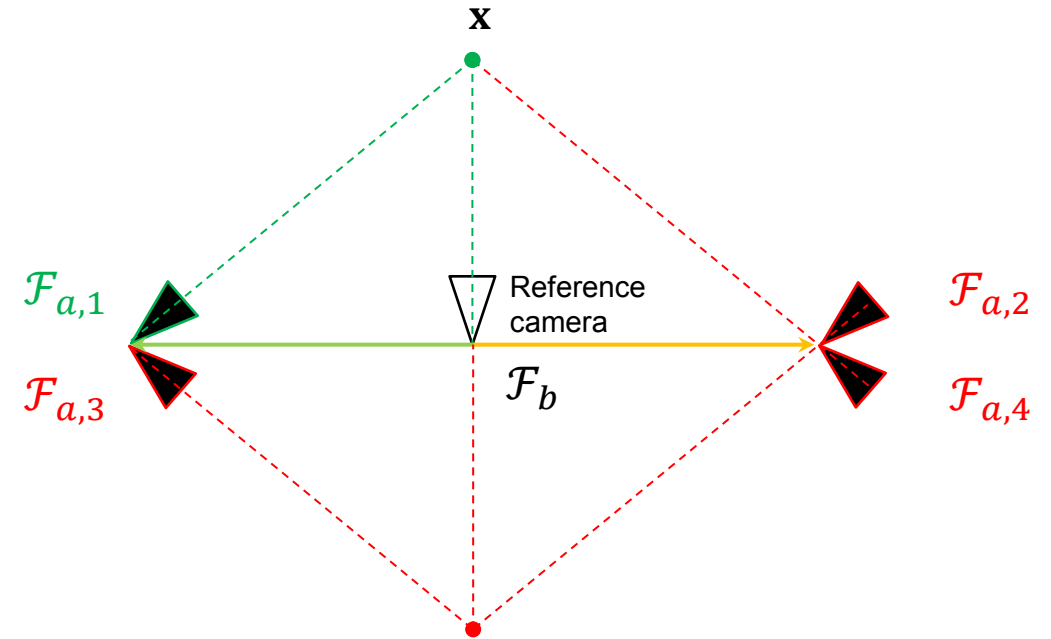
The reason for it being four solutions is that the perspective camera model does not differentiate between points in front of the camera and points that are behind it

The figure illustrates how this might look like for the case when  $\mathbf{T}_{ba,1}$  is the correct pose

$\mathbf{T}_{ba,i}$  is the pose of  $\mathcal{F}_{a,i}$  relative to  $\mathcal{F}_b$

There is no way of predicting the correct pose out of the four, but in general only one of them corresponds to  $\mathbf{x}$  being in front of both cameras

