# MAT-INF4310: Mandatory assignment #2, autumn 2017

To be handed in by Sepember 28., 14:30

You must hand in commented scripts which actually compile and work. You must also use "Devilry".

**Exercise 1**.

**a)** Implement Gaussian elimination, Gaussian elimination with pivoting and Householder triangulation to solve an equation $Ax = b$ where $x$ and $b$ are in $\mathbb{C}^n$ and $A$ is a matrix in $\mathbb{C}^{n\times n}$ for $n \geq 1$. The syntax should be "`x=`*solvertype*`(A,b)`".

If

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \text{ and } b = \begin{pmatrix} 9 \\ 9 \\ 14 \end{pmatrix}, \text{ then } x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Test you codes on this example.

**b)** Test your codes on the $2 \times 2$ matrix

$$A = \begin{pmatrix} \varepsilon & 2 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

for $\varepsilon = 10^{-12}, \varepsilon = 10^{-14}, \varepsilon = 10^{-16}$. Do this by computing the relative error

$$E_{\text{rel}} = \frac{\|Ax - b\|}{\|b\|}.$$

Comment on your results.

**c)** Since pivoting comes at "no computational cost" in the notes, we can try to check this in practice, as well a checking whether the cost of Householder triangulation scales as twice the cost of Gaussian elimination.

In order to estimate the cost of a computation, we run the computation 5 times, and record the average elapsed time for this (in Matlab, this can be done with the `tic` and `toc` commands).

Set $n = 50 \times 2^m$ for $m = 0, \ldots, 5$, let $A$ be a random $n \times n$ matrix with integer entries in the range $-1000, \ldots, 1000$, and let $x$ be a vector with random integer entries (in the same range as those of $A$) such that $b = Ax$ is defined. Write a script which for each $m$ records the "cost" of the three algorithms.

Do the actual execution times scale as the number of operations? Present your results in a table or a plot and discuss.