UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT-INF 1100 — Modelling and computations.
Day of examination: Thursday, October 13, 2005.
Examination hours: 9:00 – 11:00.
This problem set consists of 4 pages.
Appendices: Formula sheet.
Permitted aids: Certified calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Remember to fill in your candidate number below.

Candidate no.: ________

The first 10 problems each count 2 points, while the last 10 count 3 points each. The total score can therefore be at most 50 points. There are 5 alternative answers to each question, but only one of them is correct. If your answer is wrong or you do not answer a question, you get no points. In other words, there is no penalty for guessing. *Good luck!*

Problem and answer sheets

Problem 1. The binary number 10100101 is the same as the decimal number

☐ 205 ☐ 143 ☐ 301 ☐ 165 ☐ 123

Problem 2. Written in binary form the decimal number −151 becomes

☐ 1100110 ☐ −100011 ☐ −10010111 ☐ −1100101
☐ −1010110

Problem 3. The decimal number 7.25 can be written in binary form as

☐ 111.01 ☐ 111.1 ☐ 1111.111
☐ cannot be written as a binary number
☐ requires infintely many binary digits

Problem 4. The real number $\dfrac{\sqrt{3}}{3-\sqrt{3}} - \dfrac{1}{2}$ is

☐ an irrational number ☐ an imaginary number ☐ 0 ☐ $\sqrt{3}$
☐ a rational number

(Continued on page 2.)

Problem 5. The least upper bound of the set $\{x \in \mathbb{R} \mid x < 0 \text{ and } x^2 - 1 < 3\}$ is

☐ π ☐ -1 ☐ 2 ☐ 0 ☐ $\sqrt{2}$

Problem 6. A sequence is defined by $x_n = n/(n+1)$ for $n \geq 1$. What is the least upper bound for the set given by $\{x_n \mid n \geq 1\}$?

☐ 1 ☐ Does not exist ☐ 0 ☐ 1/2 ☐ π

Problem 7. Suppose that we multiply together the brackets in the expression $(a-1)^{50}$, what will the coefficient multiplying a^{48} be?

☐ 1225 ☐ 1 ☐ 50 ☐ -50 ☐ -2450

Problem 8. Which of the following claims are true?

☐ There is a 32 bit floating point number that exactly agrees with e
☐ There is a 32 bit floating point number that exactly agrees with 1/3
☐ The number of 64 bit floating point numbers is finite
☐ The number 35 has more decimal digits than binary digits
☐ Most irrational numbers can be represented exactly in terms of 64 bit floating point numbers

Problem 9. What will the contents of the variable `s` be after evaluation of the code snippet

```
int i, j, s = 0;
for (i=1; i<3; i++)
{
     j = i*i;
     s += j*j;
}
```

☐ 5 ☐ 0 ☐ `Infinity` ☐ 98 ☐ 17

Problem 10. What will the contents of the variable `p` be after evaluation of the code snippet

```
int i, p = 1;
for (i=1; i<5; i++)
     p *= 1/i;
```

☐ 4 ☐ 1 ☐ `NaN` ☐ 1/24 ☐ 0

Problem 11. The value of the function $f(x) = x^2 - 1$ is to be computed with floating point numbers for $x = 0.9999$. Approximately how many decimal digits will you lose in the computation?

☐ 2 ☐ 10 ☐ 8 ☐ 6 ☐ 4

Hint: The condition number of f at a is given by

$$\kappa(f;a) = \frac{|af'(a)|}{|f(a)|}.$$

(Continued on page 3.)

Problem 12. We define a relative of the bisection method for solving the equation $f(x) = 0$ which we call the trisection method. Instead of dividing the interval into two parts each time, we divide it into three equal parts and choose the subinterval where f has oppsite signs at the ends. If this occurs for several subintervals we choose the subinterval which is furthest to the right on the real line. We start with the interval $[0, 1]$ and know that f is continuous and only has one root in this interval, but we do not know where the root is. Which is the smallest of the given number of iterations that we need to use to be certain that the trisection method gives an absolute error less than 10^{-12}?

□ 11 □ 41 □ 18 □ 27 □ 50

Problem 13. Which one of the following difference equations is linear and has constant coefficients?