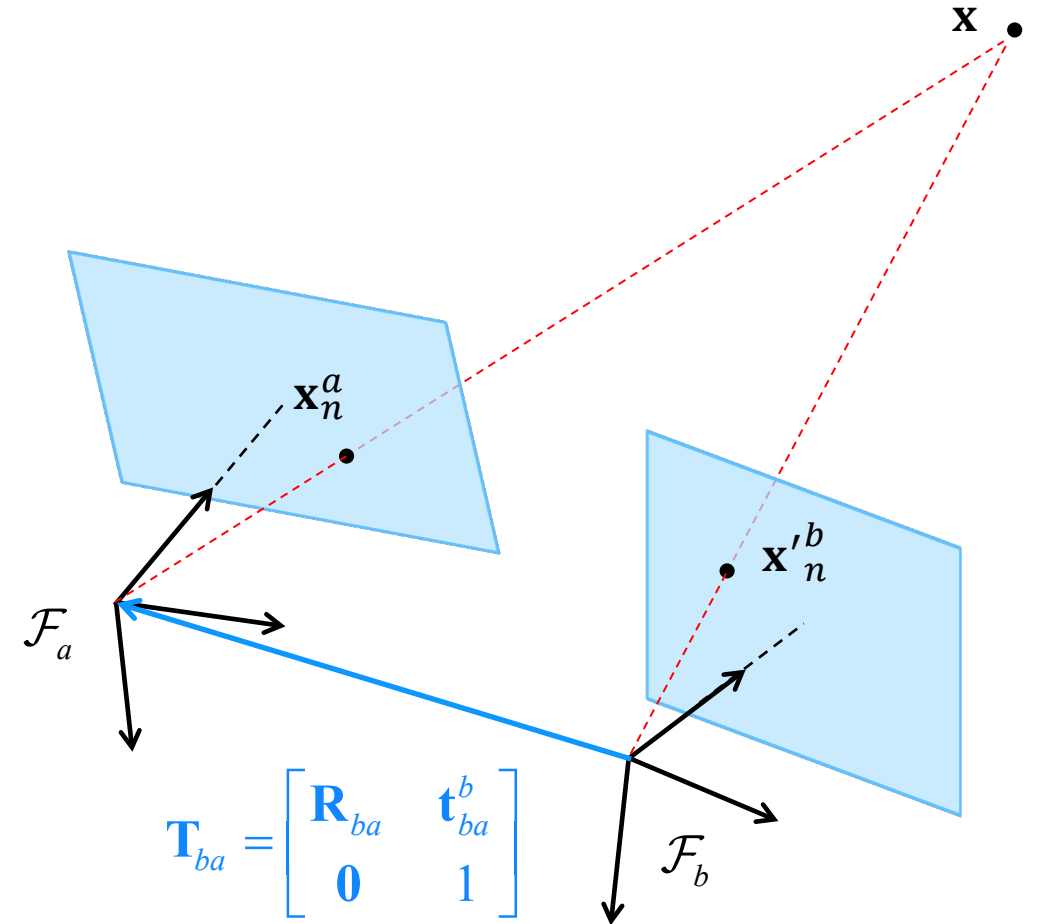
This constraint is known as **the chirality constraint** and it is tested by triangulation of at least one 3D point



# Pose from epipolar geometry

Pose between two calibrated cameras

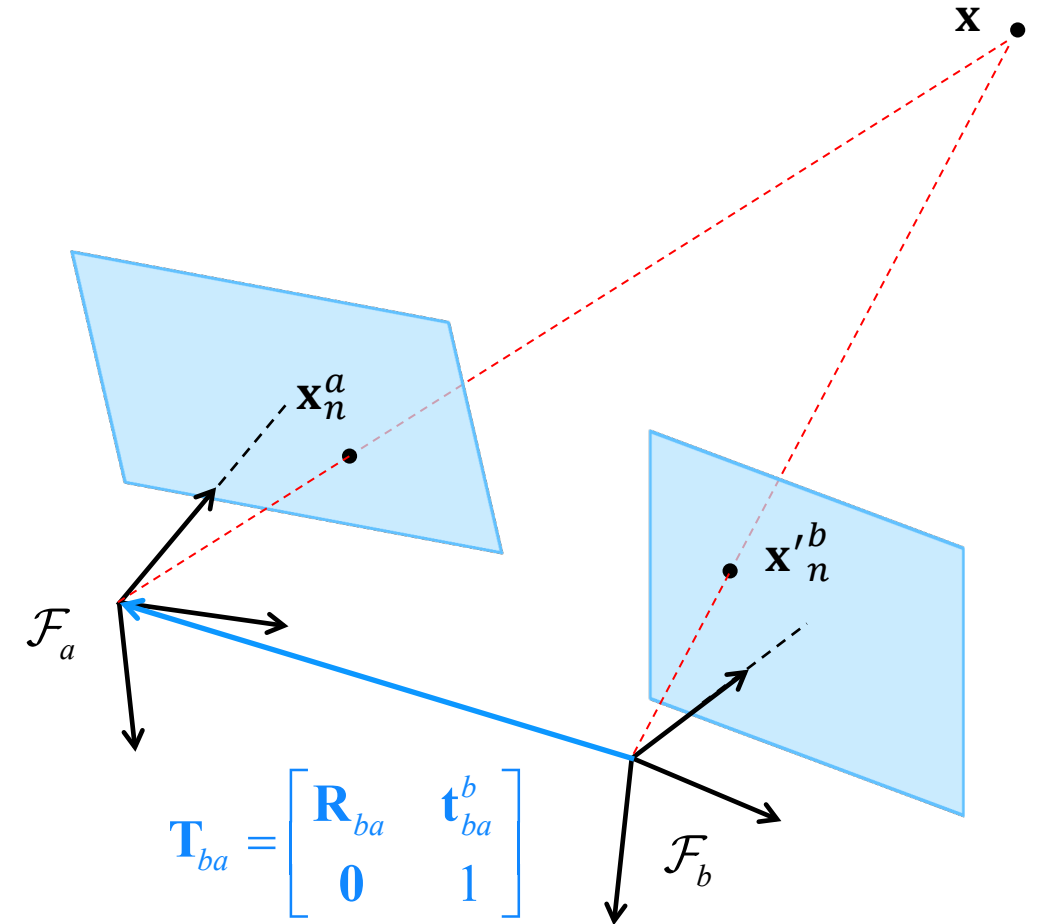
1. Establish robust correspondences  $\mathbf{u}_i^a \leftrightarrow \mathbf{u}_i^b$  between images
2. Determine correspondences  $\mathbf{x}_{n,i}^a \leftrightarrow \mathbf{x}_{n,i}^b$  using that  $\tilde{\mathbf{x}}_n = \mathbf{K}^{-1}\tilde{\mathbf{u}}$
3. Estimate the essential matrix  $\mathbf{E}_{ba}$  from correspondences  $\mathbf{x}_{n,i}^a \leftrightarrow \mathbf{x}_{n,i}^b$
4. Compute poses  $\mathbf{T}_{ba,1}, \dots, \mathbf{T}_{ba,4}$  from  $\mathbf{E}_{ba}$
5. For each pose, determine at least one 3D point  $\mathbf{x}$  by triangulation and select the pose that satisfies the chirality constraint



