

# Oppgaveløsning

$$a) R_i = \frac{S_i - S_{i,0}}{S_{i,0}}$$

|       |     |      |     |     |
|-------|-----|------|-----|-----|
| $S_1$ | 1.0 | 1.1  | 1.1 | 1.2 |
| $S_2$ | 1.6 | 0.8  | 1.2 | 1.2 |
| $R_1$ | 0   | 0.1  | 0.1 | 0.2 |
| $R_2$ | 0.6 | -0.2 | 0.2 | 0.2 |

$$\hat{r}_1 = \frac{1}{4} (0 + 0.1 + 0.1 + 0.2) = 0.1$$

$$\hat{r}_2 = \frac{1}{4} (0.6 - 0.2 + 0.2 + 0.2) = 0.2$$

b) Regn ut kovariansmatrisen C

$$C_{ij} = E[(R_i - \hat{r}_i)(R_j - \hat{r}_j)]$$

$$C_{11} = \sigma_1^2 = \frac{1}{4} \left( (0 - 0.1)^2 + (0.1 - 0.1)^2 + (0.1 - 0.1)^2 + (0.2 - 0.1)^2 \right) = \frac{1}{4} 2 \cdot 0.01 = \frac{0.01}{2} = 0.005, \sigma_1 = \sqrt{0.005} \approx 0.0707 = 7.07\%$$

$$C_{22} = \sigma_2^2 = \frac{1}{4} \left( (0.6 - 0.2)^2 + (0.2 - 0.2)^2 + (0.2 - 0.2)^2 + (0.2 - 0.2)^2 \right) = \frac{1}{4} 2 \cdot 0.16 = 0.08, \sigma_2 = \sqrt{0.08} \approx 0.2828 \approx 28.32\%$$

$$C_{12} = \frac{1}{4} \left[ (0 - 0.1)(0.6 - 0.2) + 0 + 0 + 0 \right] = \frac{1}{4} [-0.04] = -0.01$$

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} 0.005 & -0.01 \\ -0.01 & 0.08 \end{pmatrix} = \frac{1}{100} \begin{pmatrix} 0.5 & -1 \\ -1 & 8 \end{pmatrix}$$

c) Formelutvikling:  $\vec{w}^T \vec{r}$  der  $\vec{r} = \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}$   
 For oss:  $\vec{r} = \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}$   $\vec{w}_1 = \begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$ ,  $\vec{w}_2 = \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix}$

$$\mu_{\vec{w}_1} = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \end{pmatrix} \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix} = \frac{1}{4} \cdot 0.1 + \frac{3}{4} \cdot 0.2 = 0.025 + 0.15 = 0.175 = 17.5\%$$

$$\mu_{\vec{w}_2} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix} = \frac{3}{4} \cdot 0.1 + \frac{1}{4} \cdot 0.2 = 0.075 + 0.05 = 0.125 = 12.5\%$$

Standardavvik

Byttel  
om på  
wa som  
er  $\vec{w}_1$  og  
wa som er  
 $\vec{w}_2$  i  
formlingen

$$\sigma_{\vec{w}_2}^2 = \vec{w}_2^T C \vec{w}_2 = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \end{pmatrix} \frac{1}{100} \begin{pmatrix} 0.5 & -1 \\ -1 & 8 \end{pmatrix} \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix}$$

$$= \frac{1}{100} \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 0.5 \cdot \frac{3}{4} - 1 \cdot \frac{1}{4} \\ -1 \cdot \frac{3}{4} + 8 \cdot \frac{1}{4} \end{pmatrix} = \frac{1}{100} \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} +0.5 \\ 5 \end{pmatrix}$$

$$= \frac{1}{1600} (3 \cdot 1) \begin{pmatrix} +0.5 \\ 5 \end{pmatrix} = \frac{1}{1600} (3 \cdot (+0.5) + 1 \cdot 5) = \frac{1}{1600} 6.5 = 0.004063$$

$$\sigma_{\vec{w}_2} = \sqrt{0.004063} \approx 0.0637 = 6.37\%$$

$$\sigma_{\vec{w}_1}^2 = 0.04156$$

$$\sigma_{\vec{w}_1} = 20.39\%$$

d) Finn  $C^{-1}$ : Generell formel:  $C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , da er

$$C^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Var C

$$C = \frac{1}{100} \begin{pmatrix} 0.5 & -1 \\ -1 & 8 \end{pmatrix}$$

$$C^{-1} = 100 \begin{pmatrix} 0.5 & -1 \\ -1 & 8 \end{pmatrix}^{-1} = 100 \frac{1}{\underbrace{0.5 \cdot 8}_{4} - \underbrace{(-1)(-1)}_{1}} \begin{pmatrix} 8 & 1 \\ 1 & 0.5 \end{pmatrix}$$

$$= \frac{100}{3} \begin{pmatrix} 8 & 1 \\ 1 & 0.5 \end{pmatrix}$$

e)  $a = \vec{r}^T C^{-1} \vec{r} = \vec{r}^T \underline{C^{-1} \vec{r}}$   
 $b = \vec{r}^T C^{-1} \vec{r}$   
 $c = \underline{\vec{1}^T C^{-1} \vec{1}}$

