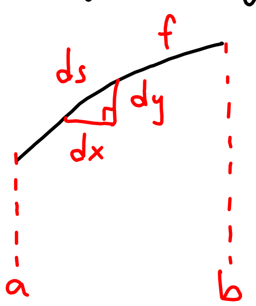
Huskeregel for graflengde



Pytagoras: $ds^2 = dx^2 + dy^2$

$$= \left[1 + \left(\frac{dy}{dx}\right)^2\right] \cdot dx^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$$

Ergo: $s = \int_a^b ds = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$

Delvis integrasjon (9.1)

Hvis $F(x)$ og $G(x)$ er funksjoner med kontinuerlige deriverte, så er

$$\int F(x)G'(x)\,dx = F(x)G(x) - \int F'(x)G(x)\,dx$$

Bevis Deriverer på begge sider:

$$F(x)G'(x) = [F(x)G(x)]' - F'(x)G(x)$$

Dette er produktregelen for derivasjon. $\square$

eks.

$$\int x e^{-x}\,dx = x(-e^{-x}) - \int 1\cdot(-e^{-x})\,dx$$

Delvis $F(x) = x$ $G'(x) = e^{-x}$
$F'(x) = 1$ $G(x) = -e^{-x}$

$$= -xe^{-x} + \int e^{-x}\,dx$$

$$= \underline{\underline{-xe^{-x} - e^{-x} + C}}$$

I - metoden (variant av delvis integrasjon)

eks.
$$\int e^x \sin x \, dx = -e^x \cos x - \int e^x(-\cos x)\, dx$$

$$\boxed{\begin{array}{ll} F(x) = e^x & G'(x) = \sin x \\ F'(x) = e^x & G(x) = -\cos x \end{array}}$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

$$= -e^x \cos x + \left[e^x \sin x - \int e^x \sin x \, dx \right]$$

$$\boxed{\begin{array}{ll} F(x) = e^x & G'(x) = \cos x \\ F'(x) = e^x & G(x) = \sin x \end{array}}$$

Kaller vi det opprinnelige integralet I, har vi nå

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x$$

$$I = -\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x.$$

Altså: $\int e^x \sin x \, dx = \underline{\underline{\frac{1}{2} e^x (\sin x - \cos x) + C}}$

"Baklengs" substitusjon (setning 9.2.3)

begrunner at vi også kan regne slik ved substitusjon:

$$\boxed{\begin{array}{l} u = u(x) \quad \text{gir} \quad x = h(u) \\ \dfrac{dx}{du} = h'(u), \quad dx = h'(u)\,du \end{array}}$$

eks.

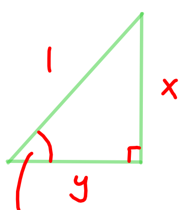
$$\int \arcsin x \, dx = \int u \cdot \cos u \, du$$

$$\boxed{\begin{array}{l} u = \arcsin x \quad \text{gir} \quad x = \sin u \\ \dfrac{dx}{du} = \cos u, \quad dx = \cos u \, du \end{array}}$$

$$= u \sin u - \int 1 \cdot \sin u \, du = u \sin u + \cos u + C$$

$$\boxed{\begin{array}{ll} F(u) = u & G'(u) = \cos u \\ F'(u) = 1 & G(u) = \sin u \end{array}}$$

$$= (\arcsin x) \cdot x + \cos(\arcsin x) + C$$



$v = \arcsin x$ fordi $x = \sin v$

$\cos v = \dfrac{y}{1} = \sqrt{1 - x^2}$

Så $\cos(\arcsin x) = \sqrt{1 - x^2}$

$$= x \arcsin x + \sqrt{1 - x^2} + C$$

Sjekk: $\dfrac{d}{dx}\left(x \arcsin x + \sqrt{1-x^2}\right) = 1 \cdot \arcsin x + \dfrac{x}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$$= \underline{\underline{\arcsin x}}$$

Ok!

eks. $\int \sqrt[3]{x+1}\,dx = \int u \cdot 3u^2\,du = 3\int u^3\,du$

$$\boxed{\begin{array}{l} u = \sqrt[3]{x+1} \quad \text{gir} \quad u^3 = x+1 \\ \qquad\qquad\qquad\qquad\quad x = u^3 - 1 \\ \frac{dx}{du} = 3u^2,\quad dx = 3u^2\,du \end{array}}$$

$= \frac{3}{4}u^4 + C$

$= \frac{3}{4}\left(\sqrt[3]{x+1}\right)^4 + C$

$= \underline{\underline{\frac{3}{4}(x+1)^{4/3} + C}}$

eks. $\int \sqrt{\frac{\arccos^3 x}{1-x^2}}\,dx = \int \frac{\sqrt{(\arccos x)^3}}{\sqrt{1-x^2}}\,dx$

$$\boxed{\begin{array}{l} u = \arccos x,\quad \frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}} \\ du = -\frac{1}{\sqrt{1-x^2}}\,dx,\quad dx = -\sqrt{1-x^2}\,du \end{array}}$$

$= -\int \sqrt{u^3}\,du$

$= -\int (u^3)^{\frac{1}{2}}\,du$

$= -\int u^{3/2}\,du$

$= -\frac{1}{\frac{5}{2}}u^{5/2} + C$

$= -\frac{1 \cdot \frac{2}{5}}{\cancel{\frac{5}{2} \cdot \frac{2}{5}}}u^{5/2} + C = -\frac{2}{5}u^{5/2} + C$

$= \underline{\underline{-\frac{2}{5}(\arccos x)^{5/2} + C}}$

$\boxed{\text{Kvadrat - teknikken}}$ (seksjon 9.3)

La $n \geq 1$ være et helt tall. Integralet

$$\int \frac{Ax + B}{(ax^2 + bx + c)^n} \, dx$$

der $ax^2 + bx + c = 0$ ikke har reelle løsninger

kan løses ved en teknikk som vi kaller "kvadrat-teknikken":

1. Se mandag...