MAT-IN3110, Autumn 2017, Compulsory assignment 1

Deadline 5 October, 14:30

Assignments should be submitted through the Devilry system.

1 LU

Write a routine, for example in Matlab, to compute the matrices L and U in the factorization LU, without pivoting, of an $n \times n$ matrix A. Test your routine on the matrices

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

2 QR

Write a routine to compute the matrices Q and R of a QR factorization of a non-singular $n \times n$ matrix A, using the Gram-Schmidt algorithm (without pivoting). Test your routine on the matrices in the previous exercise.

3 Best fitting straight line

As the result of an experiment, five (x, y) data points were obtained,

$$(1, 6), \quad (2, 5), \quad (3, 7), \quad (4, 11), \quad (5, 8).$$

We want to find the parabola $y = a + bx + cx^2$ that best fits this data in the sense of least squares. Formulate this problem as the minimization of $\|A\mathbf{x} - \mathbf{b}\|$ and find the solution, i.e., a, b, c, using the normal equations.

4 Positive-definiteness

Define what it means for a symmetric matrix to be positive definite. Show that all the diagonal elements of a positive definite matrix are positive.