

Oppgave 1 klimamodell

$$\pi R S (1-\alpha) = 4\pi R^2 \epsilon \sigma T^4$$

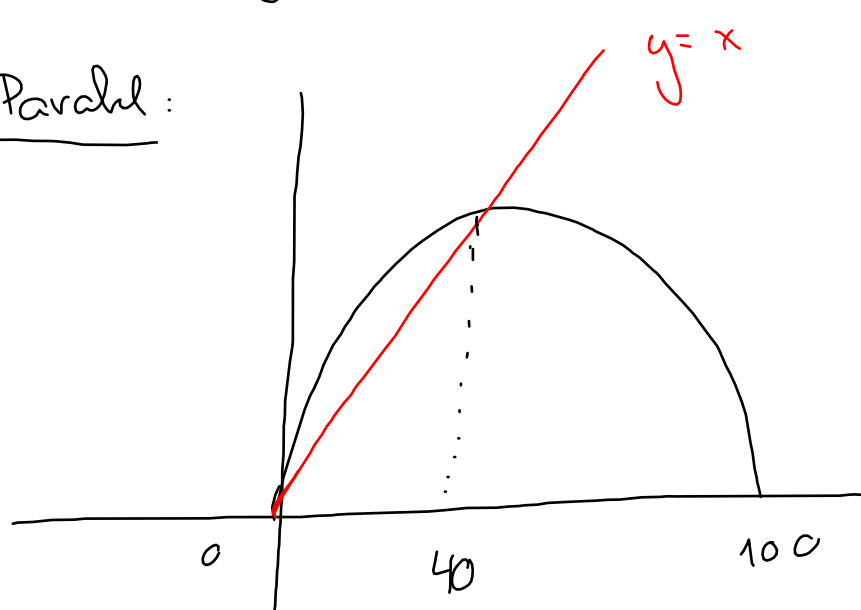
$$S(1-\alpha) = 4\epsilon\sigma T^4$$

$$\alpha = 0.2 \quad \sigma = 5.67 \cdot 10^{-8} \quad S = 392 \text{ W/m}^2 \quad \epsilon = 1$$

Reell temperatur 210 K

Oppgave 2: $g(B) = \frac{S}{3} B \left(1 - \frac{B}{100}\right)$ logistisk modell

a) Parabel:



Likevektspunkt B: $g(B)=B$

$$B = \frac{S}{3} B \left(1 - \frac{B}{100}\right)$$

$$3 = S \left(1 - \frac{B}{100}\right) = 5 - \frac{B}{20}$$

$$\frac{B}{20} = 5 - 3 \Rightarrow \underline{B = 40}$$

b) $n=0: B_0 = 70 \text{ tonn}$

$$n=1: B_1 = g(B_0) = \frac{S}{3} \cdot 70 \left(1 - \frac{70}{100}\right) = 35 \text{ tonn}$$

$$n=2: B_2 = g(B_1) = 37.92 \text{ tonn}$$

$$n=3: B_3 = g(B_2) = 39.23 \text{ tonn}$$

Hypotese: $B_n \rightarrow B$ likevekt

c) Bestanden er 15 tonn. Hvor mye må vi fiske for å holde den der. Fangst h, så vil vi året etter ha en bestand på

$$g(15-h) = 15$$

Løsning

$$15 = \frac{S}{3} (15-h) \left(1 - \frac{15-h}{100}\right) \quad | \quad \frac{3}{S}$$

$$9 = (15-h) - \frac{(15-h)^2}{100} \quad | \quad 100$$

$$900 = \overset{100}{(15-h) - (h-15)^2}$$

$$(h-15)^2 + 100(h-15) + 900 = 0$$

abc-formel: $h - 15 = \frac{-100 \pm \sqrt{100^2 - 4 \cdot 1 \cdot 900}}{2} = \frac{-100 \pm 80}{2} = \begin{cases} -10 \\ -90 \end{cases}$

Siden h må være positiv, velger vi den andre løsningen

og får $h = 15 - 10 = \underline{5 \text{ tonn}}$

Oppgave 3:

$$r_1 = 0.4, r_2 = 0.4, r_3 = 0.5$$

$$\sigma_1 = 0.2, \sigma_2 = 0.25, \sigma_3 = 0.3$$

$$\rho_{12} = 0.4, \rho_{13} = -0.1, \rho_{23} = -0.2$$

Da diagonalen har i variansene:

$$C_{11} = \sigma_1^2 = 0.2^2 = 0.04, C_{22} = \sigma_2^2 = 0.25^2 = 0.0625$$

$$C_{33} = \sigma_3^2 = 0.3^2 = 0.09$$

Utskr diagonalen har i al $C_{ij} = \rho_{ij} \sigma_i \sigma_j$

$$C_{12} = \rho_{12} \sigma_1 \sigma_2 = 0.4 \cdot 0.2 \cdot 0.25 = 0.02$$

$$C_{13} = \rho_{13} \sigma_1 \sigma_3 = -0.1 \cdot 0.2 \cdot 0.3 = -0.006$$

$$C_{23} = \rho_{23} \sigma_2 \sigma_3 = -0.2 \cdot 0.25 \cdot 0.3 = -0.015$$

$$C = \begin{pmatrix} 0.04 & 0.02 & -0.006 \\ 0.02 & 0.0625 & -0.015 \\ -0.006 & -0.015 & 0.09 \end{pmatrix}$$

