

Alternative optimeringsbetingelser

Hilf: Givet en demand $B_{t+1} = g(B_t)$. Også

(i) Bestem en intholdings $h = \sigma(B)$ slik

$$B = g(B - \sigma(B))$$

(ii) Find den B-værdi som maksimerer $\sigma(B)$.

Alternativ til (i):

Indledning: $R(B, h) = \underbrace{ph}_{\text{salgsindtækt}} - \underbrace{C(B, h)}_{\text{kostnader}}$

Også er optimer indledning, der maksimerer

$$R(B, \sigma(B)) = p\sigma(B) - C(B, \sigma(B))$$

Må ha

$$\begin{aligned} 0 &= \frac{d}{dB} R(B, \sigma(B)) = (p\sigma(B) - C(B, \sigma(B)))' \\ &= p\sigma'(B) - \frac{\partial C}{\partial B}(B, \sigma(B)) - \frac{\partial C}{\partial h}(B, \sigma(B))\sigma'(B) \end{aligned}$$

Husk at $\sigma'(B) = \frac{g'(B - \sigma(B)) - 1}{g'(B - \sigma(B))}$, og sæt i dette

$$0 = p \frac{g'(B - \sigma(B)) - 1}{g'(B - \sigma(B))} - \frac{\partial C}{\partial B}(B, \sigma(B)) - \frac{\partial C}{\partial h}(B, \sigma(B)) \frac{g'(B - \sigma(B)) - 1}{g'(B - \sigma(B))}$$

Ganger med $g'(B - \sigma(B))$:

$$0 = p \underline{g'(B - \sigma(B))} - p - \frac{\partial C}{\partial B}(B, \sigma(B)) \underline{g'(B - \sigma(B))} - \frac{\partial C}{\partial h}(B, \sigma(B)) \underline{g'(B - \sigma(B))} + \frac{\partial C}{\partial h}(B, \sigma(B))$$

$$p - \frac{\partial C}{\partial h}(B, \sigma(B)) = g'(B - \sigma(B)) \left(p - \frac{\partial C}{\partial B}(B, \sigma(B)) - \frac{\partial C}{\partial h}(B, \sigma(B)) \right)$$

$$g'(B - \sigma(B)) = \frac{p - \frac{\partial C}{\partial h}(B, \sigma(B))}{p - \frac{\partial C}{\partial B}(B, \sigma(B)) - \frac{\partial C}{\partial h}(B, \sigma(B))}$$

To ligninger for at bestemme B og $h = (\sigma(B))$

$$B_{t+1} = g(B_t)$$

$$B = g(B - h)$$

$$\underline{g'(B - h)} = \frac{p - \frac{\partial C}{\partial h}(B, h)}{p - \frac{\partial C}{\partial B}(B, h) - \frac{\partial C}{\partial h}(B, h)}$$

Eksempel: Ikke-profit

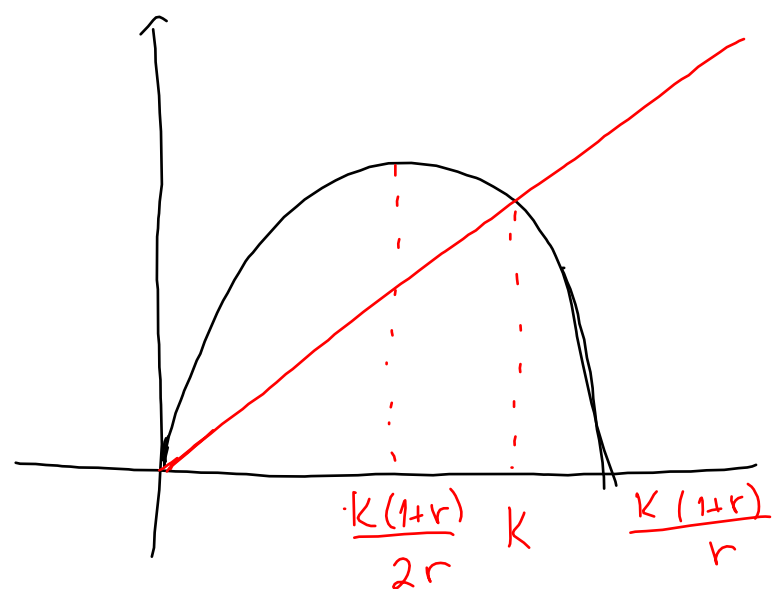
$$R(h, B) = ph - C(h, B) = 0 \quad \leftarrow$$