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$\|\mathbf{t}_{ba}^b\|$  remains unknown!



# Visual odometry – A naïve approach

Based on what we now know it is possible to use a camera for relative navigation

This is called **visual odometry** or just **VO**

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A naïve algorithm can look something like this

1. Capture two frames  $img_0$  and  $img_1$  and establish correspondences  $\mathbf{x}_{n,i}^0 \leftrightarrow \mathbf{x}'_{n,i}{}^1$
2. Estimate  $\mathbf{E}_{1,0}$  and determine the pose of  $\mathcal{F}_1$  relative to  $\mathcal{F}_0$
3. Determine the 3D point cloud  $\{\mathbf{x}_i\}$  by triangulating all correspondences  $\mathbf{x}_{n,i}^0 \leftrightarrow \mathbf{x}'_{n,i}{}^1$
4. Capture  $img_2$  and establish correspondences  $\mathbf{x}_{n,i}^1 \leftrightarrow \mathbf{x}'_{n,i}{}^2$
5. This yields 3D-2D correspondences  $\mathbf{x}_{n,i}^2 \leftrightarrow \mathbf{x}_i$  between  $img_2$  and the point cloud  $\{\mathbf{x}_i\}$
6. Estimate the pose of  $\mathcal{F}_2$  from 3D-2D correspondences  $\mathbf{x}_{n,i}^2 \leftrightarrow \mathbf{x}_i$

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repeat

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# Visual odometry – A naïve approach

It is common to let the first iteration define the scale by setting  $\|\mathbf{t}_{1,0}\| = 1$

The 3D point cloud will then have the same scale and “pass it on” to the next camera pose

Due to noise in measured and computed quantities it is difficult to preserve the scale over many iterations

