11.2.9 Finn $\int_0^1 \frac{1-e^{-t}}{t}\,dt$ med nøyaktighet på 10^{-3}.

$$e^x = \underbrace{1 + x + \cdots + \frac{x^n}{n!}}_{T_n e^x} + \underbrace{\frac{e^c}{(n+1)!}x^{n+1}}_{R_n e^x}$$

$x = -t$

$$e^{-t} = 1 + (-t) + \cdots + \frac{(-t)^n}{n!} + \frac{e^{c(t)}}{(n+1)!}(-t)^{n+1}$$

($c(t)$ mellom 0 og $-t$)

$$\frac{1-e^{-t}}{t} = \frac{1-\left(1 - t + \cdots \frac{(-1)^n t^n}{n!} + \frac{e^{c(t)}}{(n+1)!}(-1)^{n+1}t^{n+1}\right)}{t}$$

$$= \frac{t - \cdots + (-1)^{n+1}\frac{t^n}{n!} + (-1)^{n+2}\frac{e^{c(t)}}{(n+1)!}t^{n+1}}{t}$$

$$= \underbrace{1 - \cdots + (-1)^{n+1}\frac{t^{n-1}}{n!}} + \underbrace{(-1)^{n+2}\frac{e^{c(t)}}{(n+1)!}t^n}$$

$$\int_0^1 \frac{1-e^{-t}}{t}\,dt = \underbrace{\int_0^1 \left(1 + \cdots + (-1)^{n+1}\frac{t^{n-1}}{n!}\right)dt}_{\text{tilnærming}} + \underbrace{\int_0^1 (-1)^{n+2}\frac{e^{c(t)}}{(n+1)!}t^n\,dt}_{\text{vil ha } < 10^{-3}}$$

$$\left|\int_0^1 (-1)^{n+2}\frac{e^{c(t)}}{(n+1)!}t^n\,dt\right| \le \int_0^1 \frac{e^{c(t)}}{(n+1)!}t^n\,dt \le \int_0^1 \frac{t^n}{(n+1)!}\,dt \qquad c(t)<0$$

$$= \left[\frac{t^{n+1}}{(n+1)(n+1)!}\right]_0^1 = \frac{1}{(n+1)(n+1)!}$$

Vi må altså finne en n slik at $\frac{1}{(n+1)(n+1)!} < 10^{-3}$

Prøver oss fram, og finner at $n = 5$ er minste slik n.

Tilnærming: $\int_0^1 \left(1 + \cdots + \frac{(-1)^6}{5!}t^4\right)dt$

$$= \int_0^1 \left(1 - \frac{t}{2} + \frac{t^2}{6} - \frac{t^3}{24} + \frac{t^4}{120}\right)dt = \cdots = \frac{5737}{7200}$$

$\underline{\approx 0.7968}$