b) $\vec{w} = \mu_T (C + \vec{r} \vec{r}^T)^{-1} \vec{r}$

$$\mu_T = 1000$$

$$\vec{w} = \mu_T (C + \vec{r} \vec{r}^T)^{-1} \vec{r} = 1000 \begin{pmatrix} 22.82 & -13.94 & -5.43 \\ -13.94 & 16.73 & -1.15 \\ -5.43 & -1.15 & 6.67 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.4 \\ 0.5 \end{pmatrix}$$

$$= 1000 \begin{pmatrix} 22.82 \cdot 0.4 - 13.94 \cdot 0.4 - 5.43 \cdot 0.5 \\ \cdot \cdot \cdot \\ \cdot \cdot \cdot \end{pmatrix} = \begin{pmatrix} 837 \\ 541 \\ 703 \end{pmatrix}$$

Optimal plassering 837 MWh i 1
541 " i 2
703 " i 3

Gjennomsnittlig produksjon

$$837 \cdot 0.4 + 541 \cdot 0.4 + 703 \cdot 0.5 = \underline{902.7}$$

Oppgave 4: $r_1 = 0.2, r_2 = 0.3, r_3 = 0.1$

$$C^{-1} = \begin{bmatrix} 400 & 20 & 10 \\ 20 & 500 & -40 \\ 10 & -40 & 200 \end{bmatrix}$$

a) Trenger $C^{-1}\vec{1}$ og $C^{-1}\vec{r}$

$$C^{-1}\vec{1} = \begin{bmatrix} 400 & 20 & 10 \\ 20 & 500 & -40 \\ 10 & -40 & 200 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 430 \\ 480 \\ 170 \end{bmatrix}$$

$$C^{-1}\vec{r} = \begin{bmatrix} 400 & 20 & 10 \\ 20 & 500 & -40 \\ 10 & -40 & 200 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 87 \\ 150 \\ 10 \end{bmatrix}$$

$$a = \vec{r}^T C^{-1}\vec{1} = (0.2, 0.3, 0.1) \begin{pmatrix} 430 \\ 480 \\ 170 \end{pmatrix} = 247$$

$$b = \vec{r}^T C^{-1}\vec{r} = \dots = 63.4$$

$$c = \vec{1}^T C^{-1}\vec{1} = (1, 1, 1) \begin{pmatrix} 430 \\ 480 \\ 170 \end{pmatrix} = 1080$$

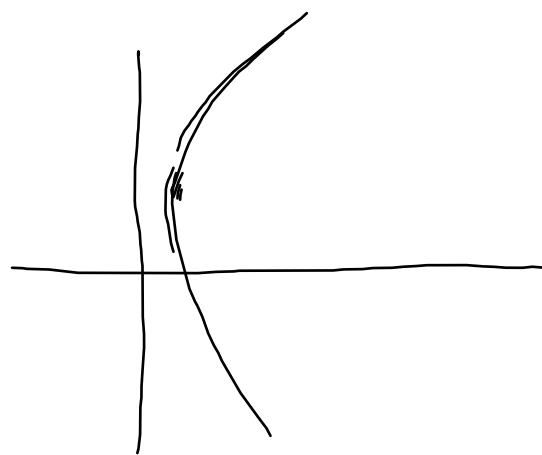
b) $\sigma^2 = c \frac{(r - \frac{a}{c})^2}{b c - a^2} + \frac{1}{c}$

$$b c - a^2 = 7463, \quad \frac{a}{c} = 0.229, \quad \frac{1}{c} = 0.000926$$

$$\sigma^2 = 1080 \frac{(r - 0.229)^2}{7463} + 0.000926 = 0.145 \underbrace{(r - 0.229)^2} + 0.000926$$

Løser for r:

$$r = 0.229 \pm \sqrt{\frac{\sigma^2}{0.145} - \frac{0.000926}{0.145}} = 0.229 \pm \sqrt{6.900^2 - 0.00639}$$



c) Ønsker forventet avkastning på 30%. Hva er minste risiko?

$$\sigma^2 = 1080 \frac{(0.3 - 0.229)^2}{7463} + 0.000926 = 0.001657$$

$$\sigma = 0.0407, \text{ dvs ca } 4.1\%$$

Hvis risikoen er 20%, hva er da størst avkastning

$$r = 0.229 \pm \sqrt{6.90 \cdot 0.2^2 - 0.00639} = 0.748 \approx \underline{\underline{75\%}}$$