Hjelperekning:  $C^{-1} \vec{r} = \frac{100}{3} \begin{pmatrix} 8 & 1 \\ 1 & 0.5 \end{pmatrix} \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}$   
 $= \frac{100}{30} \begin{pmatrix} 8 & 1 \\ 1 & 0.5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{10}{3} \begin{pmatrix} 8 \cdot 1 + 1 \cdot 2 \\ 1 \cdot 1 + 0.5 \cdot 2 \end{pmatrix} = \frac{10}{3} \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$$C^{-1} \vec{1} = \frac{100}{3} \begin{pmatrix} 8 & 1 \\ 1 & 0.5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{100}{3} \begin{pmatrix} 8 \cdot 1 + 1 \cdot 1 \\ 1 \cdot 1 + 0.5 \cdot 1 \end{pmatrix} = \frac{100}{3} \begin{pmatrix} 9 \\ 1.5 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 900 \\ 150 \end{pmatrix}$$

$$a = \vec{1}^T C^{-1} \vec{r} = (1, 1) \frac{1}{3} \begin{pmatrix} 900 \\ 150 \end{pmatrix} = \frac{1}{3} (100 + 20) = \underline{\underline{40}}$$

$$b = \vec{r}^T C^{-1} \vec{r} = (0.1, 0.2) \frac{1}{3} \begin{pmatrix} 900 \\ 150 \end{pmatrix} = \frac{1}{3} \left( \underbrace{0.1 \cdot 900}_{90} + \underbrace{0.2 \cdot 150}_{30} \right) = \frac{120}{3} = 40$$

$$c = \vec{1}^T C^{-1} \vec{1} = (1, 1) \begin{pmatrix} 300 \\ 50 \end{pmatrix} = 1 \cdot 300 + 1 \cdot 50 = \underline{\underline{350}}$$

Altså  $a = 40, b = \frac{14}{3}, c = 350$

$$f) \quad W^* = \frac{Cr - a}{b_c - a^2} C^{-1} \vec{r} + \frac{b - ar}{b_c - a^2} C^{-1} \vec{1}$$

Hilfsgleichung:  $b_c - a^2 = \frac{14}{3} \cdot 350 - 40^2 = \frac{4900}{3} - 1600$   
 $= \frac{4900}{3} - \frac{4800}{3} = \frac{100}{3}$

$$W^* = \frac{350r - 40}{\frac{100}{3}} \cdot \frac{1}{3} \begin{pmatrix} 100 \\ 20 \end{pmatrix} + \frac{\frac{14}{3} - 40r}{\frac{100}{3}} \begin{pmatrix} 300 \\ 50 \end{pmatrix}$$

$$= \frac{350r - 40}{100} \begin{pmatrix} 100 \\ 20 \end{pmatrix} + \frac{14 - 120r}{100} \begin{pmatrix} 300 \\ 50 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + r \begin{pmatrix} -10 \\ 10 \end{pmatrix}$$

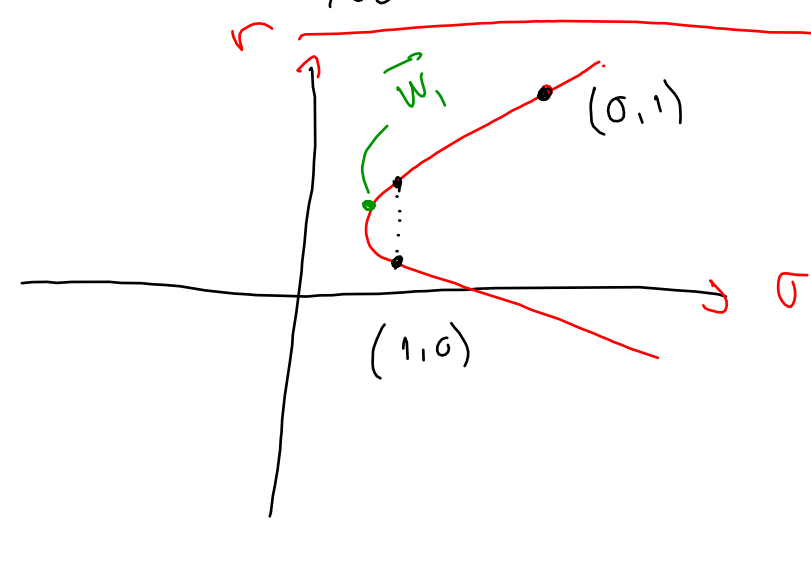
$$w_1 = 2 - 10 \cdot r \quad r = 0.1$$

$$w_2 = -1 + 10r \quad r = 0.2$$

g) Effizient portfolio Punkt:

$$\sigma^2 = c \frac{(r - \frac{a}{c})^2}{b_c - a^2} + \frac{1}{c} = 350 \frac{(r - \frac{40}{350})^2}{\frac{100}{3}} + \frac{1}{350}$$

$$= \frac{1050}{100} (r - \frac{4}{35})^2 + \frac{1}{350} = 10.5 (r - \frac{4}{35})^2 + \frac{1}{350}$$



Lösung für r:

$$\sigma^2 - \frac{1}{350} = 10.5 (r - \frac{4}{35})^2$$

$$(r - \frac{4}{35})^2 = \frac{\sigma^2 - \frac{1}{350}}{10.5}$$

$$r - \frac{4}{35} = \pm \sqrt{\frac{\sigma^2 - \frac{1}{350}}{10.5}}$$

$$r = \frac{4}{35} \pm \sqrt{\frac{\sigma^2 - \frac{1}{350}}{10.5}}$$

