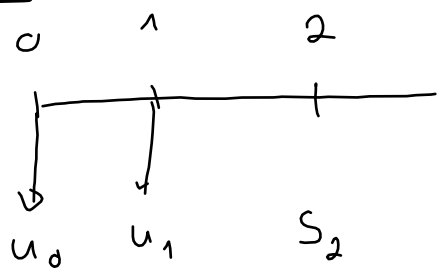


Oppgave 4

Tide fornybar ressurs



$$S_2 = S_0 - u_0 - u_1$$

$$L(u_0, u_1) = u_0^\delta + \rho u_1^\delta + \rho^2 S_2^\delta \quad \text{maksimer den funksjon}$$

Optimeringsproblemet: $f(u) = Au^\delta + B(x-u)^\delta$

Deriver: $f'(u) = A\delta u^{\delta-1} + B\delta(x-u)^{\delta-1}(-1)$

Sett $f'(u) = 0$: $A\delta u^{\delta-1} = B\delta(x-u)^{\delta-1}$ opphøyer: $\frac{1}{\delta-1}$

$$A \frac{1}{\delta-1} u = B \frac{1}{\delta-1} (x-u)$$

$$(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})u = B \frac{1}{\delta-1} x$$

$$u = \frac{B \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x, \quad x-u = \frac{A \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x$$

Maksimerer: $f(u) = A \left(\frac{B \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x \right)^\delta + B \left(\frac{A \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x \right)^\delta$

$$= \frac{x^\delta}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^\delta} (A B \frac{1}{\delta-1} + B A \frac{1}{\delta-1})$$

$$= \frac{x^\delta}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^\delta} (AB \frac{1}{\delta-1} + BA \frac{1}{\delta-1})$$

$$= \frac{AB x^\delta}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^\delta} (A \frac{1}{\delta-1} + B \frac{1}{\delta-1})$$

$$= \frac{AB x^\delta}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^{\delta-1}}$$

Side usning
 $\frac{x}{x-1} = \frac{x^{-1+1}}{x^{-1}-1}$
 $= 1 + \frac{1}{x-1}$

Ser på hva som skjer ved tidspunktet 1. Anta at ressursen er x og at i utvinnet u .

$$\rho u^\delta + \rho^2 (x-u)^\delta$$

Vil finne u som maksimerer dette uttrykket. Ser at det er problemet å løse med $\rho = A$ og $\rho^2 = B$.

Den optimale u er en

$$u^*(1) = \frac{B \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x = \frac{\rho^2 \frac{1}{\delta-1}}{\rho \frac{1}{\delta-1} + \rho^2 \frac{1}{\delta-1}} x = \frac{\rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} x$$

Maksimerer:

$$V_1(x) = \frac{AB x^\delta}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^{\delta-1}} = \frac{\rho \rho^2 x^\delta}{(\rho \frac{1}{\delta-1} + \rho^2 \frac{1}{\delta-1})^{\delta-1}}$$

$$= \frac{\rho^3 x^\delta}{(\rho \frac{1}{\delta-1} (1 + \rho^{\frac{1}{\delta-1}}))^{\delta-1}} = \frac{\rho^2 x^\delta}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}}$$

Ser på tidspunktet 0: Hvis vi har en ressurs x og

utvinnet u , da er nytten

$$u^\delta + V_1(x-u) = u^\delta + \frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} (x-u)^\delta$$

Vil finne u som gir størst nytte. Dette er et optimeringsproblem med $A=1$ og $B = \frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}}$

Beste valg

$$u^*(0) = \frac{B \frac{1}{\delta-1}}{A \frac{1}{\delta-1} + B \frac{1}{\delta-1}} x = \frac{\left(\frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} \right) \frac{1}{\delta-1}}{1 + \left(\frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} \right) \frac{1}{\delta-1}} x$$

$$= \frac{\left(\frac{\rho^{\frac{2}{\delta-1}} x}{1 + \rho^{\frac{1}{\delta-1}}} \right) (1 + \rho^{\frac{1}{\delta-1}})}{\left(1 + \frac{\rho^{\frac{2}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} \right) (1 + \rho^{\frac{1}{\delta-1}})} = \frac{\rho^{\frac{2}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} x$$

Maksimerer:

$$V_0(x) = \frac{AB}{(A \frac{1}{\delta-1} + B \frac{1}{\delta-1})^{\delta-1}} x^\delta = \frac{1 \cdot \frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} x^\delta}{(1 + \left(\frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} \right) \frac{1}{\delta-1})^{\delta-1}}$$

$$= \frac{\frac{\rho^2}{(1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} x^\delta (1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}}{\left(1 + \frac{\rho^{\frac{2}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} \right)^{\delta-1} (1 + \rho^{\frac{1}{\delta-1}})^{\delta-1}} = \frac{\rho^2 x^\delta}{(1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}})}$$

Nytt spørsmål: Hvis vi starter med en ressurs $S_0 = S$, hvor stor er da $S(2)$?

Regner først ut $S(1)$: $S(1) = S - u_0^*(S) = S - \frac{\rho^{\frac{2}{\delta-1}} S}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}}$

$$= S \left(\frac{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} - \frac{\rho^{\frac{2}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} \right)$$

$$= \frac{1 + \rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} S \leftarrow S(1)$$

Regner så ut $S(2)$: $S(2) = S(1) - u_1^*(S)$

$$= S(1) - \frac{\rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} S(1) = S(1) \left[\frac{1 + \rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} - \frac{\rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}}} \right]$$

$$= S(1) \frac{1}{1 + \rho^{\frac{1}{\delta-1}}} = \frac{1 + \rho^{\frac{1}{\delta-1}}}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} S \cdot \frac{1}{1 + \rho^{\frac{1}{\delta-1}}}$$

$$= \frac{1}{1 + \rho^{\frac{1}{\delta-1}} + \rho^{\frac{2}{\delta-1}}} S$$

Hvis $\gamma = \frac{1}{2}$, $\frac{1}{\gamma-1} = \frac{1}{\frac{1}{2}-1} = -\frac{1}{\frac{1}{2}} = -2$

$$S_2 = \frac{1}{1 + \rho^{-2} + \rho^{-4}}$$

$$f(x) = \frac{1}{1 + x^{-2} + x^{-4}} = \frac{x^4}{x^4 + x^2 + 1}$$