Oppgave 5: a) $A, B, x > 0, 0 < r < 1$

$$f(u) = A^{r-1} u^r + B^{r-1} (x-u)^r \text{ finn maks.}$$

Deriver: $f'(u) = A^{r-1} r u^{r-1} + B^{r-1} r (x-u)^{r-1} (-1)$

$$f'(u) = 0 \text{ når } A^{r-1} u^{r-1} = B^{r-1} (x-u)^{r-1} \text{ opphøyer i } \frac{1}{r-1}$$

$$Au = B(x-u)$$

$$(A+B)u = Bx$$

$$u_1^*(x) = u = \frac{B}{A+B} x$$

$$x-u = \frac{A+B}{A+B} x - \frac{B}{A+B} x = \frac{A}{A+B} x$$

Maksverdi: $f_{\max} = f\left(\frac{B}{A+B}x\right) = A^{r-1} \left(\frac{B}{A+B}x\right)^r + B^{r-1} \left(\frac{A}{A+B}x\right)^r$

$$= x^r \frac{A^{r-1} B^r (B+A)}{(A+B)^r} = \frac{A^{r-1} B^r}{(A+B)^{r-1}} x^r = \left(\frac{AB}{A+B}\right)^{r-1} x^r$$

b) $S_0, u_0, u_1, S_2 = S_0 - u_0 - u_1$

Optimer: $u_0 + \beta u_1 + \beta^2 S_2, 0 < \beta < 1, \text{ diskonteringsfaktor}$

Finn først $V(1, x)$. Her er gitt et uttak u , en nyttig

$$\beta u^r + \beta^2 (x-u)^r$$

Vi maksimerer $f(u) = \beta u^r + \beta^2 (x-u)^r$

$$= \alpha^{r-1} u^r + (\alpha^2)^{r-1} (x-u)^r$$

Hint: Innfør $\alpha = \beta^{\frac{1}{r-1}}$, da
 $\beta = \alpha^{r-1}$
 $\beta^2 = (\alpha^{r-1})^2 = (\alpha^2)^{r-1}$

Dette er problemet: A med $A = \alpha$ og $B = \alpha^2$

Maksimeringspunktet: $u_1^*(x) = \frac{B}{A+B} x = \frac{\alpha^2}{\alpha + \alpha^2} x = \frac{\alpha}{1+\alpha} x$

Maksimumsverdien: $V(1, x) = f_{\max}(x) = \left(\frac{AB}{A+B}\right)^{r-1} x^r = \left(\frac{\alpha \cdot \alpha^2}{\alpha + \alpha^2}\right)^{r-1} x^r$

$$= \left(\frac{\alpha^2}{1+\alpha}\right)^{r-1} x^r$$

c) Finn $V(0, x)$: Anta at u er i kilen O med en ressurss x .

Her er gitt et uttak u og deretter opplever optimalt, her er nyttig

$$u^r + V(1, x-u) = u^r + \left(\frac{\alpha^2}{1+\alpha}\right)^{r-1} (x-u)^r$$

Vi altså optimerer

$$f(u) = 1 \cdot u^r + \left(\frac{\alpha^2}{1+\alpha}\right)^{r-1} (x-u)^r$$

Dette er problemet fra a) med $A=1$ og $B = \frac{\alpha^2}{1+\alpha}$

Beste uttak: $u_0^*(x) = \frac{B}{A+B} x = \frac{\frac{\alpha^2}{1+\alpha}}{1 + \frac{\alpha^2}{1+\alpha}} x = \frac{\alpha^2}{1+\alpha+\alpha^2} x$

$$V(0, x) = f_{\max} = \left(\frac{AB}{A+B}\right)^{r-1} x^r = \left(\frac{1 \cdot \frac{\alpha^2}{1+\alpha}}{1 + \frac{\alpha^2}{1+\alpha}}\right)^{r-1} x^r = \left(\frac{\alpha^2}{1+\alpha+\alpha^2}\right)^{r-1} x^r$$

d) Finn de optimale uttakene u_0 og u_1 :

$$u_0^*(S_0) = \frac{\alpha^2}{1+\alpha+\alpha^2} S_0$$

Dette gir en gjenværende ressurss: $S_1 = S_0 - \frac{\alpha^2}{1+\alpha+\alpha^2} S_0 = \frac{1+\alpha}{1+\alpha+\alpha^2} S_0$

$$u_1^*(S_1) = \frac{\alpha}{1+\alpha} S_1 = \frac{\alpha}{1+\alpha} \frac{1+\alpha}{1+\alpha+\alpha^2} S_0 = \frac{\alpha}{1+\alpha+\alpha^2} S_0$$

Her er igjen?

$$S_2 = S_0 - u_0^*(S_0) - u_1^*(S_0) = S_0 - \frac{\alpha^2}{1+\alpha+\alpha^2} S_0 - \frac{\alpha}{1+\alpha+\alpha^2} S_0$$

$$S_0 \left(\frac{1+\alpha+\alpha^2}{1+\alpha+\alpha^2} - \frac{\alpha^2}{1+\alpha+\alpha^2} - \frac{\alpha}{1+\alpha+\alpha^2} \right) = \frac{1}{1+\alpha+\alpha^2} S_0$$