This is known as **scale drift**

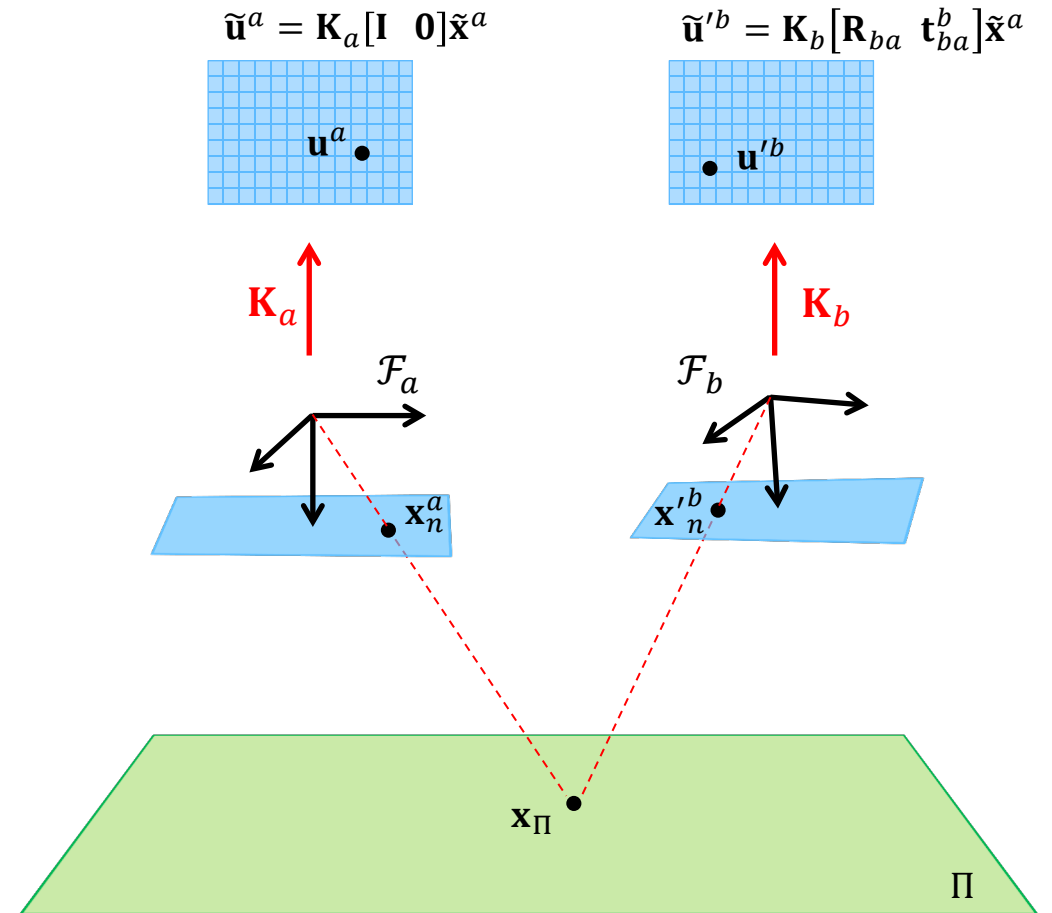
A stereo camera has a built-in scale, the baseline, and is thus better suited for VO than a single camera

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# Planar scene

For planar scenes, it is not possible to estimate the epipolar geometry,  $\mathbf{E}$  or  $\mathbf{F}$ , from correspondences

