11.1.2

a) $$f(a+h) = f(a) + f'(a)\overset{(a+h)-a}{h} + f''(a)\frac{h^2}{2} + f'''(\xi_h)\frac{h^3}{6}$$

(og deler med h) $\quad \xi_h \in [a, a+h]$

$$f'(a) - \frac{f(a+h)-f(a)}{h} = \underbrace{-\frac{h}{2}f''(a) - \frac{h^2}{6}f'''(\xi_h)}_{\text{har fått et noe mer komplisert uttrykk for feilen (må begrense } f'''(\xi_h)\text{)}}$$

b) $f(a+h) = f(a) + f'(\xi_h)h \quad \xi_h \in [a, a+h]$

men her inngår ikke $f'(a)$, så kan ikke brukes.

$\left(f'(\xi_h) = \frac{f(a+h)-f(a)}{h} \right.$ sier bare at finnes slik $\left. \xi_h \right)$.

c) Det er ikke noe feil fra a), men feilleddet er styggere.

11.5.3

velg c_1, c_2 slik at $f'(a) \approx c_1 f(a-h) + c_2 f(a+h)$

(dvs. slik at ~~tilnærmingen~~ er best mulig)

eksakt for $f(x) = 1$ $\quad f'(a) \quad c_1 f(a-h) + c_2 f(a+h)$

$$0 = c_1 \cdot 1 + c_2 \cdot 1$$

eksakt for $f(x) = x$ $\quad 1 = c_1(a-h) + c_2(a+h)$

får: $\quad 0 = c_1 + c_2 \qquad c_2 = -c_1$

$$1 = a\underbrace{(c_1 + c_2)}_{0} + h(-c_1 + c_2) = h(-2c_1) \Rightarrow c_1 = -\frac{1}{2h}$$

$$c_2 = -c_1 = \frac{1}{2h}$$

derfor $f'(a) \approx -\frac{1}{2h} f(a-h) + \frac{1}{2h} f(a+h) = \frac{f(a+h) - f(a-h)}{2h}$

er eksakt for $f(x) = 1$, $f(x) = x$ (= symmetrisk Newton)

b) anta $f(x) = cx + d \qquad f'(x) = c$

$$\frac{f(a+h) - f(a-h)}{2h} = \frac{c(a+h) + d - (c(a-h) + d)}{2h}$$

$$= \frac{\cancel{ca} + ch + \cancel{d} - \cancel{ca} + ch - \cancel{d}}{2h} = \frac{2ch}{2h} = c$$

slik at metoden er eksakt for alle polynomer av grad ≤ 1.

12.4.2

a) $\int_0^1 \frac{dx}{1+2x}$ $\quad f(x) = \frac{1}{1+2x}$ $\quad f'(x) = -2\left(\frac{1}{1+2x}\right)^2 = -2(1+2x)^{-2}$

trapesmetoden, med feil $\le 10^{-10}$: $\quad f''(x) = 4\cdot 2(1+2x)^{-3} = 8(1+2x)^{-3} \le 8$

feil $\le (b-a)\frac{h^2}{6}\max_{x\in[a,b]}|f''(x)|$ $\quad \frac{1}{1+2x} \le 1$ for $x \in [0,1]$

$= (1-0)\frac{h^2}{6}8 = \frac{4}{3}h^2$

velg h s.a. $\frac{4}{3}h^2 \le 10^{-10} \Rightarrow h \le \frac{\sqrt{3}}{2}10^{-5}$

antall intervaller $= \frac{(b-a)}{h} = \frac{1}{h} = \frac{2}{\sqrt{3}}10^{5} \approx 115470$

antall funksjonsberegninger = antall intervaller $+1 = \underline{\underline{115471}}$

b) midtpunktsmetoden $\quad$ feil $\le (b-a)\frac{h^2}{24}\max_{x\in[a,b]}|f''(x)|$

$\frac{h^2}{3} \le 10^{-10}$ $\quad h \le \sqrt{3}\,10^{-5} \Rightarrow$ antall intervaller $= \frac{1}{\sqrt{3}}10^{5} = 57735$

$\Rightarrow$ 57736 funksjonsberegninger.

c) Simpsons metode: $\quad f^{(3)}(x) = -48(1+2x)^{-4}$

$f^{(4)}(x) = 384(1+2x)^{-5}$

feil $\le (b-a)\frac{h^4}{2880}\max_{x\in[a,b]}|f^{(4)}(x)|$ $\quad |f^{(4)}(x)| \le 384$

$= \frac{h^4}{2880}\cdot 384 \le 10^{-10}$ $\quad \frac{2}{15}h^4 \le 10^{-10}$ $\quad h^4 \le \frac{15}{2}10^{-10}$

$h \le \sqrt[4]{\frac{15}{2}}\,10^{-2.5}$ $\quad h^{-1} =$ antall intervaller $= \frac{10^{2.5}}{\sqrt[4]{\frac{15}{2}}} = \frac{10^{2.5}}{\sqrt[4]{\frac{15}{2}}} \approx 191$

$\Rightarrow$ 192 funksjonsberegninger.