Eksempel $C(h, B) = c \left(\frac{h}{gB} \right)$

Udtak af
om bestanden

$$p \cancel{h} = c \frac{\cancel{h}}{gB} \Rightarrow gB = \frac{c}{p} \Rightarrow \underline{\underline{B = \frac{c}{pg}}}$$

Oppgave 3: a) $q(B) = B + rB(1 - \frac{B}{K}) = B + rB - \frac{rB^2}{K} = \frac{(1+r)B - \frac{rB^2}{K}}{\dots}$



Finnes her den deriverte er 0:

$$q'(B) = (1+r) - \frac{2rB}{K}$$

Setter lik 0: $1+r = \frac{2rB}{K} \Rightarrow B = \frac{K(1+r)}{2r}$

Maksimumsverdi:

$$q\left(\frac{K(1+r)}{2r}\right) = \frac{K(1+r)^2}{4r}$$

b) Vi ønsker å holde bestanden i likertall ved å høste h . Dette

gir bare mening når $B \leq K$ (når $B > K$ er det bedre å seig selv)

Vi finner h slik at

$$\underline{B} = q(B-h) = (1+r)(B-h) - \frac{r(B-h)^2}{K}$$

altså

$$\frac{r}{K}(B-h)^2 - (1+r)(B-h) + B = 0$$

eller

$$\frac{r}{K}(h-B)^2 + (1+r)(h-B) + B = 0$$

Lösen for $h-B$:

$$h-B = \frac{-(1+r) \pm \sqrt{(1+r)^2 - 4 \frac{r}{K} B}}{2 \frac{r}{K}}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vi bruke + for å unngå at h blir negativ

altså

$$\begin{aligned} h &= B + \frac{-(1+r) + \sqrt{(1+r)^2 - \frac{4r}{K} B}}{2 \frac{r}{K}} \\ &= B + \frac{\sqrt{(1+r)^2 - \frac{4r}{K} B} - (1+r)}{2 \frac{r}{K}} \end{aligned}$$

Hva skjer når $B \rightarrow K$

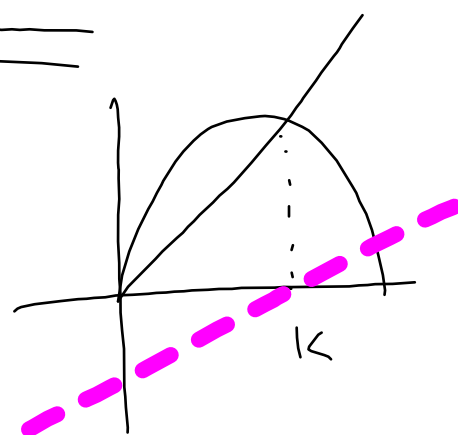
Setter inn $B=K$ i uttrykket:

$$h = K + \frac{\sqrt{(1+r)^2 - 4r} - (1+r)}{2 \frac{r}{K}}$$

$$= K + \frac{\sqrt{1+2r+r^2-4r} - (1+r)}{2 \frac{r}{K}}$$

$$= K + \frac{\sqrt{(r-1)^2 - (1+r)} - (1+r)}{2 \frac{r}{K}}$$

$$= K + \frac{r-1 - (1+r)}{2 \frac{r}{K}} = K - \frac{2}{\frac{2r}{K}} = K - \frac{K \cdot 2r}{2r} = \underline{\underline{0}}$$



Oppgave 1: a) $x_{n+1} = r \cdot x_n$

$$x_0, x_1 = rx_0, x_2 = rx_1 = r^2 x_0, \dots, x_n = r^n x_0 = \underline{C r^n}$$

b) $x_{n+1} = b x_n + c x_{n-1}$

Sjekk om $x_n = C r^n$ er en løsning

$$\underline{C r^{n+1}} = \underline{b (r^n)} + \underline{c (r^{n-1})}$$
$$C r^{n+1} (r^2 - b r - c) = 0$$

Har løsning når $\underbrace{r^2 - b r - c = 0}$
den karakteristiske ligningen

c) Anta at $r_1 \neq r_2$ er røtter i den kar. lign.