- For almost planar scenes, the estimation is likely to be ill-conditioned

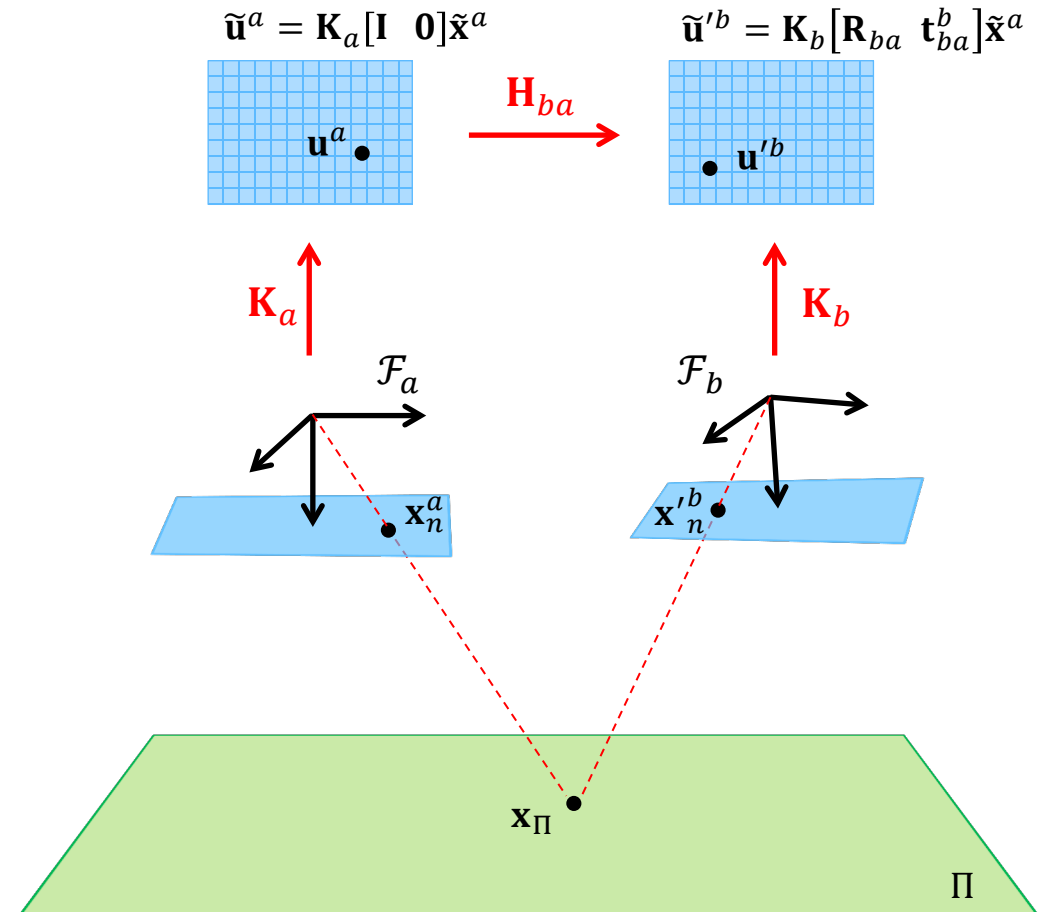


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Two images of a planar scene must however be related by a homography  $\mathbf{H}$



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Two images of a planar scene must however be related by a homography  $\mathbf{H}$