□ $x_{n+1} + x_n/n = 1$ □ $x_{n+2} + 6x_{n+1} + 1/x_n = 0$
□ $x_{n+2} - \sin(x_{n+1}) + x_n = -1$ □ $x_{n+2} - x_{n+1} + x_n/3 = 2^n$
□ $x_{n+1} = x_n(1 - x_n)$

Problem 14. The difference equation

$$x_{n+2} - x_{n+1} - 2x_n = n^2$$

has a particular solution in form

□ $x_n = A \sin n$ □ $x_n = An^2 + Bn + C$ □ $x_n = A2^n + Bn2^n$
□ $x_n = A/n + B$ □ $x_n = An^2$
where A, B and C are arbitrary real numbers.

Problem 15. A difference equation is given with two initial values,

$$3x_{n+2} + 5x_{n+1} - 2x_n = 0, \quad x_0 = 7/6, \quad x_1 = 0.$$

What is the solution?

□ $x_n = 7(1 - n)/6$ □ $x_n = 7/6$ □ $x_n = 3^{-n} + (-2)^n/6$
□ $x_n = 7(2^n - n)/6$ □ $x_n = A \sin n$ where A is an arbitrary constant

Problem 16. A difference equation and an initial value are given,

$$x_{n+1} = \frac{x_n}{n+1}, \quad n \geq 0, \quad x_0 = a$$

where a is a real number. What is the solution?

□ $x_n = a/n!$ □ $x_n = an!$ □ $x_n = a/n$ □ There is no solution
□ $x_n = a/n^2$

Problem 17. The difference equation

$$x_{n+1} - 2x_n = n - 1, \quad n \geq 1$$

with initial value $x_1 = 1$ has the solution

□ $x_n = n$ □ $x_n = n^2$ □ $x_n = 2^n - n$ □ $x_n = 2^{n-1}$
□ $x_n = 2^{n+1} - 1$

(Continued on page 4.)

Problem 18. We have written a (correct) Java program that simulates first and second order difference equations with floating point numbers. For which of the following problems will the solution produced by Java tend to zero when $n \to \infty$?

☐ $x_{n+2} + \frac{15}{4} x_{n+1} - x_n = 0, \quad x_0 = 2, \quad x_1 = \frac{1}{2}$
☐ $x_{n+2} + 5x_{n+1}/6 + x_n/6 = 0, \quad x_0 = 1, \quad x_1 = 4$
☐ $x_{n+1} = 2x_n, \quad x_0 = 1$
☐ $x_{n+2} - x_{n+1} - x_n = -1, \quad x_0 = 0, \quad x_1 = -1$
☐ $x_{n+1} - \frac{1}{3} x_n = n, \quad x_0 = 0$

Problem 19. We have the difference equation with initial values given by

$$x_{n+2} + 4x_n = 0, \quad n \geq 0, \quad x_0 = 0, \quad x_1 = 1.$$

What is the solution?

☐ $x_n = n2^n/2$ ☐ $x_n = 2^{1/2} \sin\left(\frac{n\pi}{4}\right)$ ☐ $x_n = \sin\left(\frac{n\pi}{2}\right)$
☐ $x_n = 2^{n-1} \sin\left(\frac{n\pi}{2}\right)$ ☐ $x_n = n$

Problem 20. We are given the claim P_n for $n = 1, 2, \ldots$

$$P_n : \quad |a_1 + \cdots + a_n| = |a_1| + \cdots + |a_n|,$$

which is supposed to hold whenever $a_1, a_2, \ldots, a_n$ are arbitrary real numbers. We try to prove this by induction:

1. P_1 reduces to $|a_1| = |a_1|$ which is obviously true.

2a. Given that $P_1, \ldots, P_n$ are true we must show that

$$P_{n+1} : \quad |a_1 + \cdots + a_{n+1}| = |a_1| + ... + |a_{n+1}|$$

is true. To do this we define $b_1 = a_1 + \cdots + a_n$ and $b_2 = a_{n+1}$. From the induction hypothesis it then follows that

$$|a_1 + \cdots + a_{n+1}| = |b_1 + b_2| = |b_1| + |b_2| \tag{1}$$

by making use of P_2.

2b. From P_n we then find that

$$|b_1| = |a_1 + \cdots + a_n| = |a_1| + \cdots + |a_n|.$$

Inserting this into (1) we obtain

$$|a_1 + \cdots + a_{n+1}| = |a_1| + \cdots + |a_{n+1}|$$

as required, and the proof is complete.

One of the following claims is true, which one?

☐ The proof is wrong because the numbering of a_n is not unique
☐ The claim and the proof are both correct
☐ Part 1 of the proof is wrong
☐ There is an error in part 2a of the proof
☐ There is an error in part 2b of the proof

The end!