MAT-INF4130: Compulsory exercise 1

Deadline: 14/9/17 kl. 14:30

Information

Submit through Devilry system only!

1. Solve exercise 2.11. Note that $I(k:n,k)$ is matlab notation, with I being the $n \times n$-identity matrix. In particular, convince yourself that

$$I(k:n,k) = \boldsymbol{e}_0 \in \mathbb{R}^{n-k+1} \text{ and } I(1:k,k) = \boldsymbol{e}_k \in \mathbb{R}^k.$$

Hint: For the upper triangular case, and for $A\boldsymbol{b}_k = \boldsymbol{e}_k$, let the block in the upper left have size $k \times k$. Note that, by lemma 1.35, B is upper/lower triangular when A is upper/lower triangular. Thus, when A is upper trianglular, $\boldsymbol{b}_k$ can only have k nonzero entries.

2. How many arithmetic operations are needed to invert a lower/upper triangular matrix using the approach from exercise 2.11?
Hint: It may help to set up an integral for counting these operations, as was done at the bottom of page 66.

3. Comment the following matlab code.

```
n = 8;
A = rand(n);
A = triu(A);
U=A;
for k=n:-1:1
    U(k,k) = 1/U(k,k);
    for r=k-1:-1:1
```

```
        U(r, k) = -U(r,r+1:k)*U(r+1:k,k)/U(r,r);
    end
end
U*A
```

If you prefer python, comment the following code instead.

```
from numpy import *

n = 8
A = matrix(random.random((n,n)))
A=triu(A)
U=A.copy()
for k in range(n-1,-1,-1):
    U[k,k] = 1/U[k,k]
    for r in range(k-1,-1,-1):
        U[r, k] = -U[r,(r+1):(k+1)]*U[(r+1):(k+1),k]/U[r,r]
print U*A
```

What do the `r` and `k` variables represent? Also, explain the matrix product which is computed.
Hint: It may help to look at algorithm 2.6 (`rforwardsolve`), but note the difference in that `rforwardsolve` assumes a banded matrix. What will the code above print to the display? Are there any benefits with this code?