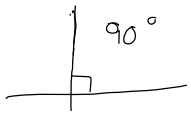
Vinkler

Grader:



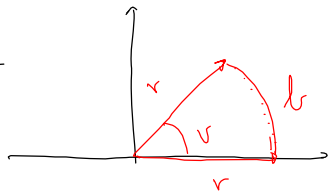




Radianer:

Förhållandet i v i radianer:

$$v = \frac{l}{r}$$

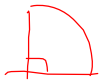


$v = 360°$

$$v = \frac{2\pi r}{r} = 2\pi$$



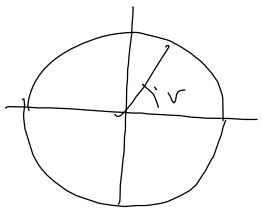
$v = 2\pi$

$90°$ 

$$v = \frac{2\pi}{4} = \frac{\pi}{2}$$

$45°$ $\quad v = \frac{\pi}{4}$

Generell omskrivning:



Vinkel: $v°$ grader

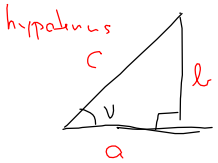
$$\frac{v}{2\pi} = \frac{v°}{360°}$$

$$v = 2\pi \frac{v°}{360°} = \frac{\pi}{180} v°$$

$v° = 30°$ $\quad v = \frac{\pi}{180} \cdot 30 = \frac{\pi}{6}$

$v° = 60°$ $\quad v = \frac{\pi}{180} \cdot 60 = \frac{\pi}{3}$

Trekantberegninger



motstående katet

hosliggende katet

Sinus: $\sin v = \frac{\text{motstående}}{\text{hypotenus}} = \frac{l}{c}$

Cosinus: $\cos v = \frac{\text{hosliggende}}{\text{hypotenus}} = \frac{a}{c}$

Tangens: $\tan v = \frac{\text{motstående}}{\text{hosliggende}} = \frac{l}{a}$

Cotangens: $\cot v = \frac{\text{hosliggende}}{\text{motstående}} = \frac{a}{l}$

Sammenhenger:

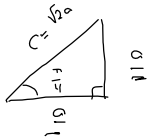
$$\frac{\sin v}{\cos v} = \frac{\frac{l}{c}}{\frac{a}{c}} = \frac{l}{a} = \tan v$$

$$\tan v = \frac{\sin v}{\cos v}$$

$$\cot v = \frac{\cos v}{\sin v}$$

Verdier til trigonometriske funksjoner

45° dvs $\frac{\pi}{4}$



likebeint

Pytagoras: $a^2 + a^2 = c^2$

$c^2 = 2a^2 \Rightarrow c = \sqrt{2}\,a$

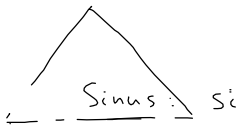
$\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$

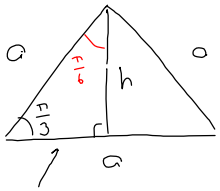
$\cos\frac{\pi}{4} = \frac{a}{\sqrt{2}\,a} = \frac{\sqrt{2}}{2}$

$\tan\frac{\pi}{4} = \frac{a}{a} = 1$

$\cot\frac{\pi}{4} = \frac{a}{a} = 1$

$V = 60°$, dvs $V = \frac{\pi}{3}$ 1ike likesidet trekant.





Pytagoras:

$\left(\frac{a}{2}\right)^2 + h^2 = a^2$

$h^2 = a^2 - \frac{a^2}{4} =$

$= \frac{3}{4}a^2$

$\Rightarrow h = \sqrt{\frac{3}{4}}\cdot a = \frac{\sqrt{3}}{2}a$

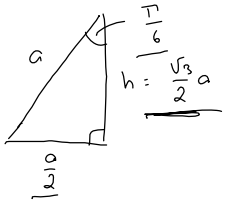
$\sin\frac{\pi}{3} = \frac{h}{a} = \frac{\frac{\sqrt{3}}{2}\cdot a}{a} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{\frac{a}{2}}{a} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \frac{h}{\frac{a}{2}} = \frac{\frac{\sqrt{3}}{2}a}{\frac{a}{2}} = \sqrt{3}$

$\cot\frac{\pi}{3} = \frac{\frac{a}{2}}{h} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

$V = 30°$, dvs $\frac{\pi}{6}$:



$\sin\frac{\pi}{6} = \frac{\text{motstående}}{\text{hypotenus}} = \frac{\frac{a}{2}}{a} = \frac{1}{2}$

$\cos\frac{\pi}{6} = \frac{\text{hosliggende}}{\text{hypotenus}} = \frac{\frac{\sqrt{3}}{2}a}{a} = \frac{\sqrt{3}}{2}$

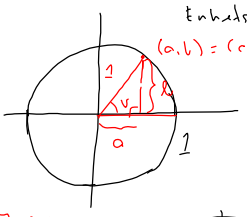
$\tan\frac{\pi}{6} = \frac{\text{motstående}}{\text{hosliggende}} = \frac{\frac{a}{2}}{\frac{\sqrt{3}}{2}a} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot\frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}a}{\frac{a}{2}} = \sqrt{3}$

Oppsummering:

$V°$	V	$\sin$	$\cos$	$\tan$	$\cot$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$

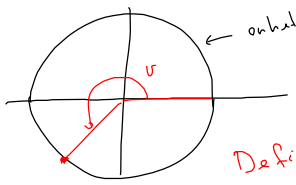
Hvað með punkta sem ekki liggja milli 0 og $\frac{\pi}{2}$?



$\sin v = \frac{b}{1} = b$

$\cos v = \frac{a}{1} = a$

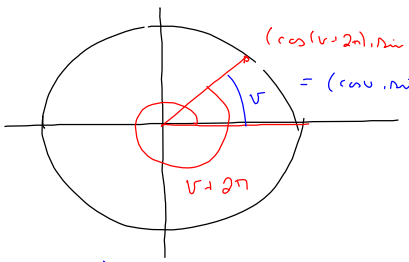
Já:



$(\cos v, \sin v)$

Skilgreining á cosinus og sinus!

Hringlaga:



$(\cos(v+2\pi), \sin(v+2\pi)) = (\cos v, \sin v)$

Reglubundið:

$\cos(v + 2\pi) = \cos v$

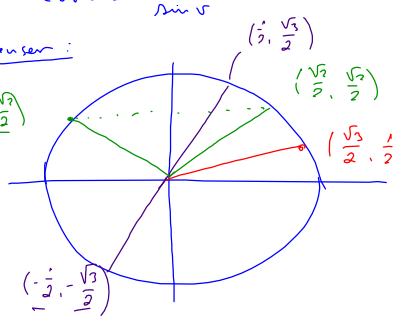
$\sin(v + 2\pi) = \sin v$

Skilgreining:

$\tan v = \frac{\sin v}{\cos v}$

$\cot v = \frac{\cos v}{\sin v}$

Fleiri hringlaga:



$v = \frac{\pi}{3}$

$v = \frac{\pi}{4}$

$v = \frac{\pi}{6}$

$v = \frac{3\pi}{4}$

$v = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$

$v^\circ = 60^\circ + 180^\circ = 240^\circ$