

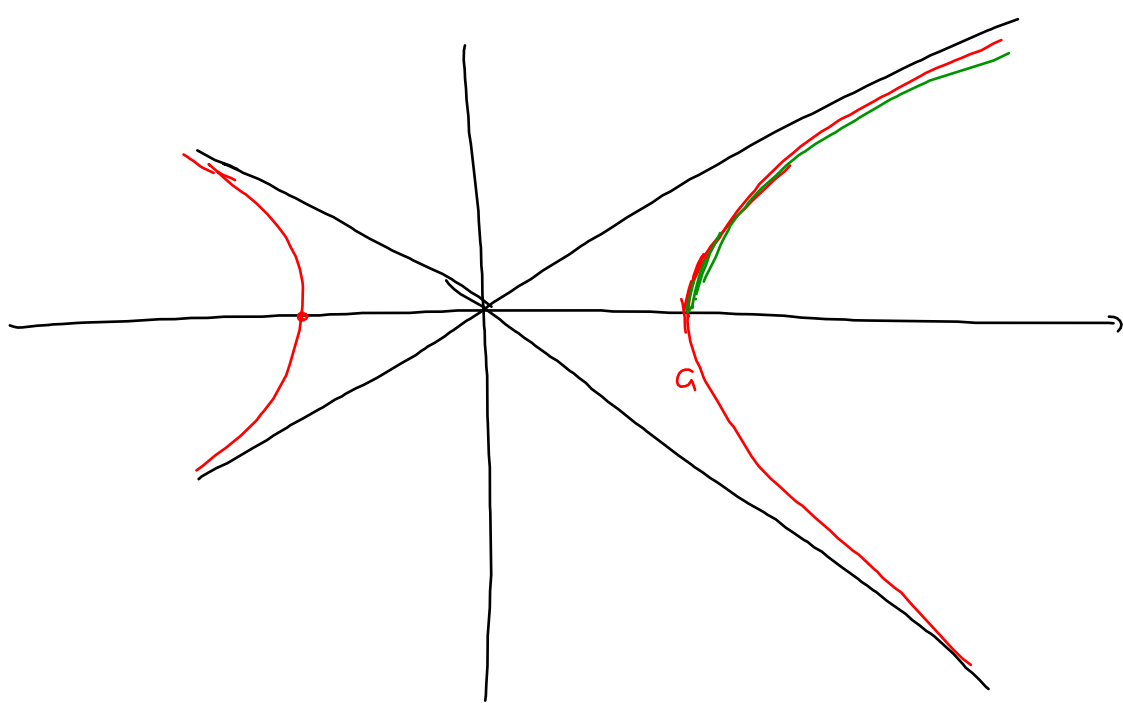
Hyperbeler

Liungung: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{a^2} - 1 = \frac{y^2}{b^2}$

$$\Rightarrow y^2 = \frac{b^2}{a^2} x^2 - b^2 \Rightarrow y = \pm \sqrt{\frac{b^2}{a^2} x^2 - b^2} \text{ for } x^2 \geq a^2$$

