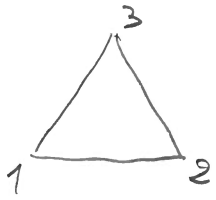
Gruppeteori

Symmetrier av en trekant (likesidet)



Permutasjoner:

Rotasjon $1 \to 2 \to 3 \to 1$ $\quad \rho$

Refleksjon $1 \to 2 \to 1$ $\quad \mu$

Andre refleksjoner:

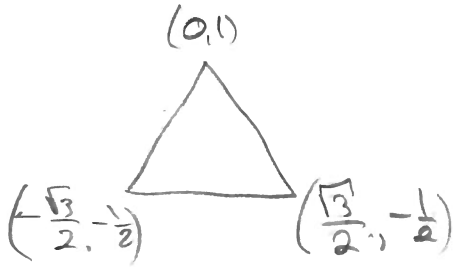
$1 \to 3 \to 1$: $\rho\mu$

$2 \to 3 \to 2$: $\rho^2\mu$

6 symmetrier : , relasjoner $\rho^3 = id$ $\mu^2 = id$ $\rho^2\mu = \mu\rho$

$$S_3 = \langle \rho, \mu \mid \rho^3 = \mu^2 = id,\ \rho^2\mu = \mu\rho \rangle$$

Representasjoner (matriseform)



$$\rho = \rho_{\frac{2\pi}{3}} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \quad \rho^2 = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\mu = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho^2\mu = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

$$\mu\rho = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

ρ, μ ortogonale : $\rho^3 = id$ betyr $\rho^2 = \rho^{-1}$, men $\rho^2 = \rho^T$.

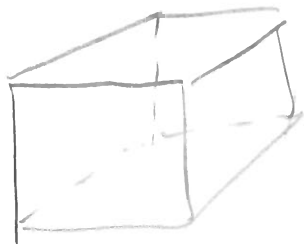
$\mu = \mu^{-1} = \mu^T$.

$$\chi_\rho(\lambda) = \begin{vmatrix} -\frac{1}{2} - \lambda & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} - \lambda \end{vmatrix} = \lambda^2 + \lambda + 1 = 0$$

$$\Rightarrow \lambda = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$\chi_\mu(\lambda) = \begin{vmatrix} -1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \quad \text{gir} \quad \lambda = \pm 1$$

Symmetrier av ~~tetning~~



Antall symmetrier: $4 \cdot 6 = 24$

Representasjoner

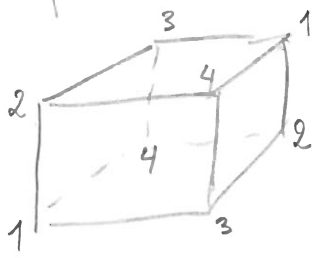
$$\rho_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \rho_y = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \rho_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\rho_x^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\rho_x^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\rho_y \rho_x = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

Permutasjoner



ρ_x: $1 \to 4 \to 3 \to 2 \to 1$ (1432)

ρ_y: $1 \to 3 \to 4 \to 2 \to 1$ (1342)

ρ_z: $1 \to 3 \to 2 \to 4 \to 1$ (1324)

$\rho_y \rho_x$: $1 \to 4 \to 2$, $2 \to 1 \to 3$, $3 \to 2 \to 1$, $4 \to 3 \to 4$ (123)

ρ_x^2: $1 \to 3 \to 2$, $2 \to 4 \to 3$, $3 \to 2 \to 1$, $4 \to 1 \to 4$

$(1324) = \rho_z$

$$\rho_y \rho_x = \rho_x \rho_z$$

$$\Downarrow$$

$$\rho_z = \rho_x^3 \rho_y \rho_x$$

$\rho_z \rho_x$: $1 \to 4 \to 1$, $2 \to 1 \to 3$, $3 \to 2 \to 4$, $4 \to 3 \to 2$

