

## Fasit til oppgaver, kapittel 1

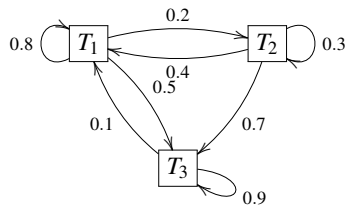
**Oppgave 1.**

$$\begin{pmatrix} 0.6 & 0.9 & 0.5 \\ 0.4 & 0 & 0 \\ 0 & 0.1 & 0.5 \end{pmatrix}$$

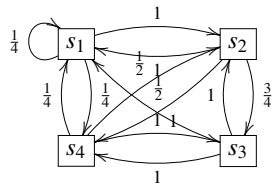
**Oppgave 2.**

$$\begin{pmatrix} 0 & 0.5 & 1.5 & 0.3 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.5 & 0.3 \end{pmatrix}$$

**Oppgave 3.**



**Oppgave 4.**



**Oppgave 5.** a)  $\begin{pmatrix} 120 \\ 180 \end{pmatrix}$  b)  $\begin{pmatrix} 0.68 & 0.32 \\ 0.32 & 0.68 \end{pmatrix}$  c)  $\begin{pmatrix} 150 \\ 150 \end{pmatrix}$

**Oppgave 6.** a)  $\begin{pmatrix} 240 \\ 30 \\ 30 \end{pmatrix}$  b)  $\begin{pmatrix} 0.66 & 0.17 & 0.17 \\ 0.17 & 0.66 & 0.17 \\ 0.17 & 0.17 & 0.66 \end{pmatrix}$

c)  $\begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}$

**Oppgave 7.**

$$M^2 = \begin{pmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{pmatrix}, \quad M^3 = \begin{pmatrix} 0.65 & 0.525 \\ 0.35 & 0.475 \end{pmatrix}$$

$$\mathbf{v}_1 = \begin{pmatrix} 110 \\ 90 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 117.5 \\ 112.5 \end{pmatrix}$$

Avhengig av summen av komponentene.

**Oppgave 8.**

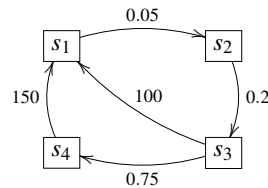
$$\begin{pmatrix} 0.998 & 0.65 & 0 \\ 0.002 & 0 & 0.1 \\ 0 & 0.35 & 0.9 \end{pmatrix}, \quad \mathbf{v}_\infty = \begin{pmatrix} 650 \\ 2 \\ 7 \end{pmatrix}$$

Forrdelingen vil være uavhengig av startverdi.

**Oppgave 9.**

$$P = \begin{pmatrix} 0 & 0 & 100 & 150 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.75 & 0 \end{pmatrix}$$

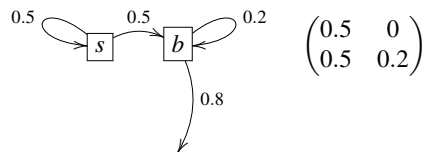
$$P^2 = \begin{pmatrix} 0 & 20 & 112.5 & 0 \\ 0 & 0 & 5 & 7.5 \\ 0.01 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 \end{pmatrix}$$



$$\mathbf{v}_0 = \begin{pmatrix} 95000 \\ 500 \\ 200 \\ 375 \end{pmatrix}$$

Etter hvert blir fordelingen mellom komponentene (ca.) 4300:174:28:17

**Oppgave 10.**



$$s_{t+1} = 0.5s_t$$

$$b_{t+1} = 0.5s_t + 0.2b_t$$

Preparatet forsvinner etter hvert ut, men vil etter hvert fordele seg med forholdet  $s : b = 3 : 5$ .

**Oppgave 11.**

## Fasit til oppgaver, kapittel 2

**Oppgave 1.** a)  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$  b)  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  c)  $\begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix}$

**Oppgave 2.** a)

$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

b)

$$\begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

d)

$$\begin{pmatrix} 2 & 0 & -2 & 4 \\ 2 & 2 & 0 & 2 \\ 4 & 2 & 0 & -6 \end{pmatrix}$$

**Oppgave 3.**

$$A^2 = \begin{pmatrix} 1 & 0 \\ 2c & 1 \end{pmatrix}, A^3 = \begin{pmatrix} 1 & 0 \\ 3c & 1 \end{pmatrix}, A^n = \begin{pmatrix} 1 & 0 \\ nc & 1 \end{pmatrix}$$

