

LESNINGER PÅ NOEN AV OPPGAVENE

FRA 25. FEBRUAR

Oppgavene 1 og 4 ble gjennomgått på forelesning
torsdag 27. februar.

2)

a)

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & -2 & 4 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + (-1) \cdot 1 + 3 \cdot 0 & 1 \cdot 1 + (-1) \cdot 4 + 3 \cdot 0 & 1 \cdot 1 + (-1) \cdot 1 + 3 \cdot 2 \\ 0 \cdot 3 + (-2) \cdot 1 + 4 \cdot 0 & 0 \cdot 1 + (-2) \cdot 4 + 4 \cdot 0 & 0 \cdot 1 + (-2) \cdot 1 + 4 \cdot 2 \\ (-2) \cdot 3 + 1 \cdot 1 + 1 \cdot 0 & (-2) \cdot 1 + 1 \cdot 4 + 1 \cdot 0 & (-2) \cdot 1 + 1 \cdot 1 + 1 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 6 \\ 2 & -8 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} \frac{22}{3} & -1 & -\frac{10}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{17}{3} & 1 & \frac{8}{3} \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 4 \\ 1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{22}{3} \cdot 1 + (-1) \cdot 3 + (-\frac{10}{3}) \cdot 1 & \frac{22}{3} \cdot 2 + (-1) \cdot (-2) + (-\frac{10}{3}) \cdot 5 & \frac{22}{3} \cdot 1 + (-1) \cdot 1 + (-\frac{10}{3}) \cdot 1 \\ (-\frac{1}{3}) \cdot 1 + 0 \cdot 3 + \frac{1}{3} \cdot 1 & (-\frac{1}{3}) \cdot 2 + 0 \cdot (-2) + \frac{1}{3} \cdot 5 & (-\frac{1}{3}) \cdot 1 + 0 \cdot 1 + \frac{1}{3} \cdot 1 \\ (-\frac{17}{3}) \cdot 1 + 1 \cdot 3 + \frac{8}{3} \cdot 1 & (-\frac{17}{3}) \cdot 2 + 1 \cdot (-2) + \frac{8}{3} \cdot 5 & (-\frac{17}{3}) \cdot 1 + 1 \cdot 1 + \frac{8}{3} \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ som er identitetsmatrisen.}$$

c) Likhningssystemet på matrise-vektorform blir

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 4 \\ 1 & 5 & 1 \end{bmatrix}}_{=B} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 10 \\ -3 \end{bmatrix}$$

Multipliser begge sider med matrisen A (multipliser for venstre!)

$$\underbrace{\begin{bmatrix} \frac{22}{3} & -1 & -\frac{10}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{17}{3} & 1 & \frac{8}{3} \end{bmatrix}} \begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 4 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{22}{3} & -1 & -\frac{10}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{17}{3} & 1 & \frac{8}{3} \end{bmatrix} \begin{bmatrix} -1 \\ 10 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{22}{3}(-1) + (-1) \cdot 10 + (-\frac{10}{3})(-3) \\ (-\frac{1}{3}) \cdot (-1) + 0 \cdot 10 + \frac{1}{3}(-3) \\ (-\frac{17}{3})(-1) + 1 \cdot 10 + \frac{8}{3}(-3) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{22}{3} \\ -\frac{2}{3} \\ \frac{23}{3} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -22 \\ -2 \\ 23 \end{bmatrix}$$

Sjåder at dette er løsningen:

$$-\frac{22}{3} + 2 \cdot \left(-\frac{2}{3}\right) + \frac{23}{3} = -\frac{26}{3} + \frac{23}{3} = -1 \quad \text{ok!}$$

$$3 \cdot \left(-\frac{22}{3}\right) - 2 \cdot \left(-\frac{2}{3}\right) + 4 \cdot \frac{23}{3} = -\frac{66}{3} + \frac{4}{3} + \frac{92}{3} = \frac{30}{3} = 10 \quad \text{ok!}$$

$$-\frac{22}{3} + 5 \cdot \left(-\frac{2}{3}\right) + \frac{23}{3} = -\frac{16}{3} + \frac{1}{3} = -\frac{15}{3} = -5 \quad \text{ok!}$$

