UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in	MAT-INF 1100 — Modelling and computations.
Day of examination:	Thursday, October 12, 2006.
Examination hours:	9:00 – 11:00.

This problem set consists of 4 pages.

Appendices:	Formula sheet.
Permitted aids:	Certified calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Remember to fill in your candidate number below.

Candidate no.: ________

The first 10 problems each count 2 points, while the last 10 count 3 points each. The total score can therefore be at most 50 points. There are 5 alternative answers to each question, but only one of them is correct. If your answer is wrong or you do not answer a question, you get no points. In other words, there is no penalty for guessing. *Good luck!*

Problem and answer sheets

Problem 1. The binary number 1001110101 is the same as the decimal number

☐ 831 ☐ 451 ☐ 629 ☐ 600 ☐ 527

Problem 2. Written in binary form the decimal number −481 becomes

☐ −111100001 ☐ −111110011 ☐ −10010111 ☐ −11100101
☐ −10101101

Problem 3. The decimal number 1.2 can be written in binary form as

☐ 1.00110011 ☐ 1.0011 ☐ 1.01 ☐ 1.001
☐ requires infintely many binary digits

Problem 4. The real number $\dfrac{(\sin(\pi/3)-1)^2+\sqrt{3}}{31}$ is

☐ an irrational number ☐ an imaginary number ☐ 0
☐ does not exist ☐ a rational number

Problem 5. A sequence is defined by $x_n = n^2$ for $n \geq 1$. What is the greatest lower bound for the set given by $\{x_n \mid n \geq 1\}$?

☐ 1 ☐ Is not defined ☐ 0 ☐ 1/2 ☐ ∞

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Problem 6. The least upper bound of the set $\{x \in \mathbb{R} \mid x^3 + 4 < 6\}$ is

☐ π ☐ $2^{1/3}$ ☐ 1 ☐ 0 ☐ $\sqrt{2}$

Problem 7. Suppose that we multiply together the brackets in the expression $(b + \sqrt{2})^{30}$, what will the coefficient multiplying b^{28} be?

☐ 435 ☐ 30 ☐ 28 ☐ 870 ☐ 1740

Problem 8. Which of the following claims are true?

☐ Using integers on a computer always leads to round-off errors

☐ In Java there is a number of type `int` that is exactly equal to 4983874

☐ There are infinitely many integers with 10^{10} digits

☐ All rational numbers can be represented exactly with a finite expansion in binary digits

☐ If the square of an irrational number a is an integer, then a must be an even number

Problem 9. What will the contents of the variable `s` be after evaluation of the code snippet

```
int i, j, s = 0;
for (i=1; i<3; i++)
{
     j = i*i;
     s += j/i;
}
```

☐ 3 ☐ 0 ☐ `Infinity` ☐ 5 ☐ 1

Problem 10. What will the contents of the variable `p` be after evaluation of the code snippet

```
int i;
float j, p = 1;
for (i=0; i<5; i++)
{
     j = i;
     p *= (j*j)/j;
}
```

☐ 1 ☐ `NaN` ☐ The program aborts ☐ 24 ☐ 0

Problem 11. The value of the function $f(x) = \ln x$ is to be computed with floating point numbers of type `float` for $x = 1.0001$. Approximately how many decimal digits will be lost in the computation?

☐ None ☐ 16 ☐ 8 ☐ 1 ☐ 4

Hint: We assume that the error, δ, when x is represented by a `float`, is approximately 10^{-8}. Use the relative error defined by

$$\frac{\bigl|f(x+\delta) - f(x)\bigr|}{\bigl|f(x)\bigr|}.$$

You may also make use of the condition number of f given by

$$\kappa(f; a) = \frac{|f'(a)a|}{|f(a)|}.$$

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Problem 12. We are trying to find the zeros of the function $f(x) = (x-3)(x^2-3x+2)$ using the bisection method. We start with the interval $[a,b] = [0,3.5]$, perform 1000 iterations and let x denote the last estimate for the zero. What will the result be?

☐ x close to $\sqrt{2}$ ☐ No convergence ☐ x close to 2

☐ x close to 1 ☐ x close to 3

'Close to' here means that the difference is less than 0.01.

Problem 13. Which one of the following difference equations is linear and has constant coefficients?