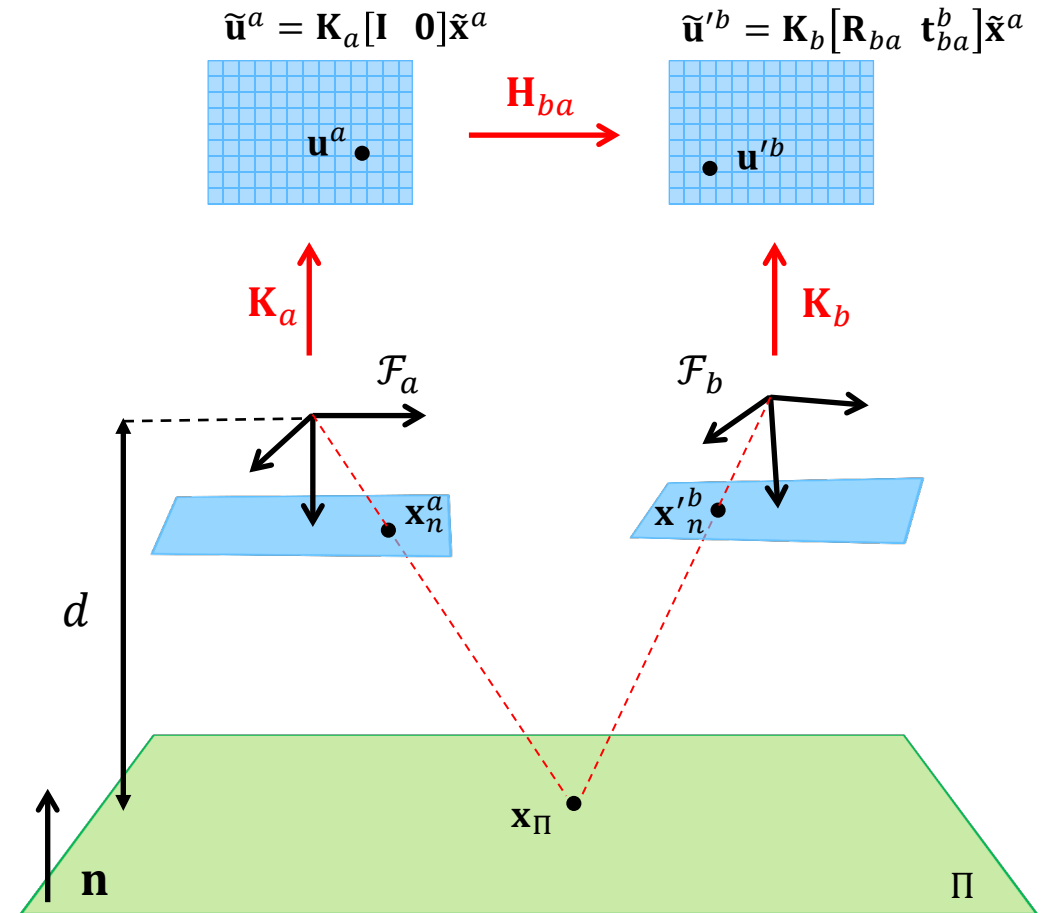
One can prove that if

$$\mathbf{T}_{ba} = \begin{bmatrix} \mathbf{R}_{ba} & \mathbf{t}_{ba}^b \\ \mathbf{0} & 1 \end{bmatrix}$$

then

$$\mathbf{H}_{ba} = \mathbf{K}_b (\mathbf{R}_{ba} - \mathbf{t}_{ba}^b (\mathbf{n}^a)^T / d) \mathbf{K}_a^{-1}$$

where  $\mathbf{n}$  is the normal vector of the plane and  $d$  is the distance of the plane relative to  $\mathcal{F}_a$



# Planar scene

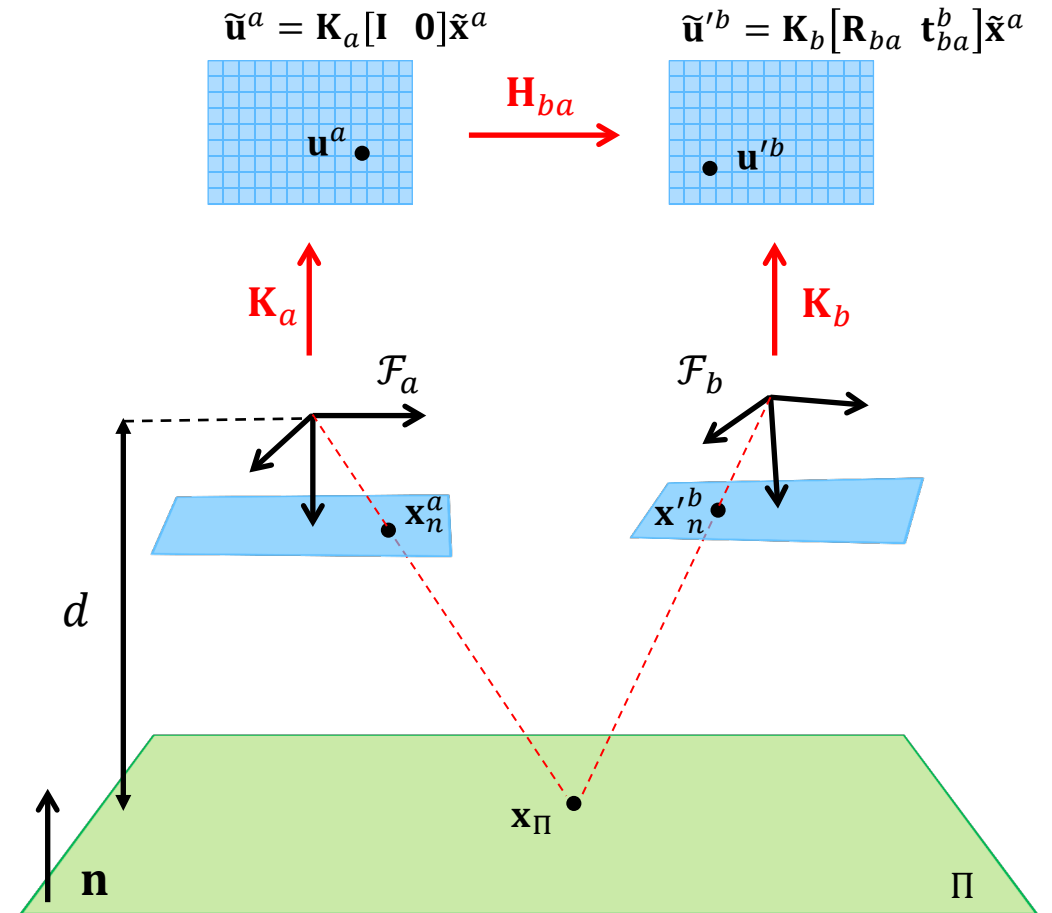
Based on the expression

$$\mathbf{H}_{ba} = \mathbf{K}_b (\mathbf{R}_{ba} - \mathbf{t}_{ba}^b (\mathbf{n}^a)^T / d) \mathbf{K}_a^{-1}$$

it is possible to estimate  $(\mathbf{R}_{ba}, \mathbf{n}^a, \frac{1}{d} \mathbf{t}_{ba}^b)$  from a known homography

