10.2.1

Veksten i befolkningen per år er $0.02\,y(t)$

Tilskudd per år på grunn av innvandring: 40000

Derfor $y'(t) = 0.02y(t) + 40000$

$$\Rightarrow \underbrace{y'(t) - 0.02y(t)}_{f(t)} = \underbrace{40000}_{g(t)} \qquad \Rightarrow F(t) = -0.02t$$

Setning 10.1.3:

$$y(t) = e^{-F(t)}\left(\int e^{F(t)} g(t)\,dt + C\right) = e^{0.02t}\left(\int e^{-0.02t}\, 40000\,dt + C\right)$$

$$= e^{0.02t}\left(\frac{40000}{-0.02} e^{-0.02t} + C\right) = \underline{-2000000 + Ce^{0.02t}}$$

Initialkrav: $y(0) = 2\,000\,000$

Setter inn $t = 0$: $2000\,000 = -2000\,000 + C$

$C = 4000\,000$

$$\Rightarrow \underline{\underline{y(t) = -2000\,000 + 4000\,000\,e^{0.02t}}}$$

10.2.10

$y(t)$ = antall liter klor i badevannet. (1000000 liter totalt)

50000 liter er $\frac{1}{20}$ av 1000 000, slik at mister

$\frac{1}{20}y(t)$ hvert døgn.

hvert døgn gir tilførsel av klor på: $\frac{0{,}001 \times 50000}{100} = 0.5$

Derfor: $y'(t) = -\frac{1}{20}y(t) + 0.5$

$$\Rightarrow y'(t) + \underbrace{\frac{1}{20}}_{f(t)} y(t) = \underbrace{0.5}_{g(t)} \qquad F(t) = \frac{1}{20}t = 0.05t$$

$$y(t) = e^{-0.05t}\left(\int e^{0.05t} 0.5\,dt + C\right) = e^{-0.05t}\left(\frac{0.5}{0.05}e^{0.05t} + C\right)$$

$$= \underline{10 + Ce^{-0.05t}}$$

initialbetingelse: $y(0) = \frac{0{,}004 \times 1000\,000}{100} = \underline{40}$

$$\Rightarrow 40 = 10 + C \Rightarrow C = 30 \Rightarrow \underline{\underline{y(t) = 10 + 30e^{-0.05t}}}$$

klorprosent er nede i 0.003% når $\frac{y(t)}{1000\,000} = \frac{0.003}{100} = 3 \times 10^{-5}$

$y(t) \times 10^{-6} = 3 \times 10^{-5} \Rightarrow \underline{y(t) = 30}$

Vi må altså løse: $10 + 30e^{-0.05t} = 30$

$$e^{-0.05t} = \frac{2}{3} \Rightarrow -0.05t = \ln\frac{2}{3} = \ln 2 - \ln 3$$

$$t = \frac{\ln 2 - \ln 3}{-0{,}05} = 20(\ln 3 - \ln 2) \approx 8.1093$$

Med andre ord: Etter litt mer enn $\underline{\underline{\text{8 dager}}}$ er klorprosenten 0.003%

10.4.18 $[f(x)]^2 = \frac{1}{x}\int_1^x f(t)\,dt \qquad (x>0)$

deriverer: $2f(x)f'(x) = -\frac{1}{x^2}\int_1^x f(t)\,dt + \frac{1}{x}f(x)$

$$= -\frac{1}{x}\left(\frac{1}{x}\int_1^x f(t)\,dt\right) + \frac{1}{x}f(x)$$

$$= -\frac{1}{x}[f(x)]^2 + \frac{1}{x}f(x)$$

setter inn $y=f(x)$: $2yy' = -\frac{1}{x}y^2 + \frac{1}{x}y$

$$yy' = -\frac{1}{2x}y^2 + \frac{1}{2x}y = \frac{y-y^2}{2x}$$

b) Vi kan nå skrive:

(separabel) $\frac{yy'}{y-y^2} = \frac{1}{2x} \Rightarrow \frac{y'}{1-y} = \frac{1}{2x}$

integrerer: $-\ln|1-y| = \frac{1}{2}\ln|x| + C = \frac{1}{2}\ln x + C$

$\ln|1-y|^{-1} = \frac{1}{2}\ln x + C \Rightarrow |1-y|^{-1} = e^{\frac{1}{2}\ln x + C}$

$\frac{1}{1-y} = De^{\frac{1}{2}\ln x} \quad (D = \pm e^C) \quad \left(e^{\frac{1}{2}\ln x} = e^{\ln x^{\frac{1}{2}}} = x^{\frac{1}{2}}\right)$

$1-y = \frac{1}{De^{\frac{1}{2}\ln x}} = \frac{1}{D\sqrt{x}} = Ex^{-\frac{1}{2}}$

$y = 1 - Ex^{-\frac{1}{2}} = \underline{\underline{1 - \frac{E}{\sqrt{x}}}}$