3

a) Vi skriver ut $L(\omega_1, \omega_2, \omega_3)$:

$$\underline{\omega}^T = [\omega_1 \ \omega_2 \ \omega_3] \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \hat{r}_3 \end{bmatrix} = \omega_1 \hat{r}_1 + \omega_2 \hat{r}_2 + \omega_3 \hat{r}_3$$

$$\underline{\omega}^T C \underline{\omega} = [\omega_1 \ \omega_2 \ \omega_3] \begin{bmatrix} \hat{\sigma}_1^2 & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{\sigma}_2^2 & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{\sigma}_3^2 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$= \begin{bmatrix} \omega_1^2 \hat{\sigma}_1^2 + \omega_2 \hat{c}_{12} + \omega_3 \hat{c}_{13} & \omega_1 \hat{c}_{12} + \omega_2 \hat{\sigma}_2^2 + \omega_3 \hat{c}_{23} & \omega_1 \hat{c}_{13} + \omega_2 \hat{c}_{23} + \omega_3 \hat{\sigma}_3^2 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$= \omega_1^2 \sigma_1^2 + \omega_1 \omega_2 \hat{C}_{12} + \omega_1 \omega_3 \hat{C}_{13} + \omega_1 \omega_2 \hat{C}_{12} + \omega_2^2 \sigma_2^2 + \omega_2 \omega_3 \hat{C}_{23} + \omega_1 \omega_3 \hat{C}_{13} \\ + \omega_2 \omega_3 \hat{C}_{23} + \omega_3^2 \sigma_3^2$$

$$= \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + \omega_3^2 \sigma_3^2 + 2\omega_1 \omega_2 \hat{C}_{12} + 2\omega_1 \omega_3 \hat{C}_{13} + 2\omega_2 \omega_3 \hat{C}_{23}$$

$$\underline{\underline{L(\omega_1, \omega_2, \omega_3) = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + \omega_3^2 \sigma_3^2 + 2\omega_1 \omega_2 \hat{C}_{12} + 2\omega_1 \omega_3 \hat{C}_{13} + 2\omega_2 \omega_3 \hat{C}_{23} \\ + \left(\omega_1 \hat{r}_1 + \omega_2 \hat{r}_2 + \omega_3 \hat{r}_3 - \hat{\mu}_E \right)^2 + \sigma_E^2}}$$

b) Finnen de partiëlderivate:

$$\underline{\underline{\frac{\partial L}{\partial \omega_1} = 2\omega_1 \hat{\sigma}_1^2 + 2\omega_2 \hat{C}_{12} + 2\omega_3 \hat{C}_{13} + 2(\omega_1 \hat{r}_1 + \omega_2 \hat{r}_2 + \omega_3 \hat{r}_3 - \hat{\mu}_E) \cdot \hat{r}_1 \\ = 2(\hat{\sigma}_1^2 + \hat{r}_1^2) \omega_1 + 2(\hat{C}_{12} + \hat{r}_1 \hat{r}_2) \omega_2 + 2(\hat{C}_{13} + \hat{r}_1 \hat{r}_3) \omega_3 \\ - 2\hat{\mu}_E \hat{r}_1}}}$$

$$\underline{\underline{\frac{\partial L}{\partial \omega_2} = 2\omega_2 \hat{\sigma}_2^2 + 2\omega_1 \hat{C}_{12} + 2\omega_3 \hat{C}_{23} + 2(\omega_1 \hat{r}_1 + \omega_2 \hat{r}_2 + \omega_3 \hat{r}_3 - \hat{\mu}_E) \cdot \hat{r}_2 \\ = 2(\hat{C}_{12} + \hat{r}_1 \hat{r}_2) \omega_1 + 2(\hat{\sigma}_2^2 + \hat{r}_2^2) \omega_2 + 2(\hat{C}_{23} + \hat{r}_2 \hat{r}_3) \omega_3 \\ - 2\hat{\mu}_E \hat{r}_2}}}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \omega_3} &= 2\omega_3 \hat{\sigma}_3^2 + 2\omega_1 \hat{c}_{13} + 2\omega_2 \hat{c}_{23} + 2(\omega_1 \hat{r}_1 + \omega_2 \hat{r}_2 + \omega_3 \hat{r}_3 - \hat{\mu}_E) \cdot \hat{r}_3 \\ &= 2(\hat{c}_{13} + \hat{r}_1 \hat{r}_3) \omega_1 + 2(\hat{c}_{23} + \hat{r}_2 \hat{r}_3) \omega_2 + 2(\hat{\sigma}_3^2 + \hat{r}_3^2) \omega_3 \\ &\quad - 2\hat{\mu}_E \hat{r}_3 \end{aligned}$$

c) Setter $\frac{\partial \mathcal{L}}{\partial \omega_1} = 0$, $\frac{\partial \mathcal{L}}{\partial \omega_2} = 0$ og $\frac{\partial \mathcal{L}}{\partial \omega_3} = 0$

