10.5.5 $f(x) = x e^{-2x}$ Løsning

a) må være rot i kar. likning

-2 (rot) er dobbeltrot

kar. likning må ta formen $a(r+2)^2 = 0$

sett $a=1$: $(r+2)^2 = r^2 + 4r + 4 = 0$

En differensiallikning blir $\underline{y'' + 4y' + 4y = 0}$

b) gen. løsning: $\underline{y(x) = Ae^{-2x} + Bxe^{-2x}}$

$y'(x) = -2Ae^{-2x} + Be^{-2x} - 2Bxe^{-2x}$

Løsning der $y(0) = 0$, $y'(0) = 1$

$y'(0) = -2A + B$

setter inn $x = 0$: $A = 0$ $(y(0) = 0)$

$-2A + B = 1 \Rightarrow B = 1$

$\Rightarrow \underline{\underline{y(x) = xe^{-2x}}}$, som er løsningen i a)

10.5.11 $\quad x(t) = N_1(t) - 300$

$y(t) = N_2(t) - 10000$

$x'(t) = b\,y(t) \qquad y'(t) = -c\,x(t).$

a) $x(t)$ er røydyr, siden den bidrar negativt i vekstraten for y, som da er byttedyr

b) $x''(t) = b\,y'(t) = b(-c\,x(t)) = \underline{-bc\,x(t)}$

c) $x(0) = x_0,\ y(0) = y_0$

$x''(t) = -bc\,x(t) \Rightarrow x''(t) + bc\,x(t) = 0$

kar. likning: $r^2 + bc = 0 \Rightarrow r = \pm\sqrt{bc}\,i$

$\Rightarrow x(t) = C\cos(\sqrt{bc}\,t) + D\sin(\sqrt{bc}\,t)$

$(b\,y(t) =)\ x'(t) = -C\sqrt{bc}\sin(\sqrt{bc}\,t) + D\sqrt{bc}\cos(\sqrt{bc}\,t)$

setter inn $t = 0$: $\quad x_0 = C \quad \Rightarrow \quad C = x_0$

$b\,y_0 = D\sqrt{bc} \qquad D = \sqrt{\frac{b}{c}}\,y_0$

$$x(t) = \underline{\underline{x_0\cos(\sqrt{bc}\,t) + \sqrt{\frac{b}{c}}\,y_0\sin(\sqrt{bc}\,t)}}$$

$$y(t) = \frac{1}{b}x'(t) = \frac{1}{b}\left(-x_0\sqrt{bc}\sin(\sqrt{bc}\,t) + \sqrt{\frac{b}{c}}\,y_0\sqrt{bc}\cos(\sqrt{bc}\,t)\right)$$

$$= \underline{\underline{-\sqrt{\frac{c}{b}}\,x_0\sin(\sqrt{bc}\,t) + y_0\cos(\sqrt{bc}\,t)}}$$

d) med verdiene for b og c: $\sqrt{bc} = \sqrt{4.2}$, $\sqrt{\frac{c}{b}} = \sqrt{20 \times 84} = \sqrt{1680}$

$N_1(0) = 300 \quad N_2(0) = 1400$:

$x_0 = N_1(0) - 300 = 300 - 300 = 0$

$y_0 = N_2(0) - 10\,000 = 1400 - 10\,000 = -8600$

$\Rightarrow N_1(t) = x(t) + 300 = -\frac{8600}{\sqrt{1680}}\sin(\sqrt{4.2}\,t) + 300$

$N_2(t) = y(t) + 10000 = -8600\cos(\sqrt{4.2}\,t) + 10\,000.$

periode T: $\quad \sqrt{4.2}\,T = 2\pi \Rightarrow T = \frac{2\pi}{\sqrt{4.2}} \approx 3.0659$

$N_1(t)$ varierer mellom $300 + \frac{8600}{\sqrt{1680}} \approx 509.185$

og $300 - \frac{8600}{\sqrt{1680}} \approx 90.1815$

$N_2(t)$ varierer mellom $8600 + 10\,000 = 18600$ og $-8600 + 10000 = 1400$

topper og bunner for de 2 forskjøvet med $T/4$ i forhold til hverandre.

14.62 $x' = e^{-t^2}$ $x(0) = 0$ g. i fasit $\left(f(t,x) = e^{-t^2} \right)$

$x'(0) = 1$

$x''(t) = -2te^{-t^2} \Rightarrow x''(0) = 0$

$x'''(t) = (-2 + 4t^2)e^{-t^2} \Rightarrow x'''(0) = -2$

$x^{(iv)}(t) = (8t + 4t - 8t^3)e^{-t^2} = (12t - 8t^3)e^{-t^2}$

a) $x(h) \approx x(0) + hx'(0)$

$0 + \frac{1}{2} \cdot 1 = 0.5$ (g. i fasit)

feil $\leq \frac{h^2}{2} \max |x''(\xi)|$ $\xi \in [0, 0.5]$

$\max |x''(\xi)| = \max\limits_{\xi \in [0,0.5]} |-2\xi e^{-\xi^2}| \leq 1$

$\leq \frac{1}{4} \cdot \frac{1}{2} \cdot 1 = \frac{1}{8} = 0.0625$

b) $x(h) \approx x(0) + hx'(0) + \frac{h^2}{2} x''(0)$

$= 0 + \frac{1}{2} \cdot 1 + \frac{1}{8} \cdot 0 = \frac{1}{2}$

$\max |x'''(\xi)| = 2$

feil $\leq \frac{h^3}{6} \max\limits_{x \in [0,0.5]} |\cancel{x}'''(\xi)| \leq \frac{1}{8} \cdot \frac{1}{6} \cdot 2 = \frac{1}{24} \approx 0.042$

c) $x(h) \approx x(0) + hx'(0) + \frac{h^2}{2} x''(0) + \frac{h^3}{6} x'''(0)$

$= 0 + \frac{1}{2} \cdot 1 + \frac{1}{8} \cdot 0 + \frac{1}{48}(-2) = \frac{1}{2} - \frac{1}{24} = \frac{12-1}{24} = \underline{\underline{\frac{11}{24}}} = \underline{\underline{0.4583}}$

feil $\leq \frac{h^4}{24} \max\limits_{\xi \in [0,0.5]} |x^{(iv)}(\xi)| = \frac{h^4}{24} \underbrace{\max\limits_{\xi \in [0,0.5]} |(12\xi - 8\xi^3)e^{-\xi^2}|}$

$\leq \frac{1}{16} \cdot \frac{1}{24} \, 5 = \frac{5}{16 \cdot 24} \approx \underline{\underline{0.0130}}$

deriverer: $12 - 24\xi^2 \geq 0$ på $[0, 0.5]$

$\rightarrow$ voksende

$\Rightarrow$ maks for $\xi = \frac{1}{2}$

som gir $12 \cdot \frac{1}{2} - 8\left(\frac{1}{2}\right)^3 = 6 - 1 = 5$

g. i fasit