**Oppgave 4.** a)

$$AB = \begin{pmatrix} -2 & -1 & -2 & 2 \\ 4 & 0 & 0 & 6 \\ 8 & 2 & 4 & 2 \\ 2 & 0 & 0 & 3 \end{pmatrix}, BA = \begin{pmatrix} 5 & -2 \\ 7 & 0 \end{pmatrix}$$

b)

$$AB = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & -2 \end{pmatrix}$$

*Produktet  $B \cdot A$  har ingen mening.*

c)

$$AB = (2), BA = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 2 & 3 \end{pmatrix}$$

d)  $AB = BA = (6)$

**Oppgave 5.** a) 22 b) 2 c) -1 d) 1

**Oppgave 6.** a)  $\lambda^2 + \lambda - 2$ , Egenverdier: 1 og -2

b)  $\lambda^2 - 5\lambda - 12$ , Egenverdier:  $\frac{5 \pm \sqrt{73}}{2}$

c)  $\lambda^2 - \lambda - 1$ , Egenverdier:  $\frac{1 \pm \sqrt{5}}{2}$

d)  $\lambda^2 - 2\lambda - 8$ , Egenverdier: 4 og -2

e)  $\lambda^2 - 1$ , Egenverdier: 1 og -1

**Oppgave 7.** a)

$$\begin{pmatrix} \frac{3}{2} & 1 \\ -1 & -1 \end{pmatrix}$$

b)

$$-\frac{1}{12} \begin{pmatrix} 1 & -8 \\ -2 & 4 \end{pmatrix}$$

c)

$$\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

d)

$$-\frac{1}{8} \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}$$

e)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

**Oppgave 8.**

$$A^{-1} = \frac{1}{6} \begin{pmatrix} 3 & 6 & 0 \\ 0 & 6 & 0 \\ -1 & -2 & 2 \end{pmatrix}$$

**Oppgave 9.** a)  $x = 1, y = 1$

b)  $x = -\frac{5}{2}, y = 2$

c)  $x = 2, x = 1$

**Oppgave 10.**

**Oppgave 11.** *Produktet av to ortogonale matriser er en ortogonal matrise fordi*

$$(PQ)^{-1} = Q^{-1}P^{-1} = Q^T P^T = (PQ)^T$$

**Oppgave 12.**

$$\det(A) = 4 \neq 0 \quad A^{-1} = \frac{1}{4} \begin{pmatrix} 3 & -2 \\ -1 & 2 \end{pmatrix}$$

$$\lambda^2 - 5\lambda + 4$$

Egenverdier: 1 og 4

**Oppgave 13.**

$$\det(A) = 1 \quad A^{-1} = A$$

$$\chi_A(\lambda) = -\lambda^3 - \lambda^2 + \lambda + 1$$

Eigenverdier: 1 og -1

**Oppgave 14.**

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

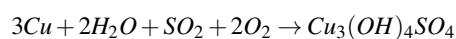
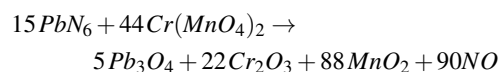
**Oppgave 15.** Underbestemt

## Fasit til oppgaver, kapittel 3

**Oppgave 1.****Oppgave 2.****Oppgave 3.****Oppgave 4.**

**Oppgave 5.** a)  $\mathbf{w} = -\mathbf{v}_1 + \frac{1}{2}\mathbf{v}_2$  b)  $\mathbf{w} = \frac{2}{3}\mathbf{v}_1 + \frac{7}{3}\mathbf{v}_2 - \frac{1}{3}\mathbf{v}_3$   
c) Nei

**Oppgave 6.** a) Nei b)  $2\mathbf{v}_1 - \mathbf{v}_2 - \mathbf{v}_3 = \mathbf{0}$  c) Nei

**Oppgave 7.****Oppgave 8.****Oppgave 9.****Oppgave 10.****Oppgave 11.****Oppgave 12.****Oppgave 13.****Oppgave 14.** 2**Oppgave 15.** 2

**Oppgave 16.** a) 2 b) 1 c) 0

**Oppgave 17.** a) 3 b) 1 c) 2

**Oppgave 18.** a) Ja b) Nei c) Nei

**Oppgave 19.****Oppgave 20.**

**Oppgave 21.** a)  $\left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle$  b)  $\left\langle \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \right\rangle$

**Oppgave 22.** a)  $x(1, 1)$  b)  $x_1(1, 1, 1)$  c)  $bx$

**Oppgave 23.**

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

**Oppgave 24.** Dimensjonen til  $V$  er 8, kan ikke si noe om dimensjonen til  $W$ .

**Oppgave 25.**  $n$

**Oppgave 26.** a)  $\lambda^2 - 1$ , egenverdier: 1 og -1

b)  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  og  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

**Oppgave 27.** a)  $\lambda^2 - 2\lambda + 1$ , egenverdi  $\lambda = 1$

b) Egenvektor  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ , ingen basis

**Oppgave 28.** a)

$$\begin{aligned} T(af + bg) &= \frac{1}{2}x \frac{d}{dx}(af + bg) \\ &= \frac{1}{2}ax \frac{df}{dx} + \frac{1}{2}bx \frac{dg}{dx} \\ &= aT(f) + bT(g) \end{aligned}$$

b)

$$T(ax^2 + bx + c) = \frac{1}{2}x(2ax + b)$$

c)  $ax^2$  og  $bx$

## Fasit til oppgaver, kapittel 4

**Oppgave 1.** a)  $\frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$  b)  $\frac{1}{2} \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$

c)  $\frac{\sqrt{2}}{2} \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$  d)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