ρ_x^2: $1 \to 2 \to 1$, $2 \to 4 \to 3$, $3 \to 1 \to 4$, $4 \to 3 \to 2$

$(1243) = \rho_y^3$

$$\rho_z \rho_x = \rho_x \rho_y^3$$

$$\Downarrow$$

$$\rho_z = \rho_x \rho_y^3 \rho_x^3$$

$$\rho_x^3 \rho_y \rho_x = \rho_x \rho_y^3 \rho_x^3$$

$$\Downarrow$$

$$\rho_y \rho_x = \rho_x^2 \rho_y^3 \rho_x^3 \Rightarrow \rho_y \rho_x^2 = \rho_x^2 \rho_y^3$$

$$\mathrm{Sym}(\square) = \langle \rho_x, \rho_y \mid \rho_x^4 = \rho_y^4 = id \quad \rho_y \rho_x^2 = \rho_x^2 \rho_y^3 \rangle$$

$$= \langle \rho, \mu, \sigma \mid \rho^4 = \mu^3 = \sigma^2 = id,\ \mu\rho = \rho^2\mu^2\sigma,\ \sigma\mu = \mu^2\sigma,\ \sigma\rho = \rho^3\mu^2 \rangle$$

$\rho = (1234)$

$\mu = (123)$

$\sigma = (12)$

$\mu\rho$: $1\to2\to3$, $2\to3\to1$, $3\to4\to4$, $4\to1\to2$

$\rho^2\mu^2\sigma$: $1\to2\to3\to1\to2\to3$, $2\to1\to2\to3\to4\to1$, $3\to3\to1\to2\to3\to4$, $4\to4\to4\to4\to1\to2$

$\sigma\mu$: $1\to2\to1$, $2\to3\to3$, $3\to1\to2$, $4\to4\to4$

$\mu^2\sigma$: $1\to2\to3\to1$, $2\to1\to2\to3$, $3\to3\to1\to2$, $4\to4\to4\to4$

$\sigma\rho$: $1\to2\to1$, $2\to3\to3$, $3\to4\to4$, $4\to1\to2$

$\rho^2\mu^2$: $1\to2\to3\to4\to1$, $2\to3\to1\to2\to3$, $3\to1\to2\to3\to4$, $4\to4\to4\to1\to2$

ρ : Rotasjon om x-aksen

μ : Rotasjon om diagonalen gjennom 4-punktene

σ : Rotasjon om midtpunktet på 1-2-kanten

Visualize-simetria.web.uah.es

Grupper av lav orden

orden 1: $\{\Phi 3$

orden 2: $\{1, -1\}$ $Sym(\bullet\ \bullet)$

orden 3: Symmetri av ulikefarget trehånd
$\{id, a, a^2\} \cong \mathbb{Z}_3$

orden 4: To grupper $\mathbb{Z}_4$ og $\mathbb{Z}_2 \times \mathbb{Z}_2$
$\parallel$
alle par $(\pm 1, \pm 1)$

orden 5: $\mathbb{Z}_5$ ulikefarget 5-kant

orden 6: S_3, $\mathbb{Z}_6$
↑ ↑
ulike abelsk abelsk, syklisk

Alle elementer må ha orden som deler orden til gruppa

orden 6 $x^3 = 1$ $y^2 = 1$

6 elementa $x^m y^n$ $m = 0, 1, 2$ $n = 0, 1$

Hva med yx og yx^2 ?

$yx = x \Rightarrow y = id$

$yx = xy \Rightarrow (xy)^2 = x^2 (xy)^3 = y$ $\mathbb{Z}_6$

$yx = x^2 y \Rightarrow S_3$ $x = (123)$ $y = (12)$

S_3: symmetri av 3-hånd

$\mathbb{Z}_6$: symmetri av ulikefarget 6-kant

yx: $1 \mapsto 2 \mapsto 1$
$2 \mapsto 3 \mapsto 3$
$3 \mapsto 1 \mapsto 2$

$x^2 y$: $1 \mapsto 2 \mapsto 3 \mapsto 1$
$2 \mapsto 1 \mapsto 2 \mapsto 3$
$3 \mapsto 3 \mapsto 1 \mapsto 2$