Oppg 8.

$$f(x) = \frac{x}{1+x^2} \qquad f'(x) = \frac{1\cdot(1+x^2) - 2x\cdot x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \qquad f'(0) = 1$$

$$f''(x) = \frac{-2x\,(1+x^2)^{\cancel{2}} - 4x(\cancel{1+x^2})(1-x^2)}{(1+x^2)^{\cancel{4}\,3}} = \frac{-6x+2x^3}{(1+x^2)^3} \qquad f''(0) = 0$$

$$f'''(x) = \frac{(-6+6x^2)(1+x^2)^3 - 6x(1+x^2)^2(-6x+2x^3)}{(1+x^2)^6}$$

$$f'''(0) = \frac{-6\cdot 1 - 0}{1} = -6$$

$$T_3(x) = \cancel{f(0)} + f'(0)x + \cancel{\frac{f''(0)}{2}}x^2 + \frac{f'''(0)}{6}x^3$$

$$= x - \frac{6}{6}x^3 = \underline{\underline{x - x^3}}$$

Oppg. 10 $\quad f(x) = \sin x - \frac{2x}{c+x^2} \qquad f(0) = 0$

$$f'(x) = \cos x - \frac{2(c+x^2) - (2x)\cdot(2x)}{(c+x^2)^2} = \dots$$

$$f'(0) = 1 - \frac{2c}{c^2} = 1 - \frac{2}{c}$$, skal være lik $= 0$

(siden Taylorrekken skal bli $x^3/3$)

$1 - \frac{2}{c} = 0 \iff c = 2$ (og stopp der)

Oppg 11 $\sum_{i=0}^{\beta-1} i\beta^i$ er et tall i β-systemet, med β siffer.

Vi vet at $(\beta-1)\beta^{\beta-1} \leq \sum_{i=0}^{\beta-1} i\beta^i < \beta^\beta$

bare tatt med summand $\beta-1$

$\beta = 3$: $< 3^3 = 27$, så ikke denne

$\beta = 4$: $< 4^4 = 256$, så ikke denne

$\beta = 5$: $4 \cdot 5^4 = 2500 \leq$ $< 5^5 = 3125$, så må være denne.

$\beta = 6$: $(\beta-1)\beta^{\beta-1} = 5 \cdot 6^5 > 10000$

Oppg 12.

$f(x) = xe^{-x}$ grad n om $a=0$

$= T_n(x) + R_{n+1}(x)$ $\qquad R_{n+1}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}x^{n+1}$

$f(x) = x\left(1 - x + \frac{x^2}{2!} + \dots + (-1)^n\frac{x^{n-1}}{(n-1)!} + \frac{R_n(\xi)}{n!}x^n\right)$ $\qquad e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$

$= \underbrace{x\left(1 - x + \dots + (-1)^n\frac{x^{n-1}}{(n-1)!}\right)}_{T_n(x)} + \frac{x^{n+1}}{n!}R_n(\xi)$ $\qquad e^{-x} = 1 - x + \frac{x^2}{2!} + \dots + (-1)^n\frac{x^n}{n!} + \dots$

$R_{n,e^{-x}}(x)$

har vist at: $R_{n+1,f}(x) = x R_{n,e^{-x}}(x)$

$|R_{n+1,f}(x)| \le 0.01 \iff |R_{n,e^{-x}}(x)| \le 0.01$

$\left|\frac{f^{(n)}(\xi)}{n!}x^{n+1}\right| \le \left|\frac{e^{-\xi}}{n!}x^{n+1}\right|$

$\le \frac{1}{n!} \le 0.01$

$\frac{1}{n!} \le 0.01 \iff n! \ge 100$

$n \ge 5$