**Oppgave 2.** a)  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  b)  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  c)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

**Oppgave 3.** a)  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

b)  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  c)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

**Oppgave 4.** a)  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

b)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}$  c)  $\frac{1}{2} \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix} \mathbf{x} +$

$\frac{1}{2} \begin{pmatrix} 1 \\ 2 - \sqrt{3} \end{pmatrix}$  d) Identiteten

**Oppgave 5.** a)  $\mathbf{x} + \begin{pmatrix} 0 \\ -2 \end{pmatrix}$  b)  $\mathbf{x} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}$  c)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x}$

**Oppgave 6.**  $\begin{pmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & -\cos \theta \end{pmatrix}$

**Oppgave 7.** Rotasjon med  $-\theta$ .

**Oppgave 8.**  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x}$

**Oppgave 9.**  $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x}$

**Oppgave 10.**  $\begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} \mathbf{x}$

**Oppgave 11.**  $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

**Oppgave 12.**  $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

**Oppgave 13.**  $(\frac{1}{2}, \frac{1+\sqrt{2}}{2})$

**Oppgave 14.**  $(-1, 0)$

**Oppgave 15.** Linja  $x + y + 2 = 0$

**Oppgave 16.** Linja  $2x - 2y + 1 = 0$

**Oppgave 17.**  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ , hjørner:  $(0, 0)$   $(0, 1)$   $(-1, 0)$   $(-1, 1)$

## Fasit til oppgaver, kapittel 5

**Oppgave 18.**  $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ , hjørner:  $(0,0)$   $(1,1)$   $(-1,1)$   
 $(0,2)$

**Oppgave 19.**  $\mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ , hjørner:  $(1,1)$   $(2,1)$   $(1,2)$   
 $(2,2)$

**Oppgave 20.**  $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ , hjørner:  $(1,2)$   $(1,3)$   
 $(2,3)$   $(2,4)$

**Oppgave 21.**  $S(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $T(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $(T \circ S)(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

**Oppgave 22.**  $S(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $T(\mathbf{x}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ ,  $(T \circ S)(\mathbf{x}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

**Oppgave 23.**  $S(\mathbf{x}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $T(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $(T \circ S)(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

**Oppgave 24.**  $S(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x}$ ,  $T(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $(T \circ S)(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

**Oppgave 25.**  $S(\mathbf{x}) = \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ,  $T(\mathbf{x}) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{x}$ ,  $(T \circ S)(\mathbf{x}) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

**Oppgave 1.** a)  $\sqrt{6}$  b) 3 c)  $\sqrt{14}$

**Oppgave 2.** a)  $(0,1,0)$  b)  $(-2,1,0)$  c)  $(0,1,0)$  d)  $(2,3,0)$

**Oppgave 3.** a)  $(-2,1,2)$  b)  $(1,1,3)$  c)  $(-2,2,0)$  d)  $(1,4,0)$

**Oppgave 4.** a)  $-3\mathbf{k}$  b)  $-\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  c) 0 d)  $3\mathbf{i} + 4\mathbf{k} - \mathbf{k}$

**Oppgave 5.** a)  $\mathbf{w} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

b)  $\mathbf{w} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

c)  $\mathbf{w} = \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$

d)  $\mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

**Oppgave 6.** a)  $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\mathbf{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

b)  $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ ,  $\mathbf{w} = \begin{pmatrix} -4 \\ -2 \\ -5 \end{pmatrix}$

c)  $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ ,  $\mathbf{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

**Oppgave 7.** a) 2 b) -2 c) 2

**Oppgave 8.** -4

**Oppgave 9.** 0

**Oppgave 10.**

$$\frac{1}{3} \begin{pmatrix} \sqrt{6} \\ (1 - \sqrt{2})(3 - \sqrt{3}) \\ 3 - \sqrt{3} \end{pmatrix}$$

**Oppgave 11.**

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

**Oppgave 12.** Regn ut Housholder-matrisen til den oppgitte vektoren

$$a) \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

$$b) \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$c) \frac{1}{9} \begin{pmatrix} 7 & -4 & -4 \\ -4 & 1 & -8 \\ -4 & -8 & 1 \end{pmatrix}$$

**Oppgave 13.**

$$\begin{pmatrix} 7 \\ 1 \\ -3 \end{pmatrix}$$

**Oppgave 14.**

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

**Oppgave 15.**

$$\begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix}$$

**Oppgave 16.**

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

*dvs. refleksjon + rotasjon*

**Oppgave 17.** Linja gitt ved  $(1, 0, -1)$ , dvs en rotasjon