FASIT

Oppgave 1:

a)

$$A\bar{v}_1 = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1-1 \\ 1-1 \\ 2-2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\,\bar{v}_1$$

$\bar{v}_1$ har egenverdi $\lambda_1 = 0$

$$A\bar{v}_2 = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+1 \\ 1-1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = 2\,\bar{v}_2$$

$\bar{v}_2$ har egenverdi $\lambda_2 = 2$

b) Egenverdier:

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 & 1 \\ 1 & -1-\lambda & -1 \\ 2 & -2 & -\lambda \end{vmatrix}$$

$$= (1-\lambda)\begin{vmatrix} -1-\lambda & -1 \\ -2 & -\lambda \end{vmatrix} - (-1)\begin{vmatrix} 1 & -1 \\ 2 & -\lambda \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & -1-\lambda \\ 2 & -2 \end{vmatrix}$$

$$= (1-\lambda)(\lambda^2 + \lambda - 2) + (-\lambda + 2) + (-2 + 2 + 2\lambda)$$

$$= \lambda^2 + \lambda - 2 - \lambda^3 - \lambda^2 + 2\lambda - \lambda + 2 + 2\lambda$$

$$= -\lambda^3 + 4\lambda = \lambda(4 - \lambda^2) = 0 \quad \Rightarrow \quad \lambda = 0, \pm 2.$$

Må beregne egenvektoren for $\lambda_3 = -2$:

$$\begin{bmatrix} -3 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 2 & -2 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ -3 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \end{bmatrix} \begin{matrix} \\ +3R_1 \\ -2R_1 \end{matrix} \sim$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -4 & 4 & 0 \end{bmatrix} \begin{matrix} \\ \\ +2R_2 \end{matrix} \sim \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \cdot 0.5 \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \left.\begin{matrix} x - y + z = 0 \\ y - z = 0 \end{matrix}\right| \begin{matrix} z = s ,\ y = z = s \\ x = y - z = 0 \end{matrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad s \neq 0. \qquad \text{La } \bar{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

A har bare forskjellige reelle egenverdier og er derfor diagonaliserbar.

$$P = (\bar{v}_1\ \bar{v}_2\ \bar{v}_3) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{diagonaliserer A.}$$

c) Ligningssettet er $\bar{x}' = A\bar{x}$. Generell løsning er da

$$\bar{x}(t) = C e^{\lambda_1 t} \bar{v}_1 + D e^{\lambda_2 t} \bar{v}_2 + E e^{\lambda_3 t} \bar{v}_3 =$$

$$C\bar{v}_1 + D e^{2t} \bar{v}_2 + E e^{-2t} \bar{v}_3$$

Da er $\bar{x}(0) = C\bar{v}_1 + D\bar{v}_2 + E\bar{v}_3 = \bar{v}_2$. Altså $C = E = 0$, $D = 1$ så $\bar{x}(t) = e^{2t}\bar{v}_2$

Oppgave 2.

a) (I) $\frac{\partial f}{\partial x} = 2xy = 0$

(II) $\frac{\partial f}{\partial y} = x^2 + 4y - 4 = 0$

(I) gir $x = 0$ eller $y = 0$

$x = 0$ i (II) gir $4 \cdot y - 4 = 0$, dvs. $y = 1$

$y = 0$ i (II) gir $x^2 - 4 = 0$, dvs $x = \pm 2$.

Kritiske punkter: $\underline{\underline{(0,1), (2,0), (-2,0)}}$

$A = \frac{\partial^2 f}{\partial x^2} = 2y$ $\quad B = \frac{\partial^2 f}{\partial x \partial y} = 2x$ $\quad C = \frac{\partial^2 f}{\partial y^2} = 4$

$\Delta = AC - B^2 = 8y - 4x^2$

$(0,1)$: $\Delta = 8$, $A = 2$: Lokalt minimum

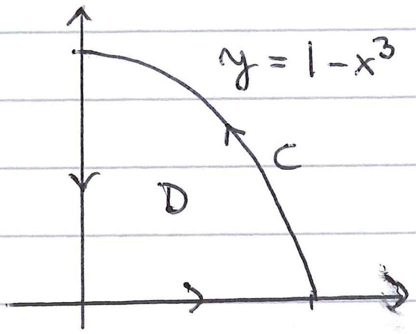
$(2,0)$: $\Delta = -16$: Sadelpunkt

$(-2,0)$: $\Delta = -16$: Sadelpunkt.

b) Vi ser fra a) at $\bar{F} = \nabla f$, altså er $\bar{F}$ konservativt. Kurven C går fra $(0,0)$ til $(1,1)$. Da har vi

$$\int_C \bar{F} \cdot d\bar{s} = f \Big|_{(0,0)}^{(1,1)} = (x^2 y + 2y^2 - 4y) \Big|_{(0,0)}^{(1,1)} = 1 + 2 - 4 = \underline{\underline{-1.}}$$

Oppgave 3.



a) $A = \int_0^1 1 - x^3 \, dx = 1 - \frac{1}{4} = \frac{3}{4}$

$$\iint_D x \, dA = \int_0^1 \left(\int_0^{1-x^3} x \, dy \right) dx = \int_0^1 x - x^4 \, dx = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

$$\iint_D y \, dA = \int_0^1 \left(\int_0^{1-x^3} y \, dy \right) dx = \int_0^1 \frac{1}{2} y^2 \Big|_0^{1-x^3} dx = \frac{1}{2} \int_0^1 (1-x^3)^2 \, dx$$

$$= \frac{1}{2} \int_0^1 1 - 2x^3 + x^6 \, dx = \frac{1}{2}\left(1 - \frac{2}{4} + \frac{1}{7}\right) = \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{7}\right) = \frac{1}{2}\left(\frac{9}{14}\right) = \frac{9}{28}$$

Tyngdepunkt :

$$\bar{x} = \frac{1}{A} \iint_D x \, dA = \frac{4 \cdot 3}{3 \cdot 10} = \underline{\underline{\frac{2}{5}}}$$

$$\bar{y} = \frac{1}{A} \iint_D y \, dA = \frac{4 \cdot 9}{3 \cdot 28} = \underline{\underline{\frac{3}{7}}}$$

b) $\bar{F} = (2x - 3xy - y,\ x^2 + 2x + 7xy) =: (p, q)$

$$\text{curl } \bar{F} = \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y} = 2x + 2 + 7y - (-3x - 1) = 3 + 5x + 7y$$

Green's teorem gir

$$\int_C \bar{F} \cdot d\bar{s} = \iint_D \text{curl } \bar{F} \, dA = \iint_D (3 + 5x + 7y) \, dA$$

$$= 3 \cdot \frac{3}{4} + 5 \cdot \frac{3}{10} + 7 \cdot \frac{9}{28} = \frac{9}{4} + \frac{3}{2} + \frac{9}{4} = \frac{24}{4} = \underline{\underline{6.}}$$