(deler på 2, og flytter over $2\hat{\mu}_E \hat{r}$ til høyre side)

$$(\hat{\sigma}_1^2 + \hat{r}_1^2) \omega_1 + (\hat{c}_{12} + \hat{r}_1 \hat{r}_2) \omega_2 + (\hat{c}_{13} + \hat{r}_1 \hat{r}_3) \omega_3 = \hat{\mu}_E \hat{r}_1$$

$$(\hat{c}_{12} + \hat{r}_1 \hat{r}_2) \omega_1 + (\hat{\sigma}_2^2 + \hat{r}_2^2) \omega_2 + (\hat{c}_{23} + \hat{r}_2 \hat{r}_3) \omega_3 = \hat{\mu}_E \hat{r}_2$$

$$(\hat{c}_{13} + \hat{r}_1 \hat{r}_3) \omega_1 + (\hat{c}_{23} + \hat{r}_2 \hat{r}_3) \omega_2 + (\hat{\sigma}_3^2 + \hat{r}_3^2) \omega_3 = \hat{\mu}_E \hat{r}_3$$

På matris-vektorforn blir dette

$$(C + \hat{r} \hat{r}^T) \underline{\omega} = \hat{\mu}_E \hat{r}$$

Fra oppgave 1 vet vi at

$$\hat{r} \hat{r}^T = \begin{bmatrix} \hat{r}_1^2 & \hat{r}_1 \hat{r}_2 & \hat{r}_1 \hat{r}_3 \\ \hat{r}_1 \hat{r}_2 & \hat{r}_2^2 & \hat{r}_2 \hat{r}_3 \\ \hat{r}_1 \hat{r}_3 & \hat{r}_2 \hat{r}_3 & \hat{r}_3^2 \end{bmatrix}$$

5

$$C^{-1} = \begin{bmatrix} \frac{1}{\delta_1^2} & 0 & 0 \\ 0 & \frac{1}{\delta_2^2} & 0 \\ 0 & 0 & \frac{1}{\delta_3^2} \end{bmatrix} \text{ er et gjett!}$$

Vi ser at

$$\begin{bmatrix} \frac{1}{\delta_1^2} & 0 & 0 \\ 0 & \frac{1}{\delta_2^2} & 0 \\ 0 & 0 & \frac{1}{\delta_3^2} \end{bmatrix} \begin{bmatrix} \hat{\delta}_1^2 & 0 & 0 \\ 0 & \hat{\delta}_2^2 & 0 \\ 0 & 0 & \hat{\delta}_3^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\delta_1^2} \cdot \hat{\delta}_1^2 + 0 \cdot 0 + 0 \cdot 0 & \frac{1}{\delta_1^2} \cdot 0 + 0 \cdot \hat{\delta}_2^2 + 0 \cdot 0 & \frac{1}{\delta_1^2} \cdot 0 + 0 \cdot 0 + 0 \cdot \hat{\delta}_3^2 \\ 0 \cdot \hat{\delta}_1^2 + \frac{1}{\delta_2^2} \cdot 0 + 0 \cdot 0 & 0 \cdot 0 + \frac{1}{\delta_2^2} \cdot \hat{\delta}_2^2 + 0 \cdot 0 & 0 \cdot 0 + \frac{1}{\delta_2^2} \cdot 0 + 0 \cdot \hat{\delta}_3^2 \\ 0 \cdot \hat{\delta}_1^2 + 0 \cdot 0 + \frac{1}{\delta_3^2} \cdot 0 & 0 \cdot 0 + 0 \cdot \hat{\delta}_2^2 + \frac{1}{\delta_3^2} \cdot 0 & 0 \cdot 0 + 0 \cdot 0 + \frac{1}{\delta_3^2} \cdot \hat{\delta}_3^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Dette er identitets-
matrisen i dimensjon 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

⇒ Vårt gjett er riktig!