Asymptoter: $y = \pm \frac{b}{a} x$

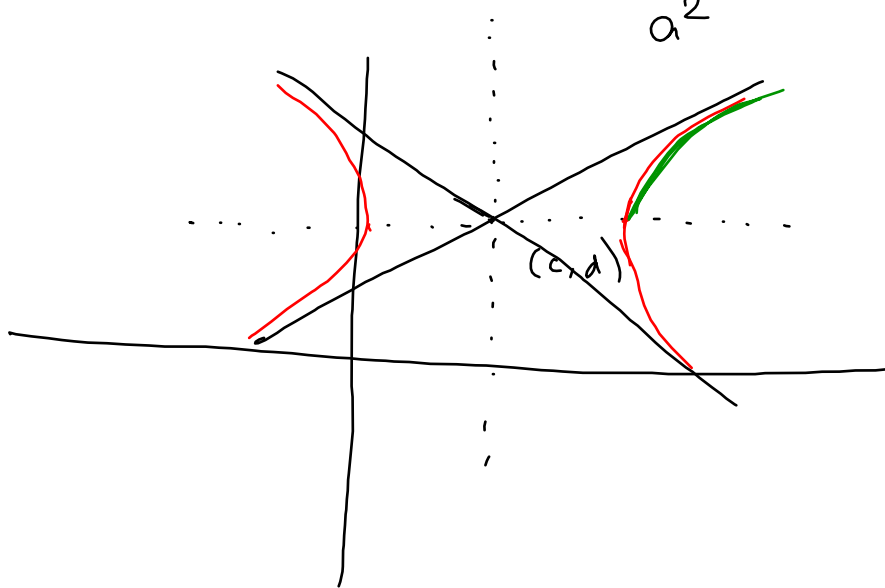
$$y = \frac{b}{a} x$$



$$y = -\frac{b}{a} x$$

Flutte "origo":

$$\frac{(x-c)^2}{a^2} - \frac{(y-d)^2}{b^2} = 1$$

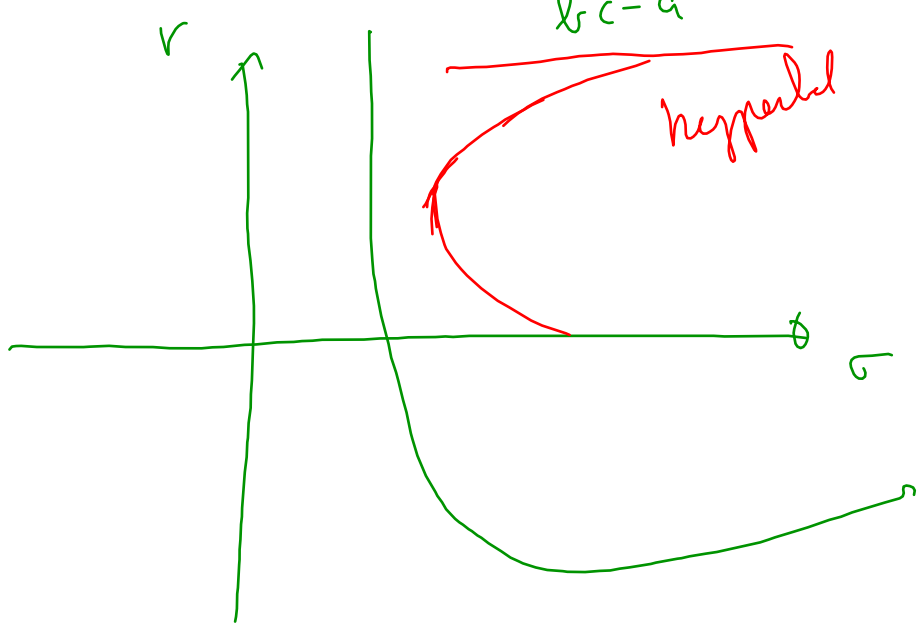


Tilbage til porteføljer

Følgende gang fandt vi formelen

$$\sigma^2 = c \frac{(r - \frac{a}{c})^2}{b c - a^2} + \frac{1}{c}$$

(En kurve i σ - r -planen)