This is known as *homography decomposition* and it has been shown to have four solutions

- Two solutions can be invalidated by requiring points to be in front of both cameras
- With some knowledge about the normal vector  $\mathbf{n}$  it is often possible to eliminate one of the two remaining solutions



$$\mathbf{H}_{ba} = \mathbf{K}_b (\mathbf{R}_{ba} - \mathbf{t}_{ba}^b (\mathbf{n}^a)^T / d) \mathbf{K}_a^{-1}$$

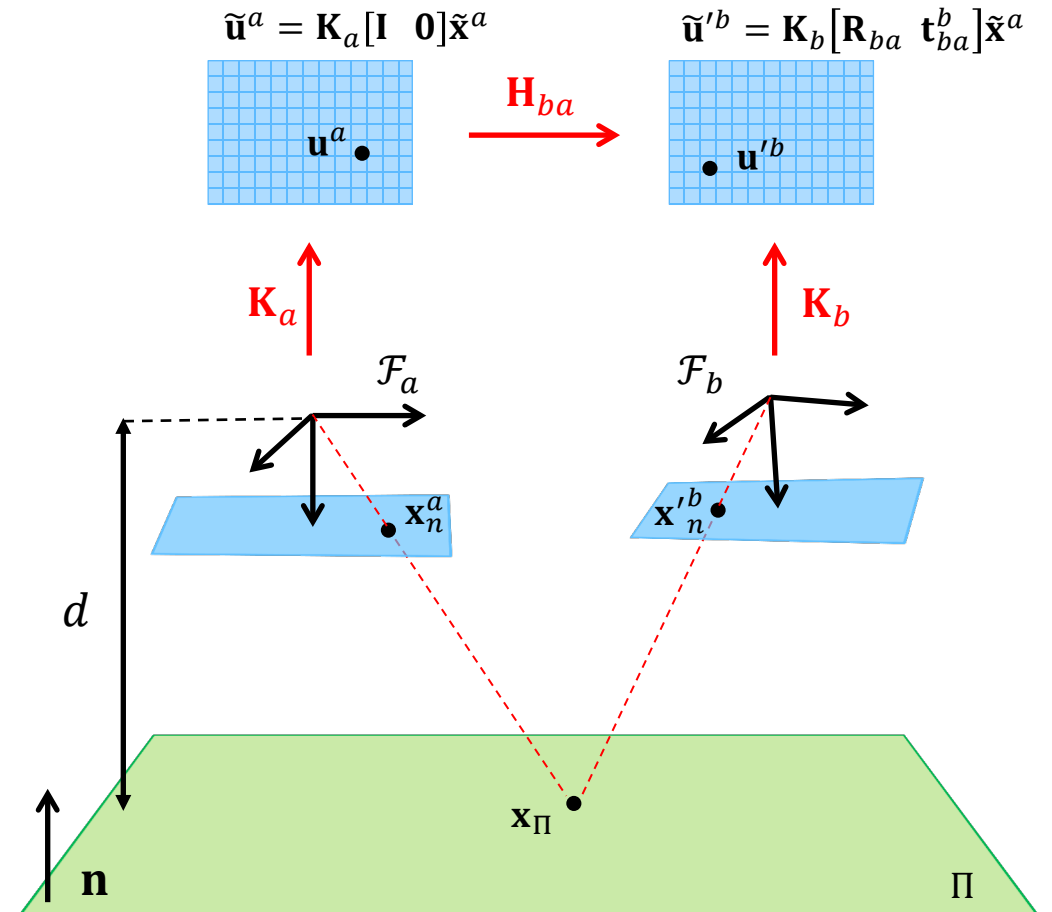


# Planar scene

This can be used for VO in cases where the scene contains planar surfaces that we can detect and know the orientation of

- Indoors
  - walls, floor, ceiling
- In city environments
  - ground, sides of buildings
- Imaging from high altitudes
  - ground