Påstår at da er $x_n = C r_1^n + D r_2^n$ en løsning av differens-
ligningen: Må sjekke at

$$0 = x_{n+1} - b x_n - c x_{n-1} = \underline{C r_1^{n+1} + D r_2^{n+1}} - b (C r_1^n + D r_2^n) - c (C r_1^{n-1} + D r_2^{n-1})$$

$$= C (r_1^{n+1} - b r_1^n - c r_1^{n-1}) + D (r_2^{n+1} - b r_2^n - c r_2^{n-1})$$

$$= C r_1^{n-1} (\underbrace{r_1^2 - b r_1 - c}_0) + D r_2^{n-1} (\underbrace{r_2^2 - b r_2 - c}_0) = 0$$

d) Er alle løsninger på formen $x_n = \underline{C r_1^n + D r_2^n}$?

Ligningen: $x_{n+1} = b x_n + c x_{n-1}$

Detta betyr at hvis vi kjenner x_0, x_1 , kan vi i prinsippet
regne ut hele løsningen.

Vi må vite at gitt x_0 og x_1 , kan vi alltid finne C og D
som oppfyller disse betingelsene, dvs

$$x_0 = C r_1^0 + D r_2^0 = C + D \quad | - r_2$$

$$x_1 = C r_1^1 + D r_2^1 = C r_1 + D r_2$$

$$\left. \begin{array}{l} -r_2 x_0 = -C r_2 - D r_2 \\ x_1 = C r_1 + D r_2 \end{array} \right\} \begin{array}{l} x_1 - r_2 x_0 = C (r_1 - r_2) \\ C = \frac{x_1 - r_2 x_0}{r_1 - r_2} \end{array}$$

Tilsvarende finner vi D

e)

$$x_{n+1} = -x_n + 6 x_{n-1}$$

Kar. lign: $r^2 = -r + 6 \Rightarrow \underline{r^2 + r - 6 = 0}$

$$(r+3)(r-2) = 0$$

$$r_1 = 2, r_2 = -3$$

Generell løsning:

$$x_n = C 2^n + D (-3)^n$$

f) Fibonacci-tallene

1, 1, 2, 3, 5, 8, 13, 21, ...
 $x_0 \quad x_1 \quad x_2$

Finn en formel for x_n

$$x_{n+1} = x_n + x_{n-1}, \quad x_0 = 1, x_1 = 1$$

Karakteristiske ligningen: $r^2 = r + 1$

altså $r^2 - r - 1 = 0$

Løsning: $r = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{5}}{2}$

Generelle løsning: $x_n = C r_1^n + D r_2^n = C \left(\frac{1+\sqrt{5}}{2} \right)^n + D \left(\frac{1-\sqrt{5}}{2} \right)^n$

Krav $x_0 = 1, x_1 = 1$

$$1 = C \left(\frac{1+\sqrt{5}}{2} \right)^0 + D \left(\frac{1-\sqrt{5}}{2} \right)^0 = C + D \Rightarrow D = 1 - C$$

$$1 = C \left(\frac{1+\sqrt{5}}{2} \right)^1 + D \left(\frac{1-\sqrt{5}}{2} \right)^1 = C \frac{1+\sqrt{5}}{2} + D \frac{1-\sqrt{5}}{2}$$

Sett inn i rekursiv ligning

$$1 = C \frac{1+\sqrt{5}}{2} + (1-C) \frac{1-\sqrt{5}}{2} = C \left(\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2} \right) + \frac{1-\sqrt{5}}{2}$$

$$= C \sqrt{5} + \frac{1-\sqrt{5}}{2}$$

$$1 - \frac{1}{2} + \frac{\sqrt{5}}{2} = C \sqrt{5}$$

$$\frac{1+\sqrt{5}}{2} = C \sqrt{5} \Rightarrow C = \frac{1+\sqrt{5}}{2\sqrt{5}}$$

J tillegg

$$D = 1 - C = 1 - \frac{1+\sqrt{5}}{2\sqrt{5}} = \frac{2\sqrt{5} - 1 - \sqrt{5}}{2\sqrt{5}} = \frac{\sqrt{5} - 1}{2\sqrt{5}}$$

Løsning:

$$x_n = C \left(\frac{1+\sqrt{5}}{2} \right)^n + D \left(\frac{1-\sqrt{5}}{2} \right)^n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{n+1}$$

$$= \frac{\sqrt{5}}{5} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} \right] \text{ Binets formel.}$$