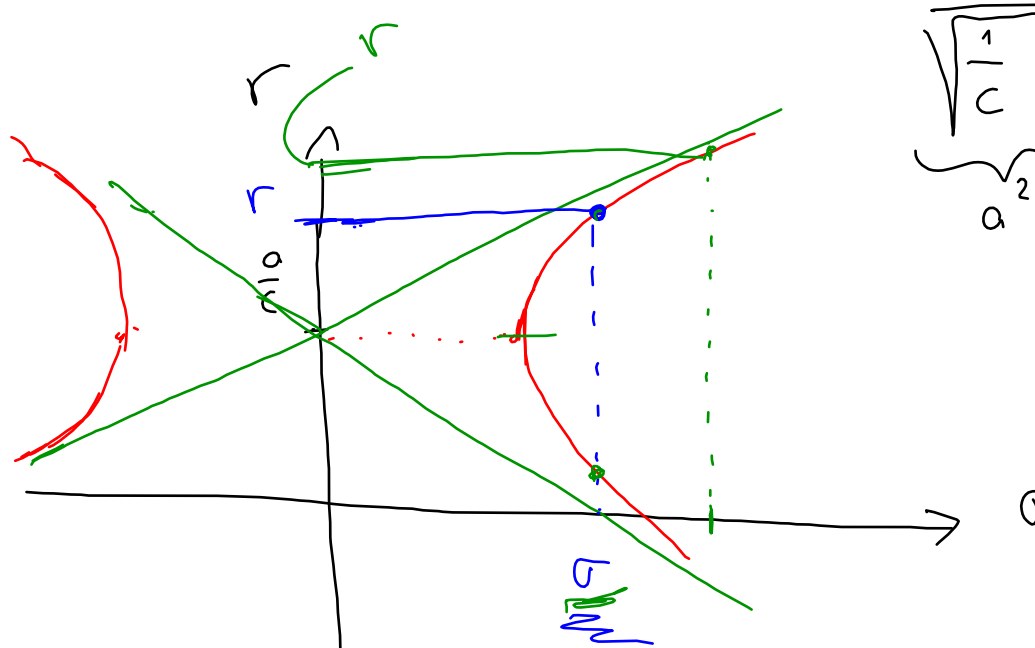


$$\frac{(\sigma - c)^2}{a^2} - \frac{(r - d)^2}{b^2} = 1$$

$$\sigma^2 - c \frac{(r - \frac{a}{c})^2}{b c - a^2} = \frac{1}{c} \quad | \cdot c$$

$$c \sigma^2 - c^2 \frac{(r - \frac{a}{c})^2}{b c - a^2} = 1$$

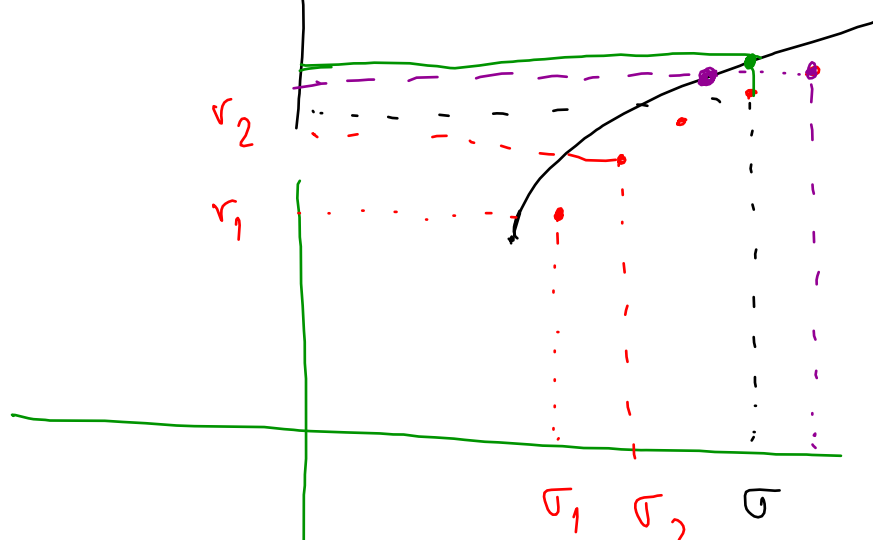
$$\frac{\sigma^2}{\underbrace{\sqrt{\frac{1}{c}}}_{a^2}}^2 - \frac{(r - \frac{a}{c})^2}{\underbrace{(\frac{\sqrt{b c - a^2}}{c})^2}_{b^2}} = 1$$