☐ $x_{n+1} + nx_n = 1$ ☐ $x_{n+2} - 4x_{n+1} + x_n^2 = 0$

☐ $x_{n+2} - x_{n+1} + x_n = -\sin(x_n)$ ☐ $x_{n+2} + 4x_{n+1} + x_n = \sin(2^n)$

☐ $x_{n+1} = n^2 x_n$

Problem 14. The difference equation

$$2x_{n+2} - x_n = n^2$$

has the particular solution

☐ $x_n = n^2$ ☐ $x_n = n^2 - 3n + 4$ ☐ $x_n = -n^2$

☐ $x_n = n^2 - 8n + 24$ ☐ $x_n = -n^2 - 4n + 12$

Problem 15. A difference equation is given with two initial values,

$$2x_{n+2} + 2x_{n+1} + x_n = 0, \quad x_0 = 1, \quad x_1 = 0.$$

What is the solution?

☐ $x_n = 2^{-n/2}\big(\cos(3n\pi/4) + \sin(3n\pi/4)\big)$

☐ $x_n = 3^{-n/2}\big(\cos(n\pi/2) + \sin(n\pi/2)\big)$ ☐ $x_n = 3^{-n} + (-2)^n/6$

☐ $x_n = 2^{n/2}\big(\cos(3n\pi)\big)$ ☐ $x_n = 2^{n/2}\big(\cos(3n\pi/4) + \sin(3n\pi/4)\big)$

Problem 16. A difference equation and an initial value are given,

$$x_{n+1} = (n+1)^2 x_n, \quad n \geq 1, \quad x_1 = 1.$$

What is the solution?

☐ $x_n = n!$ ☐ $x_n = n$ ☐ $x_n = n^2$ ☐ $x_n = (n!)^2$

☐ $x_n = \big((n-1)!\big)^2$

Problem 17. The difference equation

$$x_{n+1} - 3x_n = 2^n, \quad n \geq 1$$

with initial value $x_1 = 1$ has the solution

☐ $x_n = n$ ☐ $x_n = 3^{n-1}$ ☐ $x_n = 2^{n-1}$ ☐ $x_n = 3^n - 2^n$

☐ $x_n = (3^n + 2^n)/5$

(Continued on page 4.)

Problem 18. We have written a (correct) Java program that simulates first and second order difference equations with floating point numbers. For which of the following problems will the solution produced by Java tend to zero when $n \to \infty$?

☐ $x_{n+1} = 5x_n/2, \quad x_0 = 1/5$

☐ $12x_{n+2} - 7x_{n+1} + x_n = 0, \quad x_0 = 1, \quad x_1 = 2$

☐ $x_{n+1} - x_n = 1/(1+n^2), \quad x_0 = 0$

☐ $6x_{n+2} - 35x_{n+1} - 6x_n = 0, \quad x_0 = 6, \quad x_1 = -1$

☐ $x_{n+2} - 16x_n = -1, \quad x_0 = 0, \quad x_1 = -1$

Problem 19. A second order difference equation has the general solution

$$x_n = C_1 + C_2 8^n, \quad n \geq 0.$$

What is the difference equation?

☐ $x_{n+2} - 9x_{n+1} + 8x_n = 0$ ☐ $x_{n+2} - 9x_{n+1} - 8x_n = 0$

☐ $x_{n+2} + 7x_{n+1} - 8x_n = 0$ ☐ $x_{n+2} + 9x_{n+1} + 8x_n = 0$

☐ $x_{n+2} - 7x_{n+1} + 8x_n = 0$

Problem 20. Let P_n denote the claim

$$\sum_{j=2}^{n} j = \frac{1}{2}n(n+1).$$

A proof by induction that P_n is true for all integers $n \geq 2$ could be as follows:

1. The first claim P_2 is obviously true.
2. Suppose that we have proved that $P_2, \ldots, P_k$ are all true. To complete the proof we must show that then P_{k+1} is also true. Since P_k is true we have

$$\begin{aligned} 2 + \cdots + k + (k+1) &= \frac{1}{2}k(k+1) + k + 1 = \frac{1}{2}k^2 + \frac{3}{2}k + 1 \\ &= \frac{1}{2}(k^2 + 3k + 2) \\ &= \frac{1}{2}(k+1)(k+2). \end{aligned}$$

In other words P_{k+1} is true if P_k is true.
One of the following claims is true, which one?

☐ The claim P_n is true, but part 2 of the proof is wrong

☐ The claim P_n is wrong and part 1 of the proof is wrong

☐ The claim P_n is wrong and both part 1 and part 2 of the proof are wrong

☐ Both the claim P_n and the proof are correct

☐ The proof is correct, but it is not by induction

The end!