Then we should be able to reject 3 of the 4 poses provided by homography decomposition



$$H_{ba} = K_b (R_{ba} - \mathbf{t}_{ba}^b (\mathbf{n}^a)^T / d) K_a^{-1}$$

# Summary

## Pose from epipolar geometry

From the essential matrix  $\mathbf{E}_{ba}$  we can estimate the relative pose  $\mathbf{T}_{ba}$  between two cameras  $\mathcal{F}_a$  and  $\mathcal{F}_b$

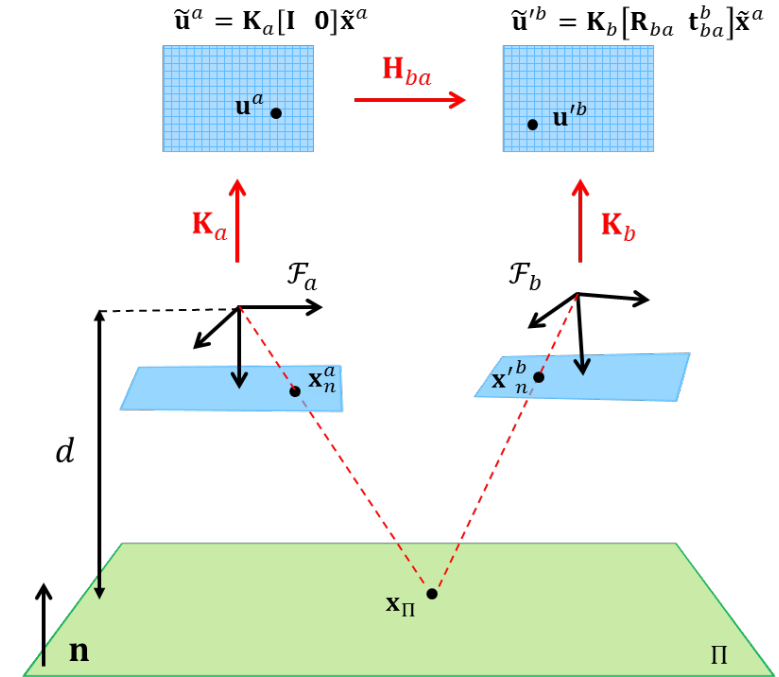
$$\mathbf{E}_{ba} \rightarrow \mathbf{T}_{ba} = \begin{bmatrix} \mathbf{R}_{ba} & \mathbf{t}_{ba}^b \\ \mathbf{0} & 1 \end{bmatrix}$$

But  $\|\mathbf{t}_{ba}^b\|$  is unknown!

## Visual Odometry

We now know methods that can be important components in a VO algorithm

- $\mathbf{u}_i^a \leftrightarrow \mathbf{u}_i^b \rightarrow \mathbf{T}_{ba}$
- $\mathbf{u}_i^a \leftrightarrow \mathbf{u}_i^b \rightarrow \mathbf{x}_i$
- $\mathbf{x}_i^a \leftrightarrow \mathbf{u}_i^b \rightarrow \mathbf{T}_{ba}$



## Planar scenes

It is possible to determine pose from decomposing the homography that exists between overlapping images, but it requires some knowledge about  $\mathbf{n}$

$$\mathbf{H}_{ba} = \mathbf{K}_b (\mathbf{R}_{ba} - \mathbf{t}_{ba}^b (\mathbf{n}^a)^T / d) \mathbf{K}_a^{-1}$$

# Further reading

- Online book by Richard Szeliski – Computer Vision: Algorithms and Applications  
[http://szeliski.org/Book/drafts/SzeliskiBook\\_20100903\\_draft.pdf](http://szeliski.org/Book/drafts/SzeliskiBook_20100903_draft.pdf)
  - Chapter 7.2 covers Two-view structure from motion
- Davide Scaramuzza, *Tutorial on Visual Odometry*, 2012  
[http://rpg.ifi.uzh.ch/docs/Visual\\_Odometry\\_Tutorial.pdf](http://rpg.ifi.uzh.ch/docs/Visual_Odometry_Tutorial.pdf)
- Ezio Malis & Manuel Vargas, *Deeper understanding of the homography decomposition for vision-based control*, 2007  
<https://hal.inria.fr/inria-00174036v3/document>