Sammenhængen: For givet

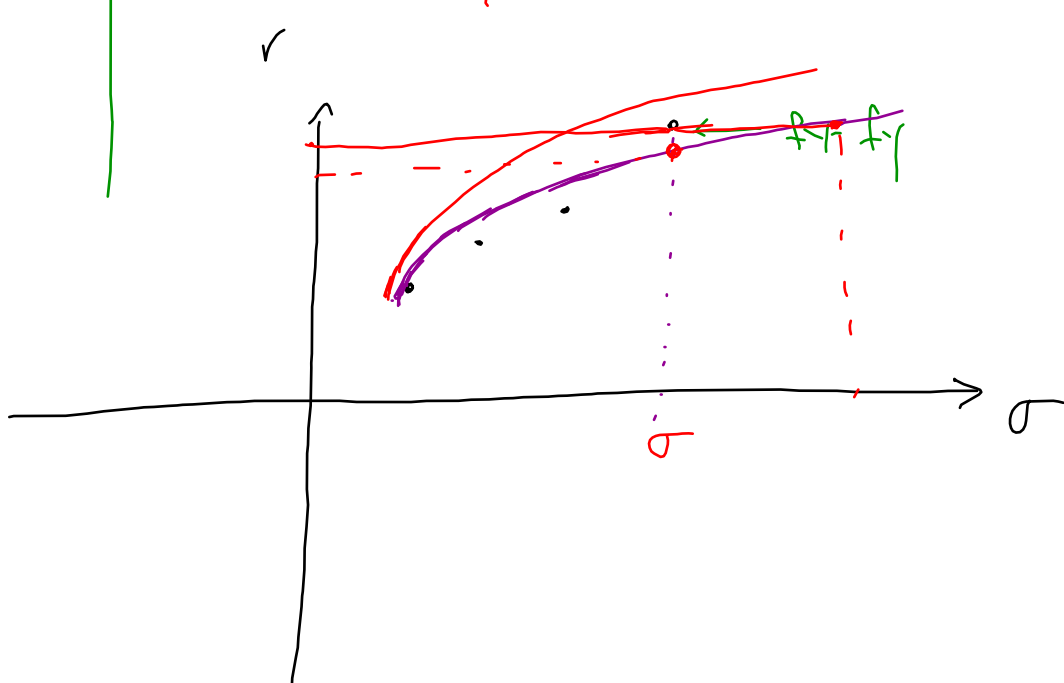
afkastning r , er σ den minimale risiko.

Omvendt: Givet σ , giver r den største afkastning i konf. med risiko σ .

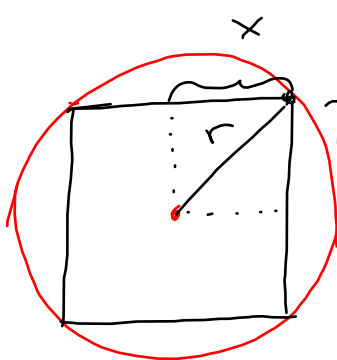


den efficients
porteføljer

4 alger



Oppgave 12:



$$f(x, y) = kxy^2 \quad \text{maksimeres}$$

$$g(x, y) = x^2 + y^2 = r^2$$

Partiell derivate:

$$\frac{\partial f}{\partial x} = ky^2, \quad \frac{\partial f}{\partial y} = 2kxy$$

$$\frac{\partial g}{\partial x} = 2x, \quad \frac{\partial g}{\partial y} = 2y$$

Lagrange: $\nabla f = \lambda \nabla g$:

$$(i) \quad ky^2 = 2\lambda x$$

$$(ii) \quad 2kxy = 2\lambda y$$

$$(iii) \quad x^2 + y^2 = r^2$$

$$\Rightarrow \frac{2ky}{ky^2} = \frac{2\lambda y}{2\lambda x}$$

$$\frac{2x}{y} = \frac{y}{x} \Rightarrow 2x^2 = y^2$$

Sett inn i (iii):

$$x^2 + 2x^2 = r^2$$

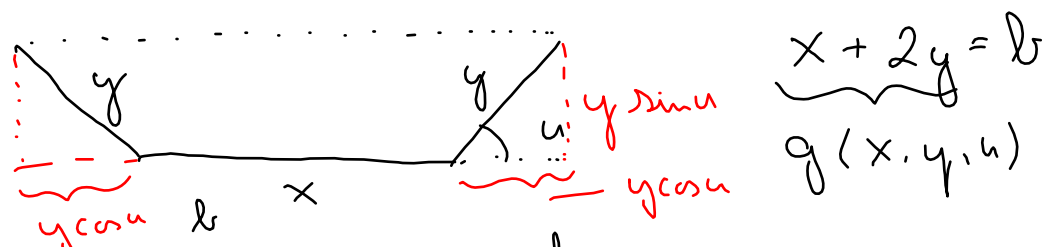
$$3x^2 = r^2$$

$$x^2 = \frac{r^2}{3} \Rightarrow x = \frac{r}{\sqrt{3}}$$

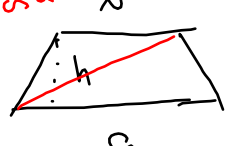
$$y^2 = 2x^2 = 2 \frac{r^2}{3} = \frac{2}{3} r^2$$

$$y = \sqrt{\frac{2}{3}} r$$

Oppgave 13



Areale til trapes:



$$A = \frac{a+b}{2} \cdot h$$

$$A = \frac{x + (x + 2y \cos u)}{2} \cdot y \sin u = (x + y \cos u) y \sin u$$

$$= xy \sin u + y^2 \sin u \cos u$$

$$f(x, y, u) = xy \sin u + y^2 \sin u \cos u$$

$$g(x, y, u) = x + 2y$$

Lagrange: $\frac{\partial f}{\partial x} = y \sin u$

$$\frac{\partial f}{\partial y} = x \sin u + 2y \sin u \cos u$$

$$\frac{\partial f}{\partial u} = xy \cos u + y^2 (\cos u \cos u - \sin u \sin u)$$

$$= xy \cos u + y^2 (2 \cos^2 u - 1)$$

$$\frac{\partial g}{\partial x} = 1, \quad \frac{\partial g}{\partial y} = 2, \quad \frac{\partial g}{\partial u} = 0$$

Lagranges betingelser: (i) $y \sin u = \lambda$

$$(ii) \quad x \sin u + 2y \sin u \cos u = 2\lambda$$

$$(iii) \quad xy \cos u + y^2 (2 \cos^2 u - 1) = 0$$

$$(iv) \quad x + 2y = b$$

For (i): $\frac{x \sin u + 2y \sin u \cos u}{y \sin u} = \frac{2\lambda}{\lambda}$

$$\frac{x}{y} + 2 \cos u = 2 \Rightarrow x = 2y - 2y \cos u$$

Sett inn i (iii): $(2y - 2y \cos u) \cos u + y(2 \cos^2 u - 1) = 0$

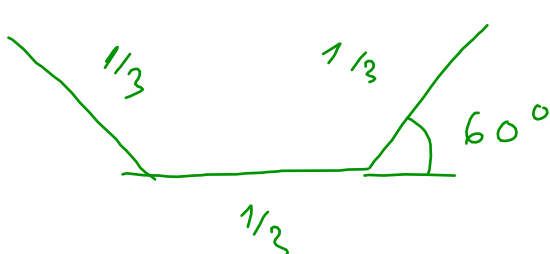
$$2y \cos u - 2y \cos^2 u + 2y \cos^2 u - y = 0$$

$$2y \cos u - y = 0 \Rightarrow y(2 \cos u - 1) = 0$$

Siden y ikke kan være null, må vi ha $\cos u = \frac{1}{2} \Rightarrow u = \frac{\pi}{3} = 60^\circ$

Dermed er: $x = 2y - 2y \cdot \frac{1}{2} = 2y - y = y$

Randbetingelser: $x + 2y = b \Rightarrow 3x = b \Rightarrow x = \frac{b}{3}$
 $y = \frac{b}{3}$



Oppgave 16:

$$P(x, y) = K x^\alpha y^\beta$$

$$\underbrace{x+y}_{g(x,y)} = S$$

Lagrange: $\frac{\partial P}{\partial x} = \alpha K x^{\alpha-1} y^\beta$

$$\frac{\partial P}{\partial y} = \beta K x^\alpha y^{\beta-1}$$

$$\frac{\partial g}{\partial x} = 1$$

$$\frac{\partial g}{\partial y} = 1$$

Betingelser:

$$\begin{aligned} \alpha K x^{\alpha-1} y^\beta &= \lambda \\ \beta K x^\alpha y^{\beta-1} &= \lambda \\ x+y &= S \end{aligned} \left\{ \begin{array}{l} \text{dividerer} \\ \text{forhold} \end{array} \right. \frac{\alpha K x^{\alpha-1} y^\beta}{\beta K x^\alpha y^{\beta-1}} = \frac{\lambda}{\lambda} = 1$$

$$\frac{\alpha}{\beta} \frac{y}{x} = 1 \Rightarrow y = \frac{\beta}{\alpha} x$$

Setter inn i $x+y=S$: $x + \frac{\beta x}{\alpha} = S \Rightarrow x(1 + \frac{\beta}{\alpha}) = S$

$$x = \frac{S}{1 + \frac{\beta}{\alpha}} \Rightarrow \frac{\alpha S}{\alpha + \beta}, y = \frac{\beta S}{\alpha + \beta}$$

b) $P(x_1, x_2, \dots, x_n) = K x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$ $\underbrace{x_1 + x_2 + \dots + x_n}_{g(x_1, x_2, \dots, x_n)} = S$

$$\frac{\partial P}{\partial x_1} = \alpha_1 K x_1^{\alpha_1-1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

$$\frac{\partial g}{\partial x_i} = 1$$

$$\frac{\partial P}{\partial x_i} = \alpha_i K x_1^{\alpha_1} \dots x_i^{\alpha_i-1} \dots x_n^{\alpha_n}$$

Lagranges betingelser:

$$\alpha_1 K x_1^{\alpha_1-1} x_2^{\alpha_2} \dots x_n^{\alpha_n} = \lambda$$

$$\alpha_i K x_1^{\alpha_1} \dots x_i^{\alpha_i-1} \dots x_n^{\alpha_n} = \lambda$$

$$\vdots$$

Deler linje 1 på linje (i)

$$\frac{\alpha_1 \cancel{K} x_1^{\alpha_1-i} \dots x_i^{\alpha_i} \dots x_n^{\alpha_n}}{\alpha_i \cancel{K} x_1^{\alpha_1} \dots x_i^{\alpha_i-1} \dots x_n^{\alpha_n}} = \frac{\lambda}{\lambda}$$

$$\frac{\alpha_1}{\alpha_i} \frac{x_i}{x_1} = 1 \Rightarrow x_i = \frac{\alpha_i}{\alpha_1} x_1$$

Setter inn i $x_1 + x_2 + \dots + x_n = S \Rightarrow$

$$x_1 + \frac{\alpha_2}{\alpha_1} x_1 + \frac{\alpha_3}{\alpha_1} x_1 + \dots + \frac{\alpha_n}{\alpha_1} x_1 = S$$

$$x_1 \left(1 + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_3}{\alpha_1} + \dots + \frac{\alpha_n}{\alpha_1} \right) = S \quad | \cdot \alpha_1$$

$$x_1 (\alpha_1 + \alpha_2 + \dots + \alpha_n) = \alpha_1 S$$

$$x_1 = \frac{\alpha_1 S}{\alpha_1 + \alpha_2 + \dots + \alpha_n}$$

$$x_i = \frac{\alpha_i}{\alpha_1} x_1 = \frac{\alpha_i}{\alpha_1} \cdot \frac{\alpha_1 S}{\alpha_1 + \alpha_2 + \dots + \alpha_n} = \frac{\alpha_i S}{\alpha_1 + \alpha_2 + \